National Center and State Collaborative

NCSC SCHEMA for Common Core State Standards Resources

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National Center and State Collaborative

The National Center and State Collaborative (NCSC) is applying the lessons learned from the past decade of research on alternate assessments based on alternate achievement standards (AA-AAS) to develop a multi-state comprehensive assessment system for students with significant cognitive disabilities. The project draws on a strong research base to develop an AA-AAS that is built from the ground up on powerful validity arguments linked to clear learning outcomes and defensible assessment results, to complement the work of the Race to the Top Common State Assessment Program (RTTA) consortia.

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IDEAs that Work
U.S. Office of Special Education Programs

The contents of this instructional resource were developed as part of the National Center and State Collaborative for a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made.

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These materials and documents were developed under the National Center and State Collaborative (NCSC) General Supervision Enhancement Grant and are consistent with its goals and foundations. Any changes to these materials are to be consistent with their intended purpose and use as defined by NCSC.

This document is available in alternative formats upon request.

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National Center and State Collaborative

NCSC is a collaborative of 18 states and five organizations.

The states include (shown in blue on map): Alaska, Arizona, Connecticut, District of Columbia, Florida, Georgia, Indiana, Louisiana, Nevada, New York, North Dakota, Pacific Assessment Consortium (PAC-6)1, Pennsylvania, Rhode Island, South Carolina, South Dakota, Tennessee, and Wyoming.

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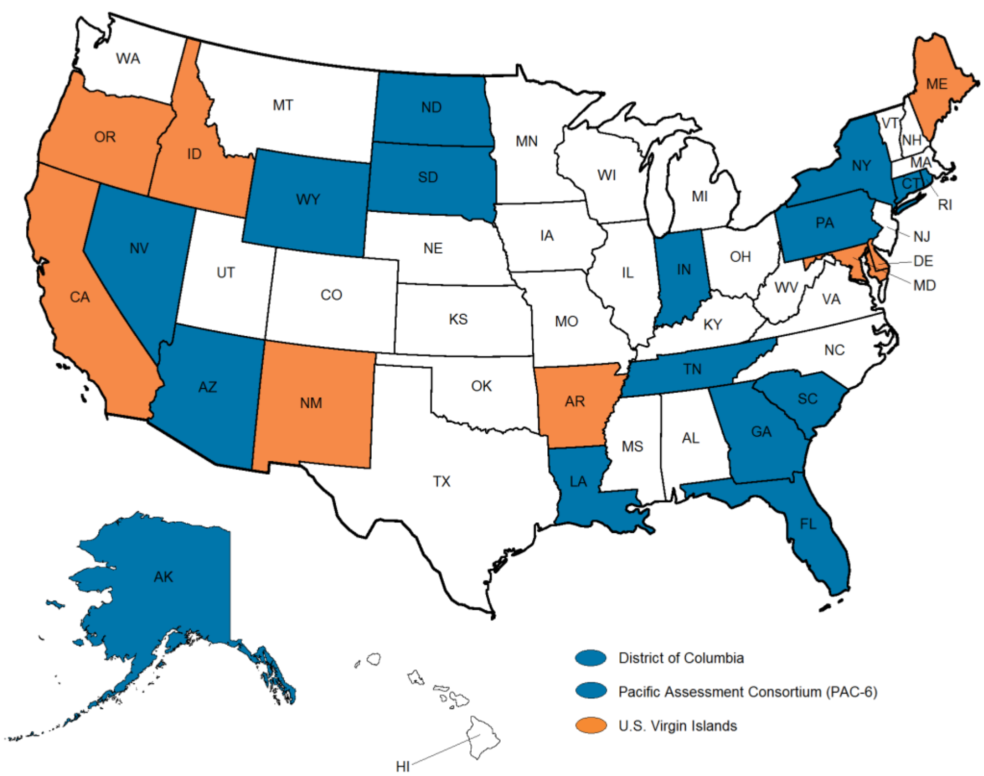

