

Polygons

An ADE Mathematics Lesson

Days 11-15

Author	ADE Content Specialists
Grade Level	10 th grade
Duration	Five days

Aligns To

Mathematics HS:

Strand 3: Patterns, Algebra, and Functions

Concept 1: Patterns

PO 1. Recognize, describe, and analyze sequences using tables, graphs, words, or symbols; use sequences in modeling.

Concept 3: Algebraic Representations

PO 1. Create and explain the need for equivalent forms of an equation or expression.

Strand 4: Geometry and Measurement

Concept 1: Geometric Shapes

PO 2. Visualize solids and surfaces in 3-dimensional space when given 2-dimensional representations and create 2-dimensional representations for the surfaces of 3-dimensional objects.

PO 3. Create and analyze inductive and deductive arguments concerning geometric ideas and relationships.

PO 4. Apply properties, theorems, and constructions about parallel lines, perpendicular lines, and angles to prove theorems.

PO 6. Solve problems using angle and side length relationships and attributes of polygons.

PO 7. Use the hierarchy of quadrilaterals in deductive reasoning.

Connects To

Mathematics HS:

Strand 1: Number and Operations

Concept 3: Estimation

PO 2. Use estimation to determine the reasonableness of a solution.

Strand 3: Patterns, Algebra, and Functions

Concept 3: Algebraic Representations

PO 2. Solve formulas for specified variables.

Strand 4: Geometry and Measurement

Concept 3: Coordinate Geometry

PO 4. Verify characteristics of a given geometric figure using coordinate formulas for distance, midpoint, and slope to confirm parallelism, perpendicularity, and congruence.

Strand 5: Structure and Logic

Concept 2: Logic, Reasoning, Problem Solving, and Proof

PO 3. Evaluate a solution for reasonableness and interpret the meaning of the solution in the context of the original problem.

Aligns To**Strand 5: Structure and Logic
Concept 2: Logic, Reasoning, Problem Solving, and Proof**

PO 1. Analyze a problem situation, determine the question(s) to be answered, organize given information, determine how to represent the problem, and identify implicit and explicit assumptions that have been made.

PO 2. Solve problems by formulating one or more strategies, applying the strategies, verifying the solution(s), and communicating the reasoning used to obtain the solution(s).

PO 4. Generalize a solution strategy for a single problem to a class of related problems; explain the role of generalizations in inductive and deductive reasoning.

PO 5. Summarize and communicate mathematical ideas using formal and informal reasoning.

PO 7. Find structural similarities within different algebraic expressions and geometric figures.

Connects To**Overview**

In this lesson you will recognize various forms of polygons and polyhedra, specifically working with comparing and contrasting the properties of regular polygons. You will also compare the properties of various types of quadrilaterals.

Purpose

You will learn the similarities and differences between regular polygons as well as between different forms of quadrilaterals. This will help you to work with irregular polygons and different forms of polygons and polyhedra.

Materials

- Regular polygon worksheets
- Quadrilateral worksheets
- Lined paper such as graph paper
- Ruler
- Compass and protractor

Objectives**Students will:**

- Create and analyze inductive and deductive arguments concerning geometric ideas and relationships.
- Solve problems using angle and side length relationships and attributes of polygons.
- Use the hierarchy of quadrilaterals in deductive reasoning.

Lesson Components

Prerequisite Skills: In earlier grades, you learned to recognize various geometric figures and to assign attributes to these simple geometric figures.

Vocabulary: *polygon, polyhedra, polyhedron, regular polygon, quadrilateral, equiangular polygon, interior angle of a polygon, exterior angle of a polygon, diagonal of a polygon, quadrilateral, trapezoid, rhombus, isosceles trapezoid, rectangle, square, parallelogram*

Session 1 (2 days)

1. Compare and contrast polygons with polyhedra. Students compare and contrast various forms of regular polygons.

Session 2 (3 days)

1. Compare and contrast various forms of quadrilaterals.

Assessment

There are two assessments that will help you pinpoint misconceptions before moving on to more complex comparisons. The first assessment follows Session 1 and assesses knowledge of polygons. The second assessment follows Session 2 and assesses knowledge of quadrilaterals.

