AzMERIT | Arizona's Statewide Achievement Assessment for English Language Arts and Mathematics

Mathematics Item Specifications

ALGEBRA II

Arizona Department of Education with American Institutes for Research - 2018

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Introduction

The Arizona Statewide Achievement Assessment for English Language Arts and Mathematics (AzMERIT) is Arizona's statewide achievement test. AzMERIT assesses the Arizona English Language Arts Standards and Arizona Mathematics Standards adopted by the Arizona State Board of Education in December 2016. AzMERIT will inform students, teachers, and parents about preparedness for college and careers upon graduating from high school. AzMERIT tests are computer-based, meaning that they can better assess students' critical thinking skills and provide them with opportunities to demonstrate a deeper understanding of the materials. Computer-based testing also allows for the use of a variety of innovative items types.

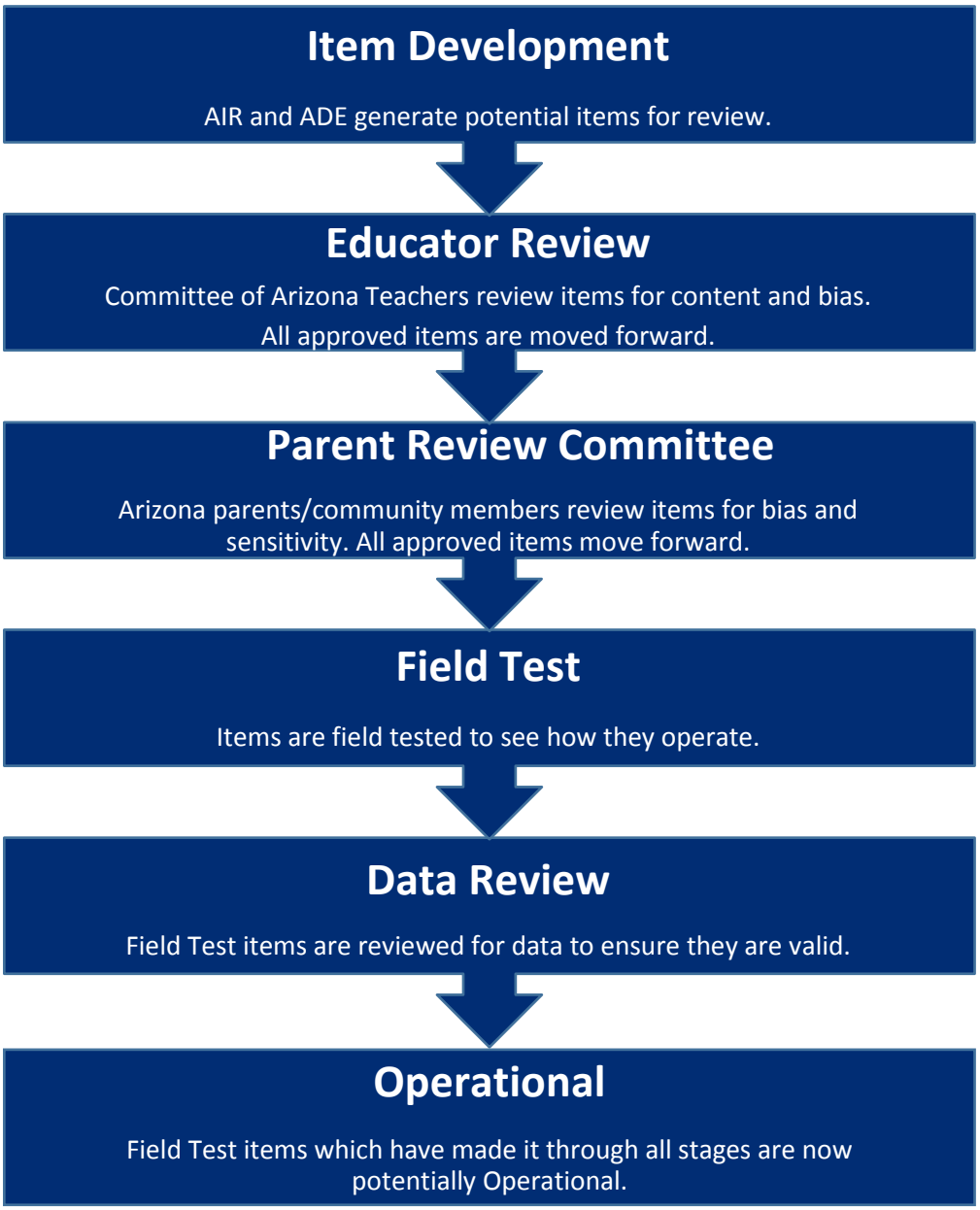
During the item-development process, all AzMERIT items are written in accordance with the Item Specifications and are reviewed and approved by a committee of Arizona educators to confirm alignment and appropriateness for inclusion in the test. AzMERIT items are generally representative of Arizona's geographic regions and culturally diverse population. Items are reviewed for the following kinds of bias: gender, racial, ethnic, linguistic, religious, geographic, and socioeconomic. Item reviews also include consideration of issues related to individuals with disabilities. Arizona community members also have an opportunity to review items for issues of potential concern to members of the community at large. Reviewers are asked to consider the variety of cultural, regional, philosophical, political, and religious backgrounds throughout Arizona, and then to determine whether the subject matter will be acceptable to Arizona students, families, and other members of Arizona communities.

This *AzMERIT Item Specifications* is a resource document that defines the content and format of the test and test items for item writers and reviewers. Each *Item Specifications* document indicates the alignment of items with the Arizona Mathematics Standards. It also serves to provide all stakeholders with information about the scope and function of assessment items. This document can also serve to assist educators to understand how assessment items are developed in alignment with the standards for English language arts and math. These item specifications for AzMERIT are intended to provide information regarding standards, item formats and response types. The descriptions of blueprints, and depth of knowledge in this document are meant to provide an overview of the test. Item specifications are meant for the purposes of assessment, not instruction. They are not intended to be tools for instruction or the basis for curricula. AzMERIT has a test blueprint that was developed by Arizona and is different from any other state or consortium test blueprint.

For the math portion of AzMERIT, all of the test questions are aligned to the mathematic content standards for these subject areas. Any item specifications that are absent for standards listed in this document may be under development. This document does not endorse the exclusion of the instruction of any grade-level content standards. The test will ask questions that check a student's conceptual understanding of math as well as their procedural skills. These items have been written to be free from bias and sensitivity, and widely vary in their degree of difficulty.

Item Development Process

AzMERIT items go through a rigorous review before they are operational. When an item is "operational" it means it is used to determine a student's score on the assessment. This is a description of the process every item must go through before it is operational on AzMERIT.



Sample tests are available online for the math portion of AzMERIT. For more information view the Guide to the Sample Tests at www.azmeritportal.org.

Test Construction Guidelines

The construction of the AzMERIT assessment is guided by the depth and rigor of the Arizona College and Career Ready Standards. Items are created to address key components of the standards and assess a range of important skills. The AzMERIT Blueprint provides an overview of the distribution of items on the AzMERIT according to the standards. The standards for Math Practices are embedded within all AzMERIT items. Further, the AzMERIT blueprint outlines the Depth of Knowledge distribution of items.

Blueprint

Algebra II AzMERIT Blueprint 2016 Standards		
Reporting Category	**Min.**	**Max.**
Algebra	**34%**	**38%**
Functions	**30%**	**34%**
Statistics and Probability and Number and Quantity	**30%**	**34%**
Statistics and Probability	17%	28%
Number and Quantity	11%	19%

Depth of Knowledge (DOK)

DOK refers to the level of rigor or sophistication of the task in a given item, designed to reflect the complexity of the Arizona Mathematics Standards. Items at DOK level 1 focus on the recall of information, such as definitions, terms, and simple procedures. Items at DOK 2 require students to make decisions, solve problems, or recognize patterns; in general, they require a greater degree of engagement and cognitive processing than items at DOK 1. Items at DOK 3 feature higher-order cognitive tasks that assess students' capacities to approach abstract or complex problems.

Percentage of Points by Depth of Knowledge (DOK) Level			
Algebra II	DOK Level 1	DOK Level 2	DOK Level 3
	10% - 20%	60% - 70%	12% - 30%

For more information on DOK go to www.azed.gov/AzMERIT.

Calculators

Arizona Desmos Graphing Calculator is permitted for both the paper-based and computer-based assessment for High School Math.