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NCSC SCHEMA for Common Core State Standards Resources

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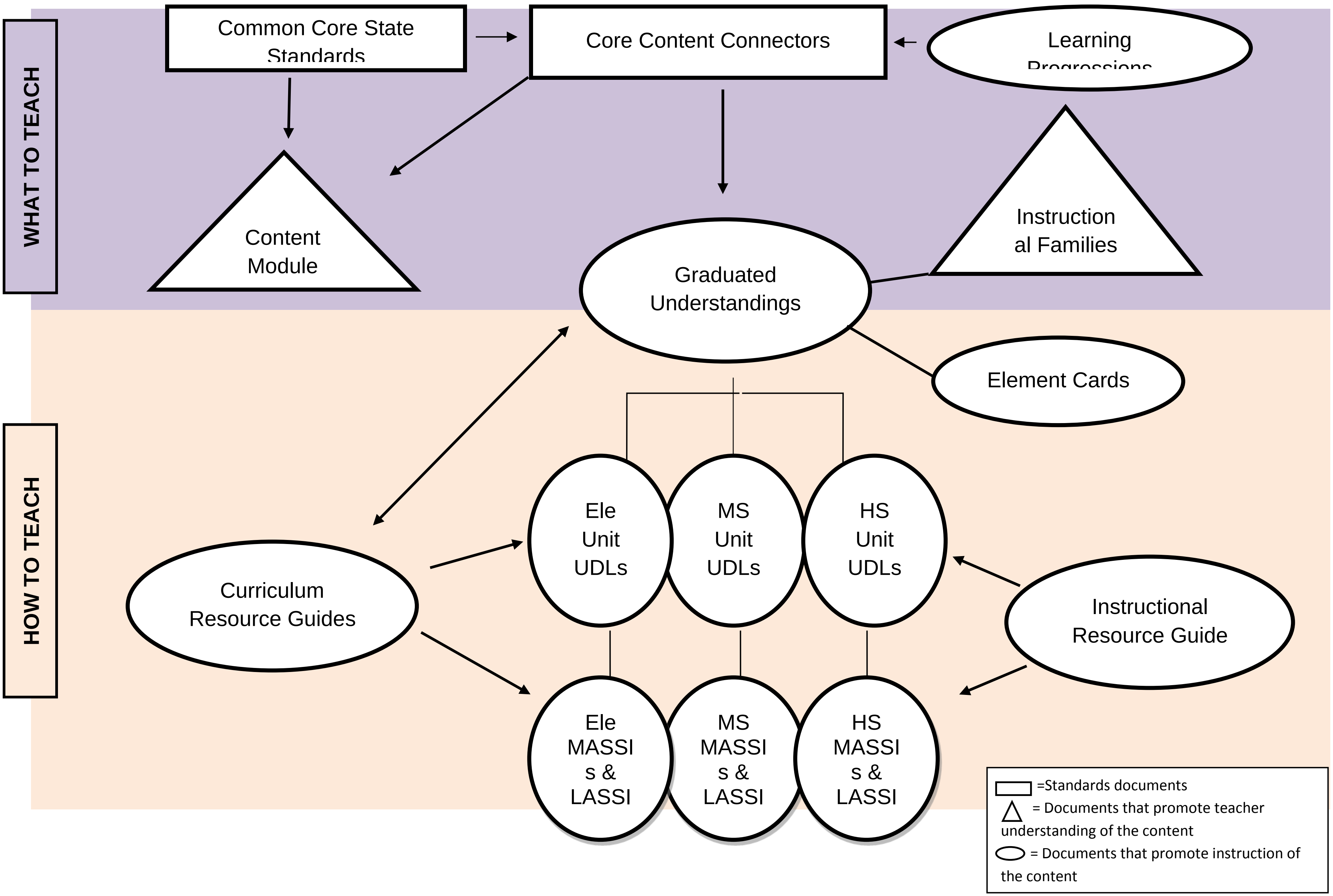
Figure 1. SCHEMA for Common Core State Standards Resources
NCSC Instructional Resources