Polygons

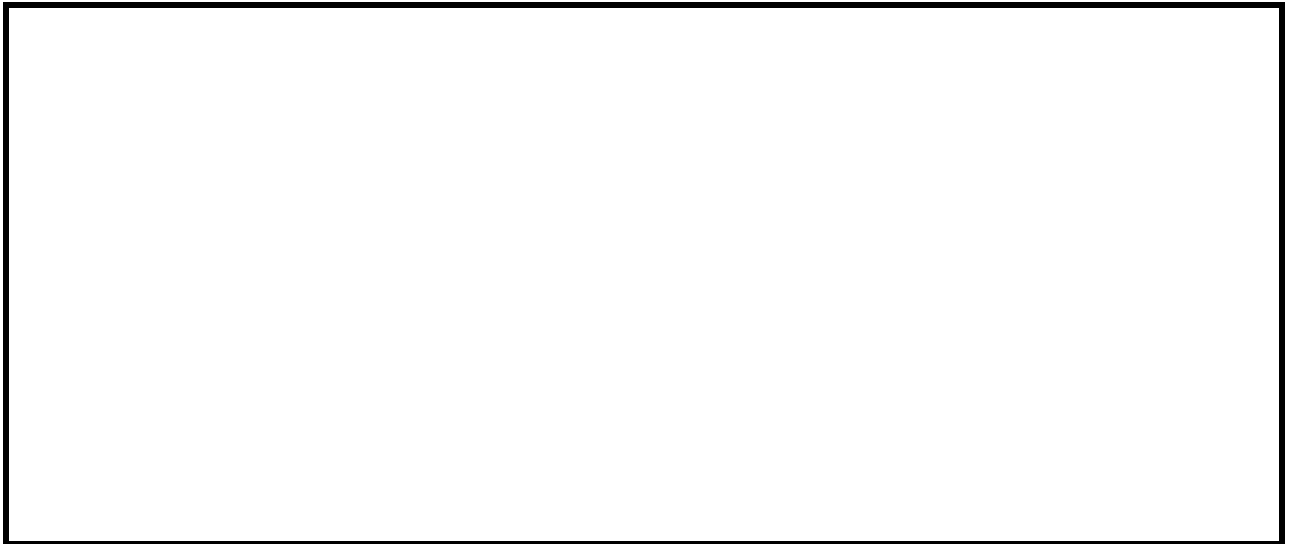
Session 1 – Regular Polygons

Discover some fascinating characteristics of polygons.

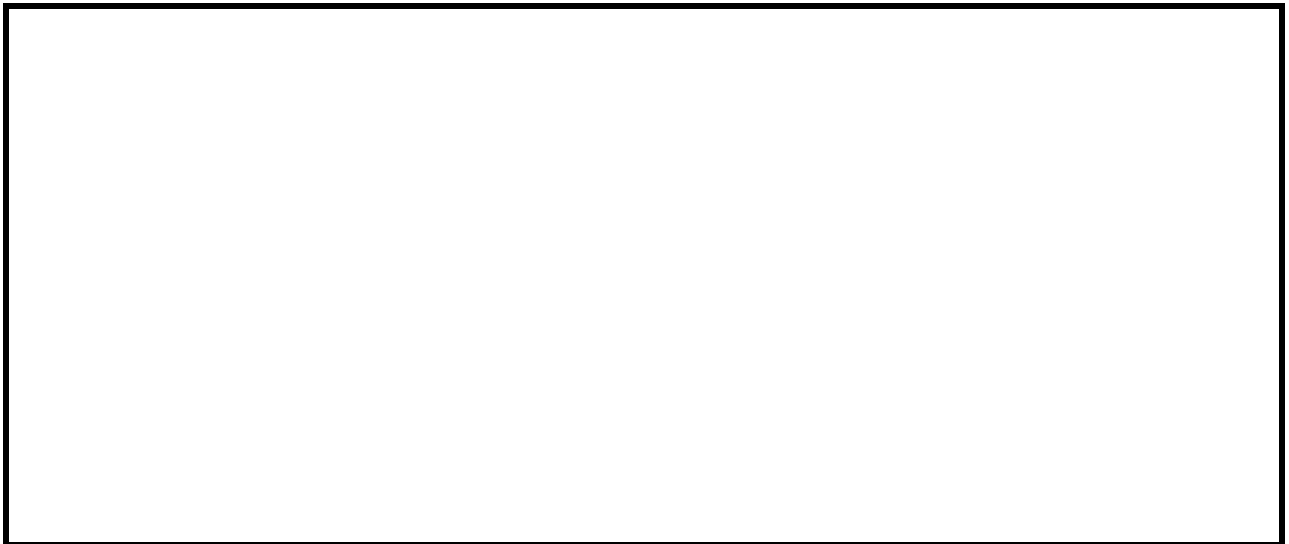
1. Polygons have special names given in relationship to the number of sides the polygon has. List all the polygons you can and include how many sides each has.