Item Formats

The AzMERIT Assessments are composed of item formats that include traditional multiple-choice response items and technology-enhanced response items (TEI). TEIs are computer-delivered response items that require students to interact with test content to select, construct, and/or support their responses. TEIs are better able to assess a deeper level of understanding.

Currently, there are nine types of TEIs that may appear on the Math computer based assessment for AzMERIT:

- Editing Tasks (ET)
- Editing Task Choice (ETC)
- Equation Editor (EQ)
- Graphic Response Item Display (GRID)
- Hot Text (HT)
    - Selectable Hot Text
    - Drag-and-Drop Hot Text
- Matching Item (MI)
- Multi-Select (MS)
- Open Response
- Table Item (TI)

For paper-based assessments (including those for students with an IEP or 504 plan that specifies a paper based accommodation), TEIs will be modified so that they can be scanned and scored electronically or hand-scored.

See the table below for a description of each TEI. In addition, for examples of each response item format described, see the AzMERIT Training Tests at www.azmeritportal.org.

Item Format	Description
Editing Task (ET)	The student clicks on a highlighted word or phrase that may be incorrect, which reveals a text box. The directions in the text box direct the student to replace the highlighted word or phrase with the correct word or phrase. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Editing Task Choice (ETC)	The student clicks a highlighted word or phrase, which reveals a drop-down menu containing options for correcting an error as well as the highlighted word or phrase as it is shown in the sentence to indicate that no correction is needed. The student then selects the correct word or phrase from the drop-down menu. For paper-based assessments, the item is modified so that it can be scanned and scored electronically. The student fills in a circle to indicate the correct word or phrase.

Item Format	Description
Equation Editor (EQ)	The student is presented with a toolbar that includes a variety of mathematical symbols that can be used to create a response. Responses may be in the form of a number, variable, expression, or equation, as appropriate to the test item. For paper-based assessments, this item type may be replaced with a modified version of the item that can be scanned and scored electronically or replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Graphic Response Item Display (GRID)	The student selects numbers, words, phrases, or images and uses the drag-and-drop feature to place them into a graphic. This item type may also require the student to use the point, line, or arrow tools to create a response on a graph. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Hot Text (HT)	**Selectable Hot Text -** Excerpted sentences from the text are presented in this item type. When the student hovers over certain words, phrases, or sentences, the options highlight. This indicates that the text is selectable ("hot"). The student can then click on an option to select it. For paper- based assessments, a "selectable" hot text item is modified so that it can be scanned and scored electronically. In this version, the student fills in a circle to indicate a selection.
	Drag-and-Drop Hot Text - Certain numbers, words, phrases, or sentences may be designated "draggable" in this item type. When the student hovers over these areas, the text highlights. The student can then click on the option, hold down the mouse button, and drag it to a graphic or other format. For paper-based assessments, drag-and-drop hot text items will be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Matching Item (MI)	The student checks a box to indicate if information from a column header matches information from a row. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Multi-Select (MS)	The student is directed to select all of the correct answers from among a number of options. These items are different from multiple-choice items, which allow the student to select only one correct answer. These items appear in the online and paper-based assessments.
Open Response	The student uses the keyboard to enter a response into a text field. These items can usually be answered in a sentence or two. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.

Item Format	Description
Table Item (TI)	The student types numeric values into a given table. The student may complete the entire table or portions of the table depending on what is being asked. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.

A2.N-RN.A **Extend the properties of exponents to rational exponents.**	**A2.N-RN.A.1**	Explain how the definition of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
	A2.N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
Quantities (N-Q)		
A2.N-Q.A **Reason quantitatively and use units to solve problems.**	**A2-N-Q.A.1**	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays, include utilizing real-world context.
	A2.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling. Include problem-solving opportunities utilizing real-world context.
	A2.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities utilizing real-world context.
The Complex Number System (N –CN)		
A2.N-CN.A **Perform arithmetic operations with complex numbers.**	**A2.N-CN.A.1**	Apply the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. Write complex numbers in the form (a+bi) with a and b real.
A2.N-CN.C **Use complex numbers in polynomial identities and equations.**	**A2.N-CN.C.7**	Solve quadratic equations with real coefficients that have complex solutions.
Algebra - A		
Seeing Structure in Expressions (A-SSE)		
A2.A-SSE.A **Interpret the structure of expressions.**	**A2.A-SSE.A.2**	Use structure to identify ways to rewrite polynomial and rational expressions. Focus on polynomial operations and factoring patterns.

A2.A-SSE.B **Write expressions in equivalent forms to solve problems.**	**A2.A-SSE.B.3**	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Include problem-solving opportunities utilizing real-world context and focus on expressions with rational exponents. c. Use the properties of exponents to transform expressions for exponential functions.
	A2.A-SSE.B.4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*
Arithmetic with Polynomials and Rational Expressions (A-APR)		
A2.A-APR.B **Understand the relationship between zeros and factors of polynomials.**	**A2.A-APR.B.2**	Know and apply the Remainder and Factor Theorem: For a polynomial *p(x)* and a number *a*, the remainder on division by (*x – a)* is *p(a)*, so *p(a)* = 0 if and only if (*x – a*) is a factor of *p(x)*.
	A2.A-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. Focus on quadratic, cubic, and quartic polynomials including polynomials for which factors are not provided
A2.A-APR.C **Use polynomial identities to solve problems.**	**A2.A-APR.C.4**	Prove polynomial identities and use them to describe numerical relationships.
A2.A-APR.D **Rewrite rational expressions.**	**A2.A-APR.D.6**	Rewrite rational expressions in different forms; write *a(x)/b(x)* in the form *q(x) + r(x)/b(x)*, where *a(x), b(x), q(x),* and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x),* using inspection, long division, or for the more complicated examples, a computer algebra system.
Creating Equations (A-CED)		
A2.A-CED.A **Create equations that describe numbers or relationships.**	**A2.A-CED.A.1**	Create equations and inequalities in one variable and use them to solve problems. Include problem-solving opportunities utilizing real-world context. Focus on equations and inequalities arising from linear, quadratic, rational, and exponential functions.
Reasoning with Equations and Inequalities (A-REI)		

A2.A-REI.A **Understand solving equations as a process of reasoning and explain the reasoning.**	**A2.A-REI.A.1**	Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Extend from quadratic equations to rational and radical equations.
	A2.A-REI.A.2	Solve rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
A2.A-REI.B **Solve equations and inequalities in one variable.**	**A2.A-REI.B.4**	Fluently solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
A2.A-REI.C **Solve systems of equations.**	**A2.A-REI.C.7**	Solve a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*
A2.A-REI.D **Represent and solve equations and inequalities graphically.**	**A2.A-REI.D.11**	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately (e.g., using technology to graph the functions, make tables of values, or find successive approximations). Include problems in real-world context. Extend from linear, quadratic, and exponential functions to cases where $f(x)$ and/or $g(x)$ are polynomial, rational, exponential, and logarithmic functions.
Functions - F		
Interpreting Functions (F-IF)		
A2.F-IF.B **Interpret functions that arise in applications in terms of the context.**	**A2.F-IF.B.4**	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Include problem-solving opportunities utilizing a real-world context. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

		Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.
	A2.F-IF.B.6	Calculate and interpret the average rate of change of a continuous function (presented symbolically or as a table) on a closed interval. Estimate the rate of change from a graph. Include problem-solving opportunities utilizing real-world context. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.
A2.F-IF.C **Analyze functions using different representations.**	**A2.F-IF.C.7**	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.
	A2.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions and classify those functions as exponential growth or decay.
A2.F-IF.C (cont.)	**A2.F-IF.C.9**	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions.). Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.
Building Functions (F-BF)		
A2.F-BF.A **Build a function that models a relationship between two quantities.**	**A2.F-BF.A.1**	Write a function that describes a relationship between two quantities. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions. Include problem-solving opportunities utilizing real-world context. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine function types using arithmetic operations and function composition.