Explanation of Schema for NCSC Instructional Resources in Mathematics

The NCSC instructional resources provide support for teachers to address the **Common Core State Standards** when teaching students with significant cognitive disabilities who participate in alternate assessment based on alternate achievement standards. Their purpose is to build the capacity of teachers to plan instruction using the **Core Content Connectors (CCCs)**. As the schema in Figure 1 indicates, the Core Content Connectors link to both the Common Core Standards and the NCSC Learning Progressions Frameworks. These connectors have been dually aligned with both the standards and framework. The CCCs retain the grade level content focus of these two resources and are not extended. The CCCs do pinpoint the primary content of the Common Core Standards and organize it in the conceptual model of the **Learning Progressions Framework**. By focusing on the CCCs, teachers will be teaching the Common Core State Standards and promoting a progression of learning.

At first, the CCCs may seem overwhelming and confusing to teachers who do not have extensive training in the content area or who do not have extensive background in adapting the content to students with significant cognitive disabilities. NCSC is preparing a collection of resources to assist teachers in both understanding the content and planning instruction.

<u>Understanding the Content.</u> Two sets of resources are offered primarily to help teachers gain a deeper understanding of the content as they prepare to develop instructional adaptations. The **Content Modules** are an online multimedia resource that provides teachers with a deeper understanding of complex concepts. These make an excellent companion resource when viewing the CCCs. For example, if a teacher is not sure what “nets” are in geometry, a content module can be used to see explanations and examples of nets. The **Curriculum Resource Guides** are a second set of resources for understanding the CCCs. These guides also offer examples of how the content is taught in general education and ideas for teaching across content areas, assessment examples, ideas for real life use, examples of modifications and adaptations for students with specific learning needs, and ways to promote college and career readiness. Each guide covers a range of CCCs for grades 3 through high school. These guides focus on five topics that were derived from the priorities identified by the NCSC Work Group 1 for the Assessment. These guides should support teachers in preparing students for the NCSC alternate assessment. Both the Content Modules and Curriculum Resource Guides were developed by special educators with extensive experience in adapting general curriculum for students with significant cognitive disabilities. These resources have been validated by mathematics content experts for accuracy and by special education teachers for clarity.

<u>Teaching the Content.</u> Teaching requires designing instructional plans at various levels of intensity. The first level of planning should be to promote universal design of learning for all students. The **Units and Lesson Plans** provide models of universally designed