2. What is the difference between a **polygon** and a **polyhedron**? Can you name some polyhedra and describe them? Draw one example of a polygon and one example of a polyhedron?

3. What is a **regular polygon**? Have you learned to construct any of them? If so, which ones?



4. Do you think that you can construct any of the other regular polygons? Try it and see what happens when you vary the number of sides of the polygon. Use your compass and ruler and just try to use some of the steps you already know.



5. What characteristics would an **equiangular polygon** exhibit? Is an equiangular polygon the same thing as a regular polygon? Support your answer.



6. Draw three different regular polygons. Either construct the polygon to ensure uniformity or use a protractor and ruler to ensure uniformity. You will record the information in chart form in question 7.
- Measure each interior angle. Label each angle using a lowercase letter.
 - Extend the sides to form the exterior angles. Measure each exterior angle. Label each angle using a number.
 - Draw all diagonals, label and count them.

Polygon 1



Polygon 2

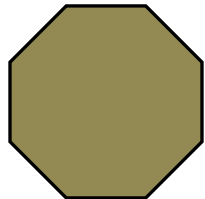
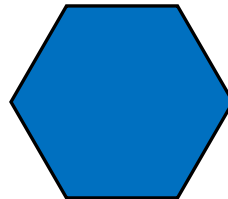
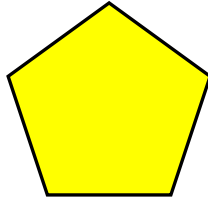
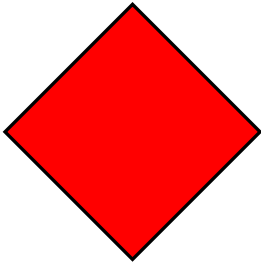
Polygon 3

7. Now consolidate all the information you obtained in question 6 in the chart provided.

Name of Polygon	Number of sides	Total Interior angle measure	Total Exterior angle measure	Number of Diagonals

8. Draw two additional regular polygons and repeat all steps in number 6 above. Add this information to your chart.

9. Experiment with all the information you obtained to see what new theorems or relationships emerge. Can you recognize three new concepts from the chart previously unknown to you? You must support your claims with visible evidence that you obtained while working on the regular polygons.



Hints

Consider relationships, for example the number of sides to number of diagonals.

Look for patterns.

Look only at the interior angles, then the exterior angles, then the diagonals. Then, look at combinations of these things.

Be creative. Allow flexibility. Be as accurate as possible with the measurements.

Arrange the entries in your chart in some type of order.

Use colored pencils and other ways of distinguishing angles, lines, diagonals.

Make sure your work is neat and that you understand what your chart is telling you.

Polygons

Assessment 1 – Regular Polygons

1. Complete the following questions. Assume all polygons are regular polygons.

The total measure of all the interior angles of a triangle is _____ degrees.

A parallelogram has _____ sides.

A rhombus has _____ diagonals.

The sum of the measure of the exterior angle of a trapezoid is _____ degrees.