	A2.F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
A2.F-BF.B **Build new functions from existing functions.**	**A2.F-BF.B.3**	Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *kf(x)*, *f(kx)*, and *f(x+k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.
	A2.F-BF.B.4	Find inverse functions. a. Understand that an inverse function can be obtained by expressing the dependent variable of one function as the independent variable of another, recognizing that functions *f* and g are inverse functions if and only if *f(x) = y* and *g(y) = x* for all values of *x* in the domain of *f* and all values of *y* in the domain of *g*. b. Understand that if a function contains a point (*a,b*), then the graph of the inverse relation of the function contains the point (b,*a*). c. Interpret the meaning of and relationship between a function and its inverse utilizing real-world context.
Linear, Quadratic, and Exponential Models (F-LE)		
A2.F-LE.A **Construct and compare linear, quadratic, and exponential models and solve problems.**	**A2.F-LE.A.4**	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithms that are not readily found by hand or observation using technology.
A2.F-LE.B **Interpret expressions for functions in terms of the situation they model.**	**A2.F-LE.B.5**	Interpret the parameters in an exponential function with rational exponents utilizing real-world context.
Trigonometric Functions (F-TF)		

A2.F-TF.A **Extend the domain of trigonometric functions using the unit circle.**	**A2.F-TF.A.1**	Understand radian measure of an angle as the length of the arc on any circle subtended by the angle, measured in units of the circle's radius.
	A2.F-TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of sine and cosine functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
A2.F-TF.B **Model periodic phenomena with trigonometric functions.**	**A2.F-TF.B.5**	Create and interpret sine, cosine and tangent functions that model periodic phenomena with specified amplitude, frequency, and midline.
A2.F-TF.C **Apply trigonometric identities.**	**A2.F-TF.C.8**	Use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and the quadrant of the angle θ to find sin(θ), cos(θ), or tan(θ) given sin(θ) or cos(θ).
Statistics and Probability - S		
Interpreting Categorical and Quantitative Data (S-ID)		
A2.S-ID.A **Summarize, represent, and interpret data on a single count or measurement variable.**	**A2.S-ID.A.4**	Use the mean and standard deviation of a data set to fit it to a normal curve, and use properties of the normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, or tables to estimate areas under the normal curve.
A2.S-ID.B **Summarize, represent, and interpret data on two categorical and quantitative variables.**	**A2.S-ID.B.6**	Represent data of two quantitative variables on a scatter plot, and describe how the quantities are related. Extend to polynomial and exponential models. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or chooses a function suggested by the context.

Algebra II Item Specifications

The Real Number System (N-RN)

A2.N-RN.A.1

Content Standards	Explain how the definition of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
Explanations	Students may explain orally or in written format.
Content Limits	This standard is aligned to Algebra II only. Rational exponents and bases
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify correct notation for radicals.	• Multiple Choice Response
Students will be required to explain the meaning of rational exponents. Context is required.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify how the properties of integer exponents extend to rational exponents, allowing for a notation for radicals in terms of rational exponents.	Understand how the definition of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
Proficient	**Highly Proficient**
Explain how the definition of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	Show how the definition of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

A2.N-RN.A.2

Content Standards	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
Explanations	Extend the properties of exponents to rational exponents.
Content Limits	Radicals and rational exponents
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to given an expression, create/identify an equivalent expression.	• Equation Response • Multiple Choice Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify expressions involving radicals and rational exponents using the properties of exponents.	Evaluate expressions involving radicals and rational exponents using the properties of exponents.
Proficient	**Highly Proficient**
Rewrite expressions involving radicals and rational exponents using the properties of exponents.	Show that two expressions involving radicals and rational exponents are equivalent using the properties of exponents.

Quantities (N-Q)

A2.N-Q.A.1

Content Standards	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays, include utilizing real-world context.
Explanations	Rational, radical, polynomial (of degree 3 or higher), trigonometric, and logarithmic formulas Include word problems where quantities are given in different units that must be converted to make sense of the problem. Graphical representations and data displays include, but are not limited to, line graphs, circle graphs, histograms, multi-line graphs, scatter plots, and multi-bar graphs.
Content Limits	Real numbers Rational, radical, polynomial (of degree 3 or higher), trigonometric, and logarithmic equations and graphs
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Given a solution, students will determine the correct units based on the context.	• Equation Response • Multiple Choice Response • Multiple Select Response • Editing Task Choice
Students will use dimensional analysis to convert one unit to another in order provide a solution within a real-world situation.	
Students will convert between different units in order to determine the solution for a real-world problem.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify units as a way to understand problems and to guide the solution of multi-step problems; identify units consistently in formulas; identify the scale and the origin in graphs and data displays, include utilizing real-world context.	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and use units consistently in formulas; determine the scale and the origin in graphs and data displays, include utilizing real-world context.
Proficient	**Highly Proficient**
Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays, include utilizing real-world context.	Use units as a way to understand problems and to justify the solution of multi-step problems; choose and justify units consistently in formulas; choose and justify the scale and the origin in graphs and data displays, include utilizing real-world context.

A2.N-Q.A.2

Content Standards	Define appropriate quantities for the purpose of descriptive modeling. Include problem-solving opportunities utilizing real-world context.
Explanations	Reason quantitatively and use units to solve problems.
Content Limits	Quadratic and rational models or situtations/models that go beyond two variables
Context	Context is required.

Sample Task Demands	**Common Item Formats**
Students will be required to use quantities appropriate to the context to solve problems.	• Equation Response • Editing Task Choice • Multiple Choice Response

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify appropriate quantities for the purpose of descriptive modeling.	Define appropriate quantities for the purpose of descriptive modeling.
Proficient	**Highly Proficient**
Define appropriate quantities for the purpose of descriptive modeling. Include problem-solving opportunities utilizing real-world context.	Define and use appropriate quantities for the purpose of descriptive modeling. Include problem-solving opportunities utilizing real-world context.

A2.N-Q.A.3

Content Standards	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities utilizing real-world context.
Explanations	Reason quantitatively and use units to solve problems.
Content Limits	Quadratic and rational models or situtations/models that go beyond two variables
Context	Context is required.

Sample Task Demands	**Common Item Formats**
Students will be required to choose a level of accuracy when reporting quantities in a real-world context.	• Editing Task Choice • Equation Response • Multiple Choice Response • Multi-Select Response

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify a level of accuracy appropriate to be reported quantities utilizing real-world context.	Identify a level of accuracy appropriate to limitations on measurement when reporting quantities utilizing real-world context.
Proficient	**Highly Proficient**
Choose a level of accuracy appropriate to limitations on measurement when reporting quantities utilizing real-world context.	Compare levels of accuracy appropriate to limitations on measurement when reporting quantities utilizing real-world context.

The Complex Number System (N-CN)

A2.N-CN.A.1

Content Standards	Apply the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. Write complex numbers in the form ($a + bi$) with a and b real.
Explanations	Perform arithmetic operations with complex numbers.
Content Limits	This standard is aligned to Algebra II only. The exponent for i should be no greater than 2. The arithmetic performed under this standard should serve to clarify the form of a single complex number (differentiating between complex numbers that are real, complex numbers with imaginary and real components, and pure-imaginary numbers).
Context	Context is not allowed.

Sample Task Demands	Common Item Formats
Students will be required to match square roots of negative numbers with complex numbers.	• Equation Response • Multiple Choice Response • Matching Item Response
Students will be required to create equivalent numbers in standard $a + bi$ form.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Know the relation i^2 = –1 and the commutative, associative, and distributive properties to add, subtract, or multiply complex numbers. Identify complex numbers in the form (a+bi) with a and b real.	Apply the relation i^2 = –1 and the commutative, associative, and distributive properties to add, subtract, or multiply complex numbers. Identify complex numbers in the form (a+bi) with a and b real.
Proficient	**Highly Proficient**
Apply the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. Write complex numbers in the form (a + bi) with a and b real.	Explain the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. Write complex numbers in the form ($a + bi$) with a and b real.

A2.N-CN.C.7

Content Standards	Solve quadratic equations with real coefficients that have complex solutions.
Explanations	Use complex numbers in polynomial identities and equations.
Content Limits	This standard is aligned to Algebra II only. Include real and complex solutions as options, but keys should be complex solutions. Equation response items for this standard are not ideal, in that there is no "+-" button, and many solutions derived from the quadratic equation are cumbersome to input. Therefore, EQ items at this standard should ask for one solution, and that solution should be simple to input.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to solve a quadratic equation.	• Equation Response • Multiple Choice Response

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify quadratic equations with real coefficients that have complex solutions.	Interpret quadratic equations with real coefficients that have complex solutions.
Proficient	**Highly Proficient**
Solve quadratic equations with real coefficients that have complex solutions.	Create quadratic equations with real coefficients that have complex solutions.