planning for an entire class of students. The Units and Lesson Plans illustrate how to target the CCCs within general education lessons. Examples are provided for planning for engagement, representation, and expression. That is, they offer a model for how to engage all students in well-designed instruction for the Common Core Standards. Many examples are offered for meeting the unique needs of students with significant cognitive disabilities. As all teachers know, even the best plans for a class may not be sufficient for some students to master specific mathematical concepts. The **MASSIs** offer intensive instruction based on evidence-based practices. These “Math Activities with Scripted Systematic Instruction” have several features. First, they target CCCs prioritized for assessment. Second, they offer a guide for instruction with increasing levels of difficulty. The first steps of the lesson are accessible to students with little to no understanding of the content. The lesson continues building understanding through a target component of the CCC. Third, the MASSIs use a real life activity to teach the concept that can be easily set up in most classrooms with inexpensive materials. That is, they bring math word problems to life using a hands-on activity. Finally, the instruction is scripted, making them easy for teachers to use, and include evidence-based practices shown to be effective in teaching mathematics skills to students with significant cognitive disabilities. The MASSIs come with data sheets that can be used for monitoring progress towards mastery and a skill test for practicing responding in a testing context. Neither the Units/Plans nor MASSIs provide everything needed to teach all CCCs at each level. Instead, they provide models for how to teach the content. In contrast, teachers may find they can apply these model plans as a way to get started in teaching the CCCs/Common Core. After teaching the model lesson plan or MASSI, teachers will gain practice in instructional strategies that are effective for teaching general mathematics content. LASSIs will serve the same purpose for ELA content as the MASSIs do for math content. LASSIs are currently in development. Consistent use of instructional strategies that have been shown to be effective when teaching students with significant cognitive disabilities will be crucial to student success. To help support teachers in using these effective teaching strategies, an Instructional Resource Guide provides guidance for teachers by explaining and providing examples on how to use these evidence-based prompting and instructional strategies. The Instructional Resource Guide will serve as a companion document to the MASSIs for teachers to reference quickly and easily and will help educators build knowledge of the essential systematic instructional methods and prompting strategies that are used in MASSIs to teach students targeted skills. Lastly, teachers will need to be prepared to teach the CCCs to a range of students with significant cognitive disabilities in a variety of educational settings. **Graduated Understandings** are made up of **Instructional Families** and **Element Cards**. Instructional Families group related CCCs into families (e.g., Counting and Representing Numbers). The Instructional Families allow teachers to view related content within and across grades. Element cards are written for select CCCs at each grade level. Each Element Card contains essential understandings. The essential understandings provide both the concrete and the symbolic (representational) understandings necessary for students to engage in the content described/identified by the CCC or a set of related CCCs. In addition, the Element Cards provide a range of instructional strategies intended to provide teachers with suggestions that will be applicable to a variety of students. Finally, each Element Card includes scaffolds and supports (e.g., use of a calculator or a raised grid) that may be necessary when teaching the content described/identified by the CCC(s).

Why This Content. Preparing all students to be ready for college, career and community opportunities after high school is critical. NCSC promotes the content already determined by the Common Core State Standards to support this goal. The priorities within the content and sequences for learning have been identified by mathematical experts. Some of this mathematical content students will use in their future jobs and life in the community. For example, a baker may need to know how to create a 4x5 array of rolls on a pan and someone who works in shipping may need to make decisions about the volume of a package. Some of the content may make the students better at problem solving in general as students learn to pose math questions and create solutions. Perhaps most importantly, our goal is to promote a full educational opportunity for all students. The challenge ahead is to make the content personally relevant and accessible to each student.

Quality Indicators. In all of the Instructional Resources, the following criteria are applied. Resources are developed, reviewed, field tested, and revised until the team considers them to meet all of these criteria.

Quality Indicators for Instructional Resources

➢ Promote Common Core State Standards
  ○ By using the Core Content Connectors
    ▪ Dually aligned with learning progressions and CCSS
➢ Set high expectations for all students
➢ Apply principles of universal design for learning
➢ Apply evidence-based teaching practices for students with SCD
➢ Use general curriculum resources and general education content experts' review
➢ Offer options for ALL students in the 1%
➢ Reflect same emphasis/ priorities being used for assessment in Work Group One
➢ Provide a teacher-friendly resource that promotes effective instruction