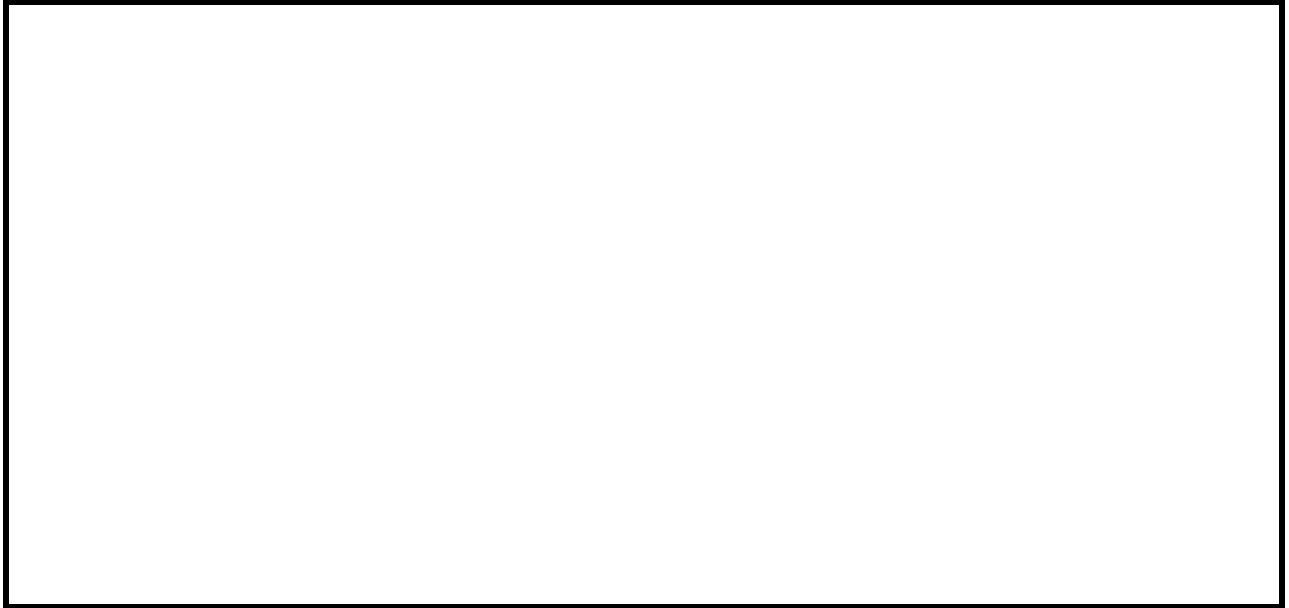
The total measure of all the interior angles of a pentagon is _____ degrees.

A hexagon has _____ diagonals.

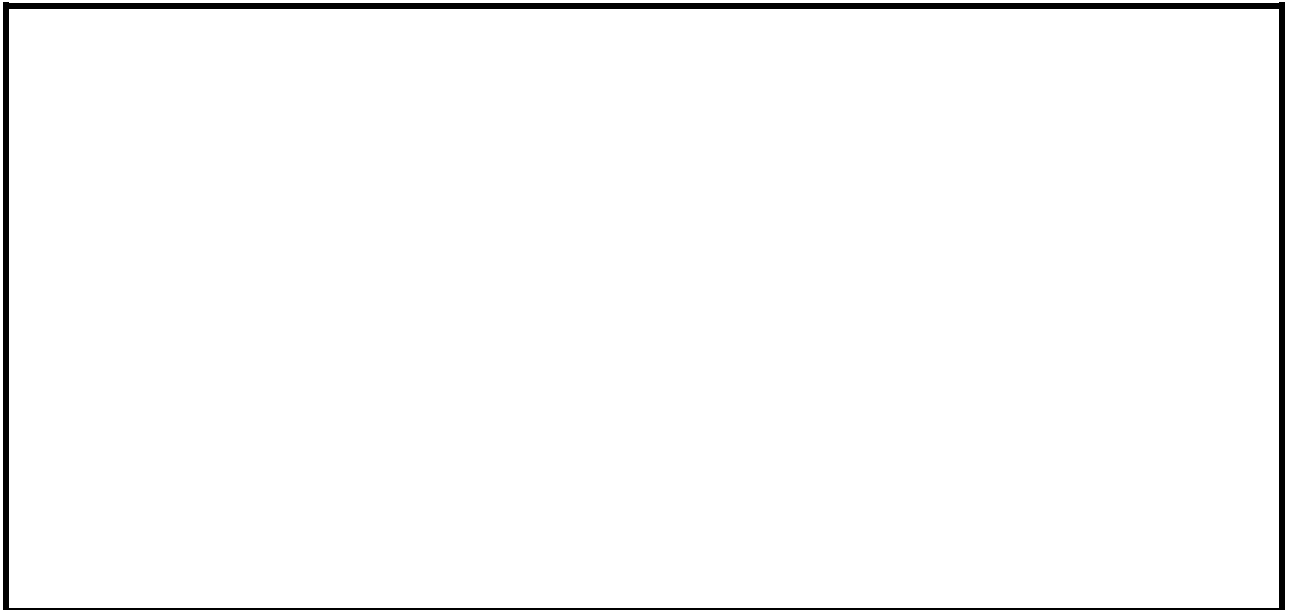
The total measure of the exterior angles of an octagon is _____ degrees.