Seeing Structure in Expressions (A-SSE)

A2.A-SSE.A.2

Content Standards	Use structure to identify ways to rewrite polynomial and rational expressions. Focus on polynomial operations and factoring patterns.
Explanations	Students should extract the greatest common factor (whether a constant, a variable, or a combination of each). If the remaining expression is quadratic, students should factor the expression further.
Content Limits	Polynomial, rational, and exponential expressions (Poly, Rat) The given expression must be in a form that allows students to use the structure to identify an equivalent expression - not simply using properties of operations.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to identify an equivalent expression.	• Equation Response • Multiple Choice Response • Multi-Select Response
Students will be required to construct a new equivalent expression from a given expression.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Use structure to identify one way to rewrite polynomials. Focus on polynomial operations.	Use structure to identify one way to rewrite polynomial and rational expressions. Focus on polynomial operations and factoring patterns.
Proficient	**Highly Proficient**
Use structure to identify ways to rewrite polynomial and rational expressions. Focus on polynomial operations and factoring patterns.	Use structure to assess ways to rewrite complex polynomial and rational expressions. Focus on polynomial operations and factoring patterns.

A2.A-SSE.B.3, A2.A-SSE.B.3c

Content Standards	A2.A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Include problem-solving opportunities utilizing real-world context and focus on expressions with rational exponents. A2.A-SSE.B.3c Use the properties of exponents to transform expressions for exponential functions.
Explanations	Students will use the properties of operations to create equivalent expressions.
Content Limits	Exponential expressions with rational or real exponents
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify forms of exponential expressions suitable for a given purpose.	• Equation Response • Multiple Choice Response
Students will be required to identify equivalent expressions.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Select an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Include problem-solving opportunities utilizing real-world context and focus on expressions with rational exponents. c. Use the properties of exponents to identify transformed expressions for exponential functions given graphs.	Produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Include problem-solving opportunities utilizing real-world context and focus on expressions with rational exponents. c. Use the properties of exponents to identify transformed expressions for exponential functions.
Proficient	**Highly Proficient**
Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Include problem-solving opportunities utilizing real-world context and focus on expressions with rational exponents. c. Use the properties of exponents to transform expressions for exponential functions.	Justify an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Include problem-solving opportunities utilizing real-world context and focus on expressions with rational exponents. c. Use the properties of exponents to transform and justify expressions for exponential functions.

A2.A-SSE.B.4

Content Standards	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*
Explanations	Write expressions in equivalent forms to solve problems.
Content Limits	This standard is aligned to Algebra II only. Finite geometric series
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to calculate the sum of a finite geometric series in context.	• Equation Response • Multiple Choice Response • Proposition Response
Students will be required to derive the formula for a finite geometric series.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the formula for the sum of a finite geometric series (when the common ratio is not 1).	Interpret the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
Proficient	**Highly Proficient**
Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve complex/multi-step problems in real-world context.

Arithmetic with Polynomials & Rational Expressions (A-APR)

A2.A-APR.B.2

Content Standards	Know and apply the Remainder and Factor Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $(x – a)$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.
Explanations	The Remainder theorem says that if a polynomial $p(x)$ is divided by $x – a$, then the remainder is the constant $p(a)$. That is, So if $p(a) = 0$ then $p(x) = q(x)(x\text{-}a)$.
Content Limits	This standard is aligned to Algebra II only. Focus should be polynomials beyond quadratics.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to given a zero of a polynomial, identify a factor, or vice versa.	• Equation Response • Multiple Choice Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Know the Remainder and Factor Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $(x – a)$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.	Know and understand the Remainder and Factor Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $(x – a)$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.
Proficient	**Highly Proficient**
Know and apply the Remainder and Factor Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $(x – a)$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.	Know and explain the Remainder and Factor Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $(x – a)$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.

A2.A-APR.B.3

Content Standards	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. Focus on quadratic, cubic, and quartic polynomials including polynomials for which factors are not provided.
Explanations	Understand the relationship between zeros and factors of polynomials.
Content Limits	Quadratic, cubic, and quartic polynomials for which factors are not provided
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to identify the zeroes of a polynomial.	• Equation Response • Multiple Choice Response • Multi-Select Response
Students will be required to given a polynomial, determine its graph.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify zeros of polynomials when suitable factorizations are available, and use the zeros to identify a rough graph of the function defined by the polynomial. Focus on quadratic, cubic, and quartic polynomials including polynomials for which factors are not provided.	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to complete a rough graph of the function defined by the polynomial. Focus on quadratic, cubic, and quartic polynomials including polynomials for which factors are not provided.
Proficient	**Highly Proficient**
Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. Focus on quadratic, cubic, and quartic polynomials including polynomials for which factors are not provided.	Interpret zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. Focus on quadratic, cubic, and quartic polynomials including polynomials for which factors are not provided.

A2.A-APR.C.4

Content Standards	Prove polynomial identities and use them to describe numerical relationships.
Explanations	Use polynomial identities to solve problems.
Content Limits	Polynomial identities
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify an expression or statement used to prove the polynomial identity.	• Equation Response • HotText Response • Multiple Choice Response • Proposition Response
Students will be required to show ways in which a polynomial identity can relate numerical values.	
Students will be required to construct a proof of polynomial identities.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify polynomial identities and use them to identify numerical relationships.	Identify polynomial identities and use them to interpret numerical relationships.
Proficient	**Highly Proficient**
Prove polynomial identities and use them to describe numerical relationships.	Prove polynomial identities and use them to create numerical relationships.

A2.A-APR.D.6

Content Standards	Rewrite rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or for the more complicated examples, a computer algebra system.
Explanations	The polynomial $q(x)$ is called the quotient and the polynomial $r(x)$ is called the remainder. Expressing a rational expression in this form allows one to see different properties of the graph, such as horizontal asymptotes.
Content Limits	This standard is aligned to Algebra II only. Rational expressions with linear and quadratic denominators
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to given a simple rational expression, identify the expression in a different form.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response
Students will be required to create or identify the quotient in the form of $q(x) + r(x)/b(x)$, or just $q(x)$ or $r(x)/b(x)$.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify rational expressions in different forms; identify $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or for the more complicated examples, a computer algebra system.	Interpret rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or for the more complicated examples, a computer algebra system.
Proficient	**Highly Proficient**
Rewrite rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or for the more complicated examples, a computer algebra system.	Create rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or for the more complicated examples, a computer algebra system.

Creating Equations (A-CED)

A2.A-CED.A.1

Content Standards	Create equations and inequalities in one variable and use them to solve problems. Include problem-solving opportunities utilizing real-world context. Focus on equations and inequalities arising from linear, quadratic, rational, and exponential functions.
Explanations	Equations can represent real world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth.
Content Limits	Exponential and rational equations (Exp, Poly, Rat, Rad)
Context	Context is subject to task demand.

Sample Task Demands	Common Item Formats
Students will be required to identify the solution for a given equation or inequality. Context is not allowed.	• Equation Response • Multiple Choice Response
Students will be required to construct an equation or inequality to model a context. Context is required.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify equations and inequalities in one variable and use them to solve problems. Include problem-solving opportunities utilizing real-world context. Focus on equations and inequalities arising from linear, quadratic, rational, and exponential functions.	Interpret equations and inequalities in one variable and use them to solve problems. Include problem-solving opportunities utilizing real-world context. Focus on equations and inequalities arising from linear, quadratic, rational, and exponential functions.
Proficient	**Highly Proficient**
Create equations and inequalities in one variable and use them to solve problems. Include problem-solving opportunities utilizing real-world context. Focus on equations and inequalities arising from linear, quadratic, rational, and exponential functions.	Justify equations and inequalities in one variable and use them to solve problems. Include problem-solving opportunities utilizing real-world context. Focus on equations and inequalities arising from linear, quadratic, rational, and exponential functions.

Reasoning with Equations and Inequalities (A-REI)

A2.A-REI.A.1

Content Standards	Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Extend from quadratic equations to rational and radical equations.
Explanations	Properties of operations can be used to change expressions on either side of the equation to equivalent expressions. In addition, adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with the same solutions. Other operations, such as squaring both sides, may produce equations that have extraneous solutions.
Content Limits	Simple rational or radical equations
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to justify a next step in a solution process (i.e., "commutative property", etc.).	• Equation Response • Graphic Response • Multiple Choice Response
Students will be required to identify a correct next step in a solution process.	
Students will be required to given a series of steps in an attempt to solve an equation identify the error(s) and the correct solution.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Identify a viable argument to justify a solution method. Extend from quadratic equations to rational and radical equations.	Show each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Evaluate a viable argument to justify a solution method. Extend from quadratic equations to rational and radical equations.
Proficient	**Highly Proficient**
Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Extend from quadratic equations to rational and radical equations.	Prove each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Justify a viable argument to justify a solution method. Extend from quadratic equations to rational and radical equations.