NCSC Instructional Resource Guide

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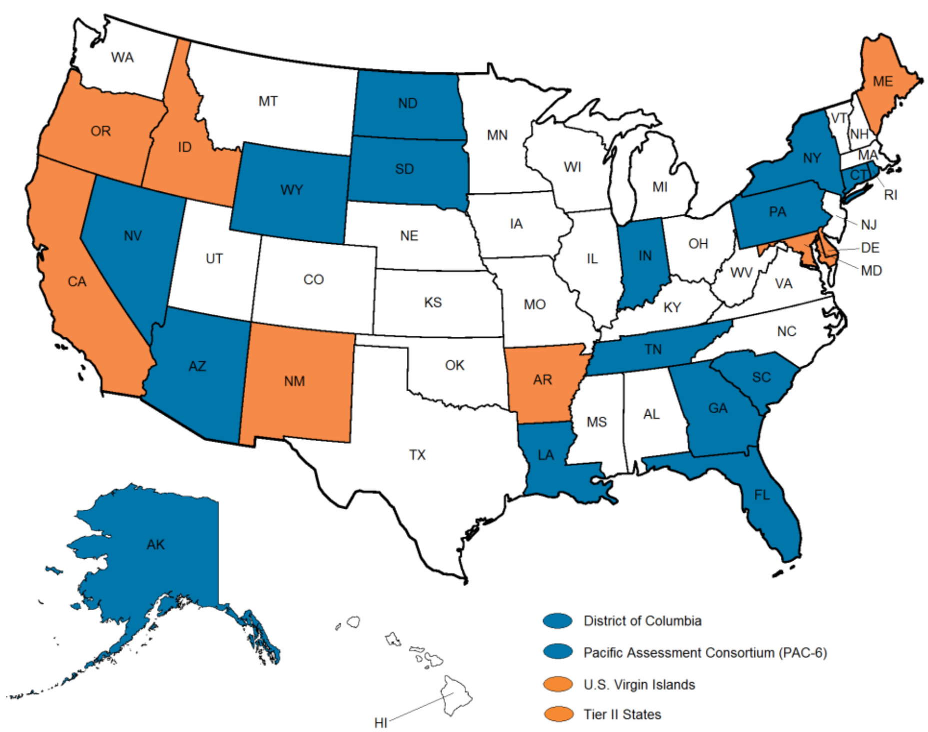

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NCSC Instructional Resource Guide

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January 2013

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Instructional Resource Guide on Prompting and Instructional Strategies

The purpose of the Instructional Resource Guide:
• To provide guidance for teachers regarding evidence-based prompting and instructional strategies to be used to teach students with significant disabilities • To serve as a companion document to the MASSIs (Math Activities with Scripted Systematic Instruction) and LASSIs (Language Arts Scripted Systematic Instruction) • To help educators build knowledge of the essential systematic instructional methods and prompting strategies that are used in the MASSIs and LASSIs to teach students targeted skills

Systematic Instruction

Teaching focused on specific, measurable responses that may either be discrete or a chained task, and that are established through the use of defined methods of prompting and feedback based on the principles and research of ABA.

Will include:

- Prompting
- Feedback
- Format of instruction
- Task Analysis
- Repeated Trial

Time Delay

There are two types of time delay, constant time delay and progressive time delay. This Instructional Resource Guide focuses on Constant Time Delay; however, it does provide a brief explanation of Progressive Time Delay.

Additional Prompting Strategies

There are additional prompting strategies that are not covered in this instructional resource guide that may be helpful when teaching your students. These strategies were not included because they are not used in the MASSIs or LASSIs. These include, but are not limited to most to least prompting, simultaneous prompting, and graduated guidance.

Additional Resources

This brief guide is meant for quick reference. The following are teacher-friendly resources for educators who would like to learn more about these procedures.

Collins, B. (2012). *Systematic instruction for students with moderate and severe disabilities.* Baltimore, MD: Paul H. Brookes.

Alberto, P., & Troutman, A. (2012). *Applied behavior analysis for teachers*. 9th Ed. Upper Saddle River, NJ: Pearson.

Finding a Response Mode

It is important to identify the best way for your student to show what they know in each lesson. Here are some options to consider:

Point to the correct response when given an array

The number of options in the array may vary depending on the student’s current skills. An array of 4 is often used with one correct answer, at least one plausible incorrect answer, and two other distractors. Be sure to vary the location of the correct answer in the array. This array can be placed on the students’ communication system.

Pull-off

Some students have difficulty pointing but may be able to make a selection when the responses are attached to a page. The array of 4 options is used, but the student pulls the correct response.

Eye gaze

Students who do not have the motor skills to point, but have vision, may be able to indicate the response by looking at the correct option. The array can be attached to each corner of a piece of see-through plexiglass (available from most hardware stores). By looking through the plexiglass, the teacher can see where the student focuses his or her eyes to indicate the answer.