2. State the relationship between the number of sides and number of diagonals for regular polygons.

3. Is there a relationship between the total measure of the interior angles and exterior angles of a regular polygon? Is this supported by the patterns that can be seen in examining quadrilaterals (parallelogram, rhombus, and trapezoid)? Why or why not?



4. Define the difference between a polygon and a polyhedron. Draw an example of 2 different polyhedra and label each of them with their name.



Polygons

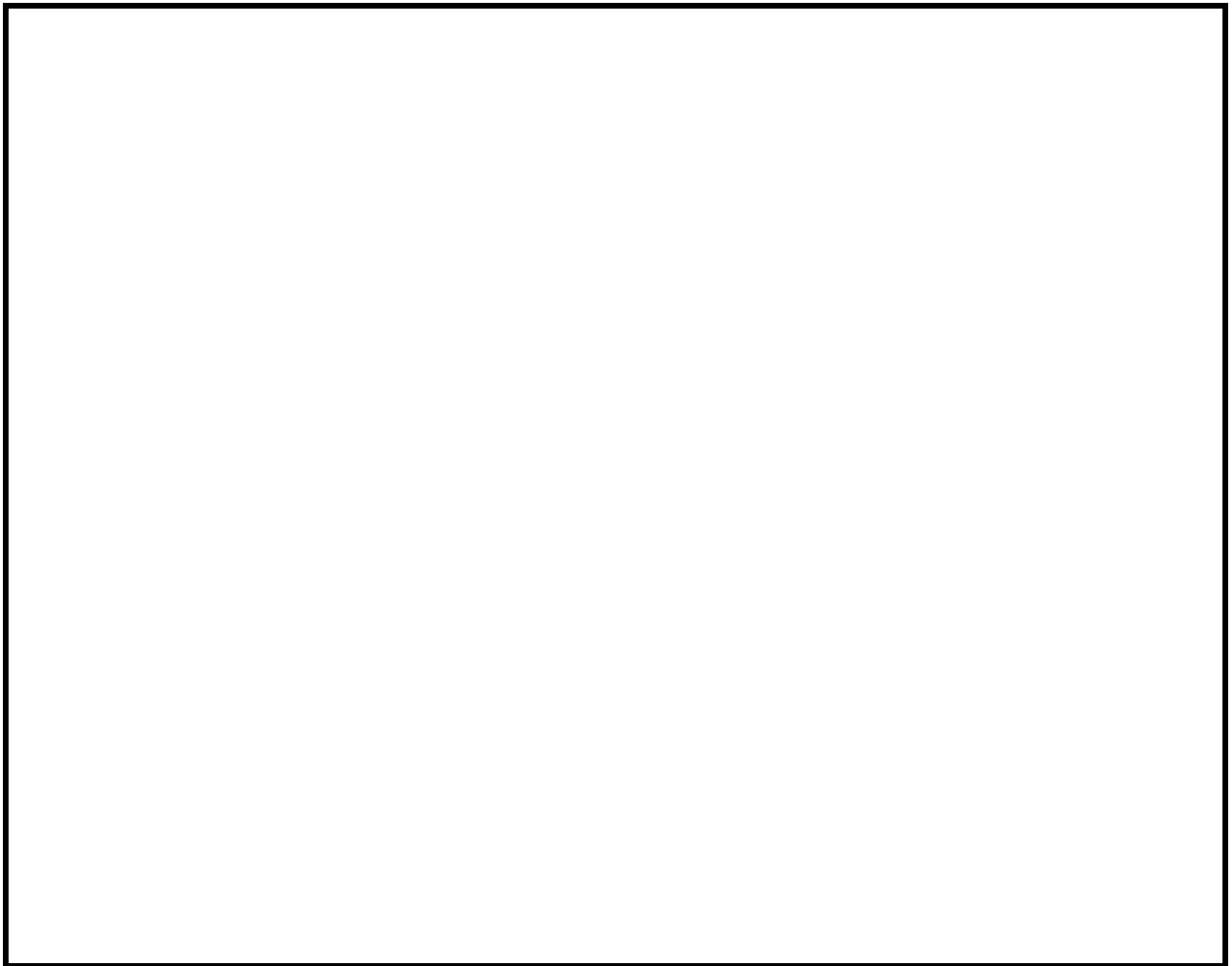
Session 2 - Quadrilaterals

1. Define each of the following terms in the space provided:

Term	Definition
Quadrilateral	
Polygon	
Vertices of a polygon	
Sides of a polygon	
Diagonals of a polygon	
Isosceles trapezoid	
Leg of a trapezoid	
Base of a trapezoid	
Base angle	

2. Define and draw examples of each of the following using a protractor and ruler or by using a construction in the space provided.

- a. rectangle
- b. square
- c. trapezoid
- d. rhombus
- e. parallelogram



3. Make a chart in which you compare the properties of all of the figures in 2a-2e. Include the number and types of angles and sides. What is the relationship of various sides of one quadrilateral? What do the figures have in common? What are the differences between the figures?

Quadrilateral	Types of Sides	Types of Angles	Similarities	Differences
Rectangle				
Square				
Trapezoid				
Rhombus				
Parallelogram				

4. Consider the following statements and determine if they are true. Justify your answer. If the answer is false, modify it to make a true statement.

a. All rhombuses are squares.

b. All trapezoids have one set of parallel sides.

c. All square are rectangles.

d. All rectangles are parallelograms.

e. The diagonals of a parallelogram are always perpendicular to each other.

f. The sides of an isosceles trapezoid are always congruent.

A large, empty rectangular box with a black border, intended for the student's answer to question f.

g. The only type of quadrilateral that does not have opposite sides parallel is a trapezoid.

A large, empty rectangular box with a black border, intended for the student's answer to question g.