A2.A-REI.A.2

Content Standards	Solve rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
Explanations	Understand solving equations as a process of reasoning and explain the reasoning.
Content Limits	This standard is aligned to Algebra II only. Radical and rational equations
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to given an equation and its solution(s), identify which solutions(s) are extraneous.	• Equation Response • Multiple Choice Response • Multi-Select Response
Students will be required to solve simple equations from context or no context.	
Students will be required to using abstract symbols, identify when an equation will have extraneous solutions.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify rational and radical equations in one variable, and identify examples showing how extraneous solutions may arise.	Interpret rational and radical equations in one variable, and identify examples showing how extraneous solutions may arise.
Proficient	**Highly Proficient**
Solve rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	Create rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A2.A-REI.B.4

Content Standards	Fluently solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
Explanations	Students should solve by factoring, completing the square, and using the quadratic formula. The zero product property is used to explain why the factors are set equal to zero. Students should relate the value of the discriminant to the type of root to expect. A natural extension would be to relate the type of solutions to $ax^2 + bx + c = 0$ to the behavior of the graph of $y = ax^2 + bx + c$.
Content Limits	Quadratic equations with complex solutions
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify which equations have complex solutions.	• Equation Response • Multiple Choice Response • Multi-Select Response
Students will be required to solve quadratic equations.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Fluently solve quadratic equations in one variable. Identify quadratic equations that can be solved by inspection (e.g., for $x^2 = 49$) and taking square roots, as appropriate to the initial form of the equation.	Fluently solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), and taking square roots, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions.
Proficient	**Highly Proficient**
Fluently solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	Fluently solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, explaining why it is appropriate to the initial form of the equation. Explain when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

A2.A-REI.B.C.7

Content Standards	Solve a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*
Explanations	Solve systems of equations.
Content Limits	A linear equation and a quadratic equation
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to given the graph of a system of equations, identify a possible solution.	• Multiple Choice Response
Students will be required to solve a system of equations.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify the solutions of a system consisting of a linear equation and a quadratic equation in two variables graphically.	Identify the solutions of a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
Proficient	**Highly Proficient**
Solve a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*	Solve and justify the solution of a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

A2.A-REI.D.11

Content Standards	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately (e.g., using technology to graph the functions, make tables of values, or find successive approximations). Include problems in real-world context. Extend from linear, quadratic, and exponential functions to cases where $f(x)$ and/or $g(x)$ are polynomial, rational, exponential, and logarithmic functions.
Explanations	Students need to understand that numerical solution methods (data in a table used to approximate an algebraic function) and graphical solution methods may produce approximate solutions, and algebraic solution methods produce precise solutions that can be represented graphically or numerically.
Content Limits	Extend from linear, quadratic, and exponential functions to cases where $f(x)$ and/or $g(x)$ are polynomial, rational, exponential, and logarithmic functions. Note that this standard is not about systems, but about the solution(s) to $f(x) = g(x)$; thus, solutions should be values of x.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to identify the solution(s) to $f(x) = g(x)$, given the graph of the two functions.	• Equation Response • Graphic Response • Multiple Choice Response • Proposition Response
Students will be required to identify the solutions to $f(x) = g(x)$.	
Students will be required to identify a possible $g(x)$, given $f(x)$ and the value(s) of x where $f(x) = g(x)$.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Find the solutions approximately to $f(x) = g(x)$ given graphs of the functions. Extend from linear, quadratic, and exponential functions to cases where $f(x)$ and/or $g(x)$ are polynomial functions.	Identify that the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately (e.g., using technology to graph the functions, make tables of values, or find successive approximations). Include problems in real-world context. Extend from linear, quadratic, and exponential functions to cases where $f(x)$ and/or $g(x)$ are polynomial, rational, exponential, and logarithmic functions.
Proficient	**Highly Proficient**
Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately (e.g., using technology to graph the functions, make tables of values, or find successive approximations). Include problems in real-world context. Extend from linear, quadratic, and exponential functions to cases where $f(x)$ and/or $g(x)$ are polynomial, rational, exponential, and logarithmic functions.	Prove why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately (e.g., using technology to graph the functions, make tables of values, or find successive approximations). Include problems in real-world context. Extend from linear, quadratic, and exponential functions to cases where $f(x)$ and/or $g(x)$ are polynomial, rational, exponential, and logarithmic functions.

Interpreting Functions (F-IF)

A2.F-IF.B.4

Content Standards	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Include problem-solving opportunities utilizing a real-world context. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.
Explanations	Students may be given graphs to interpret or produce graphs given an expression or table for the function, by hand or using technology.
Content Limits	Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions. Key features may also include domain and range
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to identify an interval on a graph where the function is increasing or decreasing.	• Equation Response • Graphic Response • Multiple Choice Response
Students will be required to identify intercepts of a function.	
Students will be required to construct the graph of a linear function with a given verbal description for the intercept and/or slope.	
Students will be required to identify key features, such as relative maximums and minimums, symmetries, and end behavior, of graphs and tables in terms of the quantities.	
Students will be required to create an exponential function that grows at a different rate than a given one.	
Students will be required to describe the meaning of key features of a function.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
For a function that models a relationship between two quantities, identify key features of graphs and tables in terms of the quantities, and match graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and relative maximums and minimums. Functions include linear, quadratic, exponential, and polynomial.	For a function that models a relationship between two quantities, define key features of graphs and tables in terms of the quantities, and identify graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.
Proficient	**Highly Proficient**
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Include problem-solving opportunities utilizing a real-world context. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and construct graphs showing key features given a verbal description of the relationship. Include problem-solving opportunities utilizing a real-world context. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.

A2.F-IF.B.6

Content Standards	Calculate and interpret the average rate of change of a continuous function (presented symbolically or as a table) on a closed interval. Estimate the rate of change from a graph. Include problem-solving opportunities utilizing real-world context. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.
Explanations	The average rate of change of a function $y = f(x)$ over an interval $[a,b]$ is $\Delta y/\Delta x=(f(b)-f(a))/(b-a)$ In addition to finding average rates of change from functions given symbolically, graphically, or in a table, Students may collect data from experiments or simulations (ex. falling ball, velocity of a car, etc.) and find average rates of change for the function modeling the situation.
Content Limits	Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to estimate the average rate of change of the graph of a given function over a given interval.	• Equation Response • Multiple Choice Response
Students will be required to calculate the average rate of change of a function expressed symbolically or as a table over a given interval.	
Students will be required to interpret the rate of change in context.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the average rate of change of a continuous function (presented symbolically or as a table) on a closed interval. Identify the rate of change from a graph. Include problem-solving opportunities utilizing real-world context. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.	Calculate the average rate of change of a continuous function (presented symbolically or as a table) on a closed interval. Calculate the rate of change from a graph. Include problem-solving opportunities utilizing real-world context. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.
Proficient	**Highly Proficient**
Calculate and interpret the average rate of change of a continuous function (presented symbolically or as a table) on a closed interval. Estimate the rate of change from a graph. Include problem-solving opportunities utilizing real-world context. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.	Interpret and explain the average rate of change of a continuous function (presented symbolically or as a table) on a closed interval. Estimate the rate of change from a graph. Include problem-solving opportunities utilizing real-world context. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.

A2.F-IF.C.7

Content Standards	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.
Explanations	Analyze functions using different representations.
Content Limits	Include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Graph a quadratic function	• Editing Task Choice • Equation Response • Multiple Choice Response • Multi-Select Response
Identify key features of an exponential function	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the graph of functions expressed symbolically. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.	Graph functions expressed symbolically and identify key features of the graph, by hand in simple cases and using technology for more complicated cases. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.
Proficient	**Highly Proficient**
Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.	Graph functions expressed symbolically and show and interpret key features of the graph, by hand in simple cases and using technology for more complicated cases. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.

A2.F-IF.C.8, A2.F-IF.C.8b

Content Standards	A2.F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. A2.F-IF.C.8b Use the properties of exponents to interpret expressions for exponential functions and classify those functions as exponential growth or decay.
Explanations	Analyze functions using different representations.
Content Limits	Exponential functions
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to create an equivalent function in a specific form that reveals characteristics of the function defined by that expression.	• Equation Response • Multiple Choice Response • Proposition Response
Students will be required to interpret parameters of a function in terms of the context.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Identify exponential functions and classify those functions as exponential growth or decay using graphs.	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Identify the properties of exponents to interpret expressions for exponential functions and classify those functions as exponential growth or decay.
Proficient	**Highly Proficient**
Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions and classify those functions as exponential growth or decay.	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Explain the properties of exponents that are used to interpret expressions for exponential functions and explain why those functions model exponential growth or decay.