Say or Type

Some students can verbalize the correct answer. This answer may be given after viewing an array of options or by generating the answer when asked a question. Other students may be able to generate the answer by typing a response. Saying or typing the answer provides students with the most flexibility to describe what they know.

Show

Some learning can be demonstrating through showing the answer. The student may be able to indicate the area of the rectangle by moving his or her hand across the shape. Or, a student may answer a comprehension question by pantomiming the answer.

Write or type on computer

Sometimes the student may be able to write the answer, for example, by writing the correct number in an equation or writing the name of the main character in a story.

Use material from the lesson

Students may be able to show the correct math answer by using a number card or plastic numbers or with other manipulatives. Similarly, in language arts, the student may use a picture on the page in the book or prop that is used with a story to answer a comprehension question.

Remember: the response mode needs to be something students can do without assistance once they learn the material.

Constant Time Delay (CTD)

CTD is a form of errorless learning that can be used with discrete responses (e.g., number ID; vocabulary words, matching). If a student makes a lot of errors through guessing, it may take longer to learn the response. CTD teaches the student to WAIT for help if unsure of the correct answer, but ANTICIPATE (answer before the prompt) when sure.

First, use a zero delay round to introduce the skill. Give the cue to respond and prompt together to ensure correct responding. The student can only make an error if he or she does not imitate this response (if this happens, a better prompt may be needed or the student may need to be reminded to attend closely).

After a few trials (or sessions), wait a few seconds before giving the prompt to allow the student to anticipate the correct answer.

Zero Delay Round

Provide the task direction and immediately give the controlling prompt to teach the child the correct response. Reinforce the child’s correct response.

For example (number identification):

1. Teacher says “Find three” while pointing to the number 3.
2. Student responds by pointing to the number 3.
3. Teacher reinforces the correct response by saying, “Good, that is three,” and records the data (prompted correct).

Time Delay Round

After several trials/sessions at zero delay, move to a 3-5 second delay (pick a delay time that is appropriate for your student to start responding, but do not vary that delay length).

The task direction is given (target stimulus); wait 3-5 seconds delay time for the student to respond.

If no response after delay, then the controlling prompt is used. After the student gives the correct response offer praise. Record Data (prompted correct: P).

If an incorrect response is given, provide error correction procedures (usually the controlling prompt to prompt a correct response) and remind the student to wait if not sure.

If multiple errors occur, return to the zero delay condition.

For example:

1. Teacher says “Find three” and waits 4 seconds (allowing the student to have a chance to answer).

2. IF the student independently points to 3, reinforce the correct response by saying “Good, that is three” and record the data (independent correct: “+”).

3. IF the student waits and does nothing, after 4 seconds the teacher points to the 3. After the student points to the 3, teacher records data (prompted correct: “P”).

4. IF the student points to the wrong answer, teacher immediately points to the correct answer, does not reinforce and records the data (error: “-“).

Sample Script for CTD (Teaching Expressive Symbol Identification)