Polygons

Assessment 2 – Quadrilaterals

1. Match each term to its definition.

_____ Quadrilateral

_____ Polygon

_____ Vertices of a polygon

_____ Sides of a polygon

_____ Diagonals of a polygon

_____ Isosceles trapezoid

_____ Leg of a trapezoid

_____ Base of a trapezoid

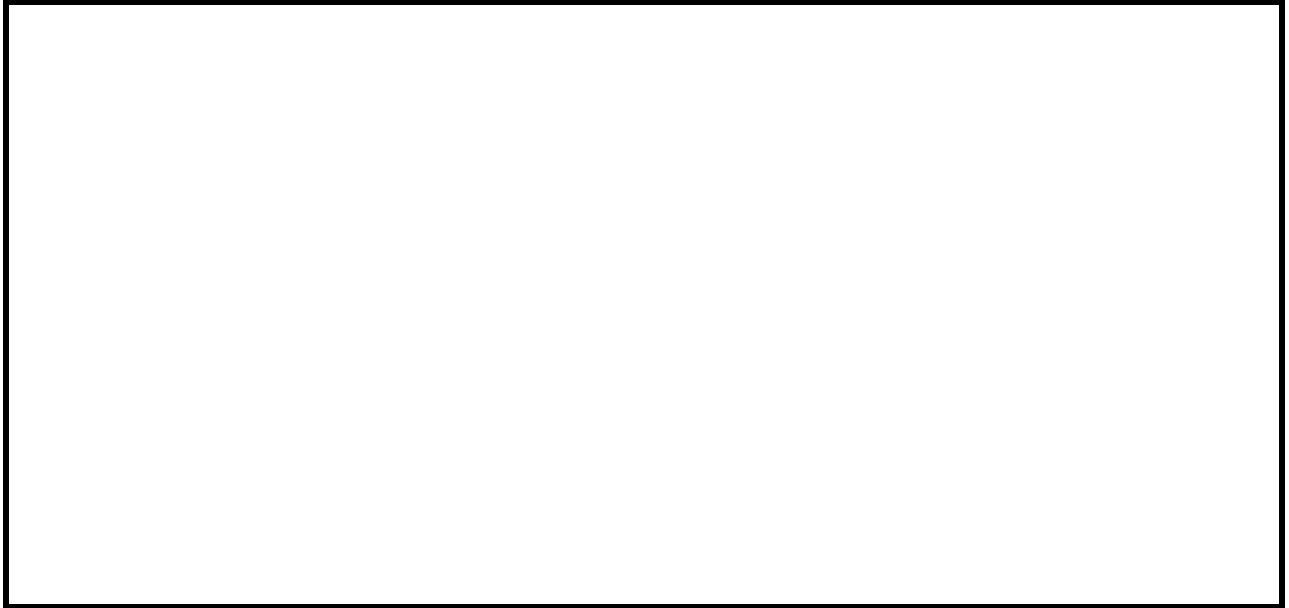
_____ Base angle

- A. the points at which two sides of a polygon meet
- B. either of the two sides which connect the parallel bases of a trapezoid
- C. the angle that is formed by the base and one leg in a polygon
- D. a polygon with four sides
- E. the segment joining two adjacent vertices in a figure
- F. a closed two-dimensional figure made up of segments which intersect only at the segment endpoints
- G. a side of a trapezoid that is perpendicular to its height
- H. a trapezoid in which the non-parallel sides are congruent
- I. a line segment joining two non-adjacent vertices of a polygon

2. Complete the following chart comparing the similarities and differences between the two types of quadrilaterals named. Justify your answers.

QUADRILATERALS	SIMILARITIES	DIFFERENCES
Square and Parallelogram		
Rhombus and Rectangle		
Trapezoid and Parallelogram		

3. Draw a graphic organizer or some other pictorial representation that shows the relationship between the various types of quadrilaterals. Make certain to include a parallelogram, trapezoid, rhombus, square, and rectangle.



4. Consider the following statements and determine if they are true or false. If the answer is false, modify it to make a true statement.
- a. All squares are rhombuses.
 - b. All trapezoids have one set of parallel sides.
 - c. All rectangles are squares.
 - d. All rectangles are parallelograms.
 - e. The legs of an isosceles trapezoid are not congruent.

Extensions

1. Use applets to further explore the relationships between interior and exterior angles of polygons.

Triangles

http://www.geogebra.org/en/upload/files/english/Barbara_Perez/Triangle_Exterior.html

Quadrilaterals

http://www.geogebra.org/en/upload/files/english/Barbara_Perez/Quadrilateral_Exterior.html

Pentagons

http://www.geogebra.org/en/upload/files/english/Barbara_Perez/Pentagon_Exterior.html

Hexagons

http://www.geogebra.org/en/upload/files/english/Barbara_Perez/Hexagon_Exterior.html

2. Work with irregular polygons to determine similarities and differences between different forms of irregular polygons.

Sources

1999 Bringing the NCTM Standards to Life, Exemplary Practices from High Schools, p. 10-14
2000 NCTM Principles and Standards, p. 308-323
2008 AZ Department of Mathematics Standards
2008 The Final Report of the National Mathematics Advisory Panel, p. 20, 29-30