A2.F-IF.C.9

Content Standards	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.
Explanations	Analyze functions using different representations.
Content Limits	Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to compare numeric values representing properties of two functions.	• Equation Response • Graphic Response • Multiple Choice Response
Students will be required to compare two functions qualitatively.	
Students will be required to construct a graph of a function for which a given comparison with another function is true.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify properties of two functions each represented in a different way (graphically or numerically in tables). Functions include linear, quadratic, exponential, and polynomial functions.	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Functions include linear, quadratic, exponential, and polynomial functions.
Proficient	**Highly Proficient**
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.	Create functions given comparisons about the properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.

Building Functions (F-BF)

A2.F-BF.A.1, A2.F-BF.A.1a, A2.F-BF.A.1b

Content Standards	A2.F-BF.A.1 Write a function that describes a relationship between two quantities. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions. Include problem-solving opportunities utilizing real-world context. A2.F-BF.A.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. A2.F-BF.A.1b Combine function types using arithmetic operations and function composition.
Explanations	Students will analyze a given problem to determine the function expressed by identifying patterns in the function's rate of change. They will specify intervals of increase, decrease, constancy, and, if possible, relate them to the function's description in words or graphically.
Content Limits	Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions. Include problem-solving opportunities utilizing real-world context.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to perform arithmetic operations to write one function that models a context for another.	• Equation Response • Multiple Choice Response
Students will be required to create a multi-faceted function to model a context.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Write a function that describes a relationship between two quantities. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions. Include problem-solving opportunities utilizing real-world context. a. Identify an explicit expression. b. Combine function types using addition and subtraction.	Write a function that describes a relationship between two quantities. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions. Include problem-solving opportunities utilizing real-world context. a. Determine an explicit expression, or steps for calculation from a context. b. Combine function types using arithmetic operations.
Proficient	**Highly Proficient**
Write a function that describes a relationship between two quantities. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions. Include problem-solving opportunities utilizing real-world context. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine function types using arithmetic operations and function composition.	Write a function that describes a relationship between two quantities. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions. Include problem-solving opportunities utilizing real-world context. a. Justify an explicit expression, a recursive process, or steps for calculation from a context. b. Combine function types using a combination of arithmetic operations and function composition.

A2.F-BF.A.2

Content Standards	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
Explanations	An explicit rule for the nth term of a sequence gives an as an expression in the term's position *n*; a recursive rule gives the first term of a sequence, and a recursive equation relates an to the preceding term(s). Both methods of presenting a sequence describe an as a function of *n*.
Content Limits	Arithmetic and geometric sequences both recursively and with an explicit formula
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to identify a formula that models a geometric or arithmetic pattern.	• Equation Response • Multiple Choice Response
Students will be required to translate a recursive function to an explicit formula.	
Students will be required to create a recursive function to model a geometric pattern described verbally.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify arithmetic sequences both recursively and with an explicit formula.	Write arithmetic sequences both recursively and with an explicit formula, and translate between the two forms.
Proficient	**Highly Proficient**
Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	Create arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

A2.F-BF.B.3

Content Standards	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.
Explanations	Students will apply transformations to functions and recognize functions as even and odd.
Content Limits	Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to show the effects of a transformation by translating a graph.	• Equation Response • Graphic Response • Multiple Choice Response
Students will be required to determine the value of k from two related functions or graphs.	
Students will be required to create a function to model a transformation of a given graph.	
Students will be required to describe the effects of k on a transformation of a function.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ and $f(x+k)$ for specific values of k (both positive and negative); identify the value of k given the graphs. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Include recognizing even and odd functions from their graphs. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.
Proficient	**Highly Proficient**
Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.	Justify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative); justify the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Functions include linear, quadratic, exponential, polynomial, logarithmic, rational, sine, cosine, tangent, square root, cube root and piecewise-defined functions.

A2.F-BF.B.4, A2.F-BF.B.4a, A2.F-BF.B.4b, A2.F-BF.B.4c

Content Standards	A2.F-BF.B.4 Find inverse functions. A2.F-BF.B.4a Understand that an inverse function can be obtained by expressing the dependent variable of one function as the independent variable of another, recognizing that functions *f* and g are inverse functions if and only if $f(x) = y$ and $g(y) = x$ for all values of *x* in the domain of *f* and all values of *y* in the domain of *g*. A2.F-BF.B.4b Understand that if a function contains a point (a,b), then the graph of the inverse relation of the function contains the point (b,a). A2.F-BF.B.4c Interpret the meaning of and relationship between a function and its inverse utilizing real-world context.
Explanations	Create and interpret inverse functions of existing functions.
Content Limits	The focus should be on functions with simple terms and coefficients. Can include functions with a finite number of points as well as linear, exponential, quadratic, absolute value, polynomial, radical, and rational functions.
Context	Context is not required for A2.F-BF.B.4a and 4b, but is required for 4c.

Sample Task Demands	Common Item Formats
Students will be required to identify the inverse relation of a given function.	• Equation Response • Multiple Choice Response • Multi-Select Response • Editing Task Choice • Table Response • Grid Response
Students will be required to recognize whether the inverse relation of a function is also a function.	
Students will be required to select which function(s) does or does not have an inverse relation that is a function over a given interval.	
Students will be required to interpret the meaning of and relationship between a function and its inverse within a real-world context.	
Given the table of values for a function, create the table values for the inverse relation of the function.	
Students will be required to graph or the select the graph of the inverse relation or the inverse function of a function.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Find inverse functions. a. Understand that an inverse function can be obtained by expressing the dependent variable of one function as the independent variable of another, given visual representations. b. Understand that if a function contains a point (*a,b*), then the graph of the inverse relation of the function contains the point (*b,a*) given visual representations. c. Identify the meaning of a function and its inverse.	Find inverse functions. a. Understand that an inverse function can be obtained by expressing the dependent variable of one function as the independent variable of another. b. Understand that if a function contains a point (*a,b*), then the graph of the inverse relation of the function contains the point (*b,a*) in concrete situations. c. Identify the meaning of and relationship between a function and its inverse utilizing real-world context.
Proficient	**Highly Proficient**
Find inverse functions. a. Understand that an inverse function can be obtained by expressing the dependent variable of one function as the independent variable of another, recognizing that functions *f* and *g* are inverse functions if and only if $f(x) = y$ and $g(y) = x$ for all values of *x* in the domain of *f* and all values of *y* in the domain of *g*. b. Understand that if a function contains a point (*a,b*), then the graph of the inverse relation of the function contains the point (*b,a*). c. Interpret the meaning of and relationship between a function and its inverse utilizing real-world context.	Find inverse functions. a. Explain that an inverse function can be obtained by expressing the dependent variable of one function as the independent variable of another, recognizing that functions *f* and *g* are inverse functions if and only if $f(x) = y$ and $g(y) = x$ for all values of *x* in the domain of *f* and all values of *y* in the domain of *g*. b. Explain that if a function contains a point (*a,b*), then the graph of the inverse relation of the function contains the point (*b,a*). c. Explain the meaning of and relationship between a function and its inverse utilizing real-world context.

Linear, Quadratic & Exponential Models (F-LE)

A2.F-LE.A.4

Content Standards	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithms that are not readily found by hand or observation using technology.
Explanations	Construct and compare linear, quadratic, and exponential models and solve problems.
Content Limits	Linear, quadratic, and exponential models
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to evaluate a logarithm.	• Equation Response
Students will be required to create an exponential equation equivalent to a logarithmic equation.	
Students will be required to create a logarithmic equation equivalent to an exponential equation.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
For exponential models, identify as a logarithm the solution to *ab*^(*ct*) = *d* where *a, c,* and *d* are numbers and the base *b* is 2, 10, or *e.*	For exponential models, express as a logarithm the solution to *ab*^(*ct*) = *d* where *a, c,* and *d* are numbers and the base *b* is 2, 10, or *e*; identify the logarithms that are not readily found by hand or observation using technology.
Proficient	**Highly Proficient**
For exponential models, express as a logarithm the solution to *ab*^(*ct*) = *d* where *a, c,* and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithms that are not readily found by hand or observation using technology.	For exponential models, express as a logarithm the solution to *ab*^(*ct*) = *d* where *a, c,* and *d* are numbers and the base *b* is 2, 10, or *e* in a real-world context; evaluate the logarithms that are not readily found by hand or observation using technology in a real-world context.

A2.F-LE.B.5

Content Standards	Interpret the parameters in an exponential function with rational exponents utilizing real-world context.
Explanations	Interpret expressions for functions in terms of the situation they model.
Content Limits	Exponential functions with domains not limited to integers
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to interpret the meaning of a parameter of a function.	• Multi-Select Response
Students will be required to interpret the meaning of a parameter in a function that combines linear and exponential terms.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the intercepts in an exponential function with rational exponents utilizing real-world context.	Identify the parameters in an exponential function with rational exponents utilizing real-world context.
Proficient	**Highly Proficient**
Interpret the parameters in an exponential function with rational exponents utilizing real-world context.	Explain the parameters in an exponential function with rational exponents utilizing real-world context.

Trigonometric Functions (F-TF)

A2.F-TF.A.1

Content Standards	Understand radian measure of an angle as the length of the arc on any circle subtended by the angle, measured in units of the circle's radius.
Explanations	Extend the domain of trigonometric functions using the unit circle.
Content Limits	This standard is aligned to Algebra II only. Positive angles, all four quadrants of the coordinate plane
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to determine the radian measure of an angle.	• Equation Response • Graphic Response • Proposition Response
Students will be required to construct an angle with a given radian measure.	
Students will be required to explain the relationship of the central angle to the arc.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify angles given radian measures.	Use radian measures to describe central angles of a circle.
Proficient	**Highly Proficient**
Understand radian measure of an angle as the length of the arc on any circle subtended by the angle, measured in units of the circle's radius.	Use the fact that a radian measure of an angle is the length of the arc on any circle subtended by the angle, measured in units of the circle's radius, to solve problems.

A2.F-TF.A.2

Content Standards	Explain how the unit circle in the coordinate plane enables the extension of sine and cosine functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
Explanations	Students may explain (orally or in written format) their understanding.
Content Limits	This standard is aligned to Algebra II only. Sine and cosine, common angles
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to evaluate trigonometric functions for common angles and their co-terminal angles.	• Equation Response • Graphic Response
Students will be required to place a point on the unit circle to show trigonometric values with given radian measures.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify how the unit circle in the coordinate plane enables the extension of sine and cosine functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	Show how the unit circle in the coordinate plane enables the extension of sine and cosine functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
Proficient	**Highly Proficient**
Explain how the unit circle in the coordinate plane enables the extension of sine and cosine functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	Explain how the unit circle in the coordinate plane enables the extension of sine and cosine functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

A2.F-TF.B.5

Content Standards	Create and interpret sine, cosine and tangent functions that model periodic phenomena with specified amplitude, frequency, and midline.
Explanations	Model periodic phenomena with trigonometric functions.
Content Limits	Trigonometric functions are used for modeling simple harmonic motion
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to determine the amplitude of a given sine or cosine function.	• Equation Response
Students will be required to create the trigonometric function with given numeric values for amplitude, midline, and frequency.	
Students will be required to create a trigonometric function given a verbal description.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Match sine, cosine and tangent functions that model periodic phenomena with specified amplitude, and midline.	Identify sine, cosine and tangent functions that model periodic phenomena with specified amplitude, frequency, and midline.
Proficient	**Highly Proficient**
Create and interpret sine, cosine and tangent functions that model periodic phenomena with specified amplitude, frequency, and midline.	Create and compare sine, cosine and tangent functions that model periodic phenomena with specified amplitude, frequency, and midline.

A2.F-TF.C.8

Content Standards	Use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and the quadrant of the angle θ to find sin(θ), cos(θ), or tan(θ) given sin(θ) or cos(θ).
Explanations	Apply trigonometric identities.
Content Limits	Trigonometric identities
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to determine the value of a trigonometric function using the Pythagorean identity.	• Equation Response • HotText Response
Students will be required to order steps in a proof of the Pythagorean identity.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and the quadrant of the angle θ as sufficient for finding sin(θ), cos(θ), or tan(θ) given sin(θ) or cos(θ).	Use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and the quadrant of the angle θ to identify sin(θ), cos(θ), or tan(θ) given sin(θ) or cos(θ).
Proficient	**Highly Proficient**
Use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and the quadrant of the angle θ to find sin(θ), cos(θ), or tan(θ) given sin(θ) or cos(θ).	Create problems that use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and the quadrant of the angle θ to find sin(θ), cos(θ), or tan(θ) given sin(θ) or cos(θ).

Interpreting Categorical and Quantitative Data (S-ID)

A2.S-ID.A.4

Content Standards	Use the mean and standard deviation of a data set to fit it to a normal curve, and use properties of the normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, or tables to estimate areas under the normal curve.
Explanations	Summarize, represent, and interpret data on a single count or measurement variable.
Content Limits	This standard is aligned to Algebra II only. If a student is required to estimate a population percentage not associated with 1, 2, or 3 standard deviations from the mean, a normal table or some equivalent mechanism must be provided. Items should state that the data are approximately normally distributed when that is not the content being assessed.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify a population percentage or area under a curve within 1, 2, or 3 standard deviations.	• Equation Response • Graphic Response • Multiple Choice Response
Students will be required to identify a population percentage for standard deviations other than 1, 2, or 3.	
Students will be required to explain why a data set should or should not be modeled with a normal distribution.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the mean and standard deviation of a data set from a normal curve.	Use the mean and standard deviation of a data set to fit it to a normal curve, and use properties of the normal distribution to estimate population percentages.
Proficient	**Highly Proficient**
Use the mean and standard deviation of a data set to fit it to a normal curve, and use properties of the normal distribution to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, or tables to estimate areas under the normal curve.	Use the mean and standard deviation of a data set to fit it to a normal curve, and use properties of the normal distribution to estimate population percentages. Explain why there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, or tables to estimate areas under the normal curve.

A2.S-ID.B.6, A2.S-ID.B.6a

Content Standards	A2.S-ID.B.6 Represent data of two quantitative variables on a scatter plot, and describe how the quantities are related. Extend to polynomial and exponential models. A2.S-ID.B.6a Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context.
Explanations	The residual in a regression model is the difference between the observed and the predicted *y* for some *x* (*y* the dependent variable and *x* the independent variable). So if we have a model $y = ax + b$ and a data point (xi, yi), the residual is for this point is $ri = yi - (axi + b)$. Students may use spreadsheets, graphing calculators, and statistical software to represent data, describe how the variables are related, fit functions to data, perform regressions, and calculate residuals.
Content Limits	Only S-ID.B.6a is aligned to Algebra II (fitting functions for Algebra I and using functions for Algebra II). Rational numbers; Bivariate data; Linear, quadratic, and exponential models for 6a
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to select a function that best represents the data given a set of data. (a)	• Equation Response • Graphic Response • Multiple Choice Response

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Represent data of two quantitative variables on a scatter plot, and describe how the quantities are related. Extend to polynomial and exponential models. a. Use functions fitted to data given scatter plots and the graphs of the functions to solve problems in the context of the data.	Represent data of two quantitative variables on a scatter plot, and describe how the quantities are related. Extend to polynomial and exponential models. a. Fit a function to the data; use functions fitted to data given scatter plots to solve problems in the context of the data. Use given functions or choose a function suggested by the context.
Proficient	**Highly Proficient**
Represent data of two quantitative variables on a scatter plot, and describe how the quantities are related. Extend to polynomial and exponential models. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context.	Represent data of two quantitative variables on a scatter plot, and describe how the quantities are related. Extend to polynomial and exponential models. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose and justify a function suggested by the context.

A2.S-ID.C.10

Content Standards	Interpret parameters of exponential models.
Explanations	To differentiate between other standards that focus on exponential equations and functions, items to this standard should focus on statistical data and contexts.
Content Limits	This standard is aligned to Algebra II only. Bivariate, exponential data
Context	Context is required.

Sample Task Demands	**Common Item Formats**
Students will interpret the growth or decay factor in an exponential model.	• Editing Task Choice • Equation Response • Multiple Choice Response • Multi-Select Response
Students will interpret the coefficient and vertical or horizontal shifts in an exponential model.	
Students will interpret changes in parameters based on the comparison of two functions in terms of a real-world context.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Match parameters of exponential models.	Identify parameters of exponential models.
Proficient	**Highly Proficient**
Interpret parameters of exponential models.	Compare parameters of exponential models.

Making Inferences & Justifying Conclusions (S-IC)

A2.S-IC.A.1

Content Standards	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
Explanations	Understand and evaluate random processes underlying statistical experiments.
Content Limits	This standard is aligned to Algebra II only. Quantities should be simple and realistic to the context so as to allow the student to show knowledge of the concept rather than computational skills.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to order steps in a statistical process or select a sample that represents a given population.	• HotText Response • Multiple Choice Response
Students will be required to describe flaws in a statistical process (i.e., not random) or recommend a correct course of action.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Understand that random sampling is necessary for making inferences about population parameters.	Understand statistics as a process for making inferences about population parameters.
Proficient	**Highly Proficient**
Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	Understand that inferences about population parameters can only be generalized based on a random sample from that population.

A2.S-IC.A.2

Content Standards	Explain whether a specified model is consistent with results from a given data-generating process.
Explanations	Possible data-generating processes include (but are not limited to): flipping coins, spinning spinners, rolling a number cube, and simulations using the random number generators. The law of large numbers states that as the sample size increases, the experimental probability will approach the theoretical probability. Comparison of data from repetitions of the same experiment is part of the model building verification process.
Content Limits	This standard is aligned to Algebra II only. Quantities should be simple and realistic to the context so as to allow the student to show knowledge of the concept rather than computational skills.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to predict the most likely results of a simulation given a probability model.	• Equation Response • Multiple Choice Response
Students will be required to select the best probability model given the results of a simulation.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify whether a specified model is consistent with results from a given data-generating process.	State whether a specified model is consistent with results from a given data-generating process.
Proficient	**Highly Proficient**
Explain whether a specified model is consistent with results from a given data-generating process.	Explain whether a specified model is consistent with results from a given data-generating process.

A2.S-IC.B.3

Content Standards	Recognize the purposes of and differences between designed experiments, sample surveys, and observational studies.
Explanations	Students should be able to explain techniques/applications for randomly selecting study subjects from a population and how those techniques/applications differ from those used to randomly assign existing subjects to control groups or experimental groups in a statistical experiment. In statistics, an observational study draws inferences about the possible effect of a treatment on subjects, where the assignment of subjects into a treated group versus a control group is outside the control of the investigator (for example, observing data on academic achievement and socio-economic status to see if there is a relationship between them). This is in contrast to controlled experiments, such as randomized controlled trials, where each subject is randomly assigned to a treated group or a control group before the start of the treatment.
Content Limits	Sample surveys, experiments, and observational studies in Algebra II context
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify a given activity as a survey sample, experiment, or observational study	• Multiple Choice Response
Students will be required to identify a substantive difference between the three types of activities, or the most appropriate activity for a given research question.	
Students will be required to design a study that correctly applies randomization given specific criteria.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify examples of designed experiments, sample surveys, and observational studies.	Recognize situations where designed experiments, sample surveys, and observational studies are the most appropriate.
Proficient	**Highly Proficient**
Recognize the purposes of and differences between designed experiments, sample surveys, and observational studies.	Compare the purposes of and differences between designed experiments, sample surveys, and observational studies.

A2.S-IC.B.4

Content Standards	Use data from a sample survey to estimate a population mean or proportion; recognize that estimates are unlikely to be correct and the estimates will be more precise with larger sample sizes.
Explanations	Make inferences and justify conclusions from experiments, and observational studies.
Content Limits	This standard is aligned to Algebra II only. The student should not be required to compute a margin of error, but identify what factors lead to larger or smaller margins of error.
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to provide a sample mean given raw data.	• Equation Response • Proposition Response • Simulator Response
Students will be required to relate margin of error to the characteristics of the sample population and the survey methodology.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Recognize that estimates are unlikely to be correct and the estimates will be more precise with larger sample sizes.	Use data from a sample survey to estimate a population mean; recognize that estimates are unlikely to be correct and the estimates will be more precise with larger sample sizes.
Proficient	**Highly Proficient**
Use data from a sample survey to estimate a population mean or proportion; recognize that estimates are unlikely to be correct and the estimates will be more precise with larger sample sizes.	Use data from a sample survey to compare population mean or proportion; recognize that estimates are unlikely to be correct and the estimates will be more precise with larger sample sizes.

Conditional Probability and the Rules of Probability (S-CP)

A2.S-CP.A.3

Content Standards	Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of A, and the conditional probability of *B* given *A* is the same as the probability of *B*.
Explanations	Understand independence and conditional probability and use them to interpret data.
Content Limits	This standard is aligned to Algebra II only. Rational positive numbers
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to compute conditional probabilites.	• Equation Response • Multiple Choice Response • Multi-Select Response
Students will be required to identify independent events given their probabilities and one conditional probability.	
Students will be required to determine the conditional probability (or vice versa) given that two events are independent and the probability of one event.	
Students will be required to interpret the events in terms of independence given the probability of an event and the conditional probability of that event with another event.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify a conditional probability as *A* given *B.*	Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and identify independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.
Proficient	**Highly Proficient**
Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.	Evaluate the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and show independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of B.

A2.S-CP.A.4

Content Standards	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
Explanations	Understand independence and conditional probability and use them to interpret data.
Content Limits	This standard is aligned to Algebra II only.
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to determine independence of events or conditional probabilities given a two-way table.	• Equation Response • Multiple Choice Response • Table Response
Students will be required to complete a two-way table to satisfy criteria related to independence or conditional probability.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Identify a missing value in two-way frequency tables of data when two categories are associated with each object being classified.	Complete and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to approximate conditional probabilities.
Proficient	**Highly Proficient**
Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.	Construct and compare two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

A2.S-CP.A.5

Content Standards	Recognize and explain the concepts of conditional probability and independence utilizing real-world context.
Explanations	Understand independence and conditional probability and use them to interpret data.
Content Limits	This standard is aligned to Algebra II only. Rational positive numbers
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to identify examples of independence and conditional probability given a scenario.	• Multiple Choice Response
Students will be required to interpret this in terms of the context given that two events are independent or a conditional probability.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Identify the concepts of conditional probability and independence utilizing real-world context.	Recognize and interpret the concepts of conditional probability and independence utilizing real-world context.
Proficient	**Highly Proficient**
Recognize and explain the concepts of conditional probability and independence utilizing real-world context.	Create examples of and explain the concepts of conditional probability and independence utilizing real-world context.

A2.S-CP.B.6

Content Standards	Use Bayes Rule to find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.
Explanations	Use the rules of probability to compute probabilities of compound events in a uniform probability model.
Content Limits	This standard is aligned to Algebra II only. Data is given by raw data and not probabilities
Context	Context is allowed.

Sample Task Demands	**Common Item Formats**
Students will be required to compute a conditional probability.	• Equation Response • Multiple Choice Response
Students will be required to find pieces of raw data given a conditional probability and some other raw data.	
Students will be required to interpret the meaning of the conditional probability given data and a conditional probability.	

Performance Level Descriptors

Minimally Proficient	**Partially Proficient**
Recognize Bayes Rule to find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*.	Use Bayes Rule to find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A* given visual models.
Proficient	**Highly Proficient**
Use Bayes Rule to find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.	Use Bayes Rule to find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and justify the answer in terms of the model.

A2.S-CP.B.7

Content Standards	Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.
Explanations	Use the rules of probability to compute probabilities of compound events in a uniform probability model.
Content Limits	Addition Rule
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to compute the probability of the union of two events.	• Equation Response • Proposition Response
Students will be required to find a missing probability using the Addition Rule given the probability of the union of two events and other probabilities.	
Students will be required to interpret the meaning of the union given a context and the probability of the union of two events.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Recognize the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.	Calculate probabilities using the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.
Proficient	**Highly Proficient**
Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.	Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and justify the answer in terms of the model.

A2.S-CP.B.8

Content Standards	Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.
Explanations	Use the rules of probability to compute probabilities of compound events in a uniform probability model.
Content Limits	Multiplication Rule
Context	Context is allowed.

Sample Task Demands	Common Item Formats
Students will be required to apply the Multiplication Rule to calculate a probability in a given context.	• Equation Response • Multiple Choice Response
Students will be required to apply the Multiplication Rule and interpret the answer in terms of the model.	

Performance Level Descriptors

Minimally Proficient	Partially Proficient
Recognize the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$.	Calculate probabilities using the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$.
Proficient	**Highly Proficient**
Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.	Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and justify the answer in terms of the model.