Materials	Teacher Says/Does	Student Response	Teacher Feedback
Zero Delay Round (Complete multiple trials/days as needed at the zero delay round)			
Card with + on it: +	"What symbol is this? Plus"	"Plus"	"Good, this is the plus sign, we use it to add."
Card with = on it: =	"What symbol is this? Equal"	"Equal"	"Good, this is the equal sign, it means the same."
Card with - on it: -	"What symbol is this? Subtraction"	"Subtraction"	"Good, this is the subtraction sign, we use it to subtract."
4 Second Delay Round			
Card with + on it: +	"What symbol is this?" Wait 4 seconds.	Student responds "plus" before additional prompting.	"Good! You got it! This is the plus sign, which we use to add."
		Student responds incorrectly before additional prompting.	"Plus, this is the plus sign. If you don't know the answer, wait and I'll help you."
		Student waits (does not respond within 4 seconds).	"Plus, say plus. Good."
Card with = on it: =	"What symbol is this?" Wait 4 seconds.	Student responds "equal" before additional prompting.	"Good! You got it! This is the equal sign, it means the same."
		Student responds incorrectly before additional prompting.	"Equal, this is the equal sign. If you don't know the answer, wait and I'll help you."
		Student waits (does not respond within 4 seconds).	"Equal, say equal. Good."
Card with - on it: -	"What symbol is this?" Wait 4 seconds.	Student responds "subtraction" before additional prompting.	"Good! You got it! This is the subtraction sign, which we use to subtract."
		Student responds incorrectly before additional prompting.	"Subtraction, this is the subtraction sign. If you don't know the answer, wait and I'll help you."
		Student waits (does not respond within 4 seconds).	"Subtraction, say subtraction. Good."

Sample Script for CTD (Teaching Receptive Word Identification)

Materials	Teacher Says/Does	Student Response	Teacher Feedback
Zero Delay Round (Complete multiple trials/days as needed at the zero delay round)			
**Note: distracters can be made very different in the beginning (e.g., a blank card or a card with a picture of an unrelated item), but eventually should be similar items, such as cards other targeted symbols (e.g., cat, dog, or hat).* ***Note: Shuffle cards and distracters between every trial.*			
Card with the word cat on it and two distracters: cat dog hat	Point to word cat and say "Touch cat."	Student touches word cat.	"Good, that says cat."
Card with the word dog on it and two distracters: hat dog cat	Point to word dog and say "Touch dog."	Student touches the word dog.	"Good, that says dog."
Card with the word hat on it and two distracters: hat dog cat	Point to word hat and say "Touch hat."	Student touches the word hat.	"Good, that says hat."
4 Second Delay Round			
Card with the word cat on it and two distracters: dog hat cat	"Touch cat." Wait 4 seconds.	Student touches cat before additional prompting.	"Good! You got it! That says cat."
		Student responds incorrectly before additional prompting.	Point to the word cat. Say: "This is cat. If you don't know the answer, wait and I'll help you."
		Student waits (does not respond within 4 seconds).	Point to the word cat. Say: "This is cat." After they point say "Good."
Card with the word dog on it and two distracters: dog cat hat	"Touch dog." Wait 4 seconds.	Student touches dog before additional prompting.	"Good! You got it! That says dog."
		Student responds incorrectly before additional prompting.	Point to the word dog. Say: "This is dog. If you don't know the answer, wait and I'll help you."
		Student waits (does not respond within 4 seconds).	Point to the word dog. Say: "This is dog." After they point say "Good."
Card with the word hat on it and two distracters:	"Touch hat." Wait 4 seconds.	Student touches hat before additional	"Good! You got it! That says hat."

Materials	Teacher Says/Does	Student Response	Teacher Feedback
cat hat dog		prompting.	
		Student responds incorrectly before additional prompting.	Point to the word hat. Say: “This is hat. If you don’t know the answer, wait and I’ll help you.”
		Student waits (does not respond within 4 seconds).	Point to the word hat. Say: “This is hat.” After they point say “Good.”

Some Tips for Using Time Delay

What do I do if my student keeps guessing/ making errors?

Progressive Time Delay. If students begin to make errors whenever the teacher delays the prompt, it may be better to use Progressive Time Delay (PTD). In this approach, the prompt is delayed by a very small increment of time (e.g., 2 seconds). Then the delay is gradually and systematically lengthened, allowing the student more time to respond independently.

Examples:

- 0 seconds, 1 second, 2 seconds, 3 seconds
- 0 second, 2 seconds, 4 seconds, 6 seconds

The teacher can also use "Wait training." Begin with blank index cards and teach the student to point where you point (or say what you say) after waiting for a specified amount of time.

What do I do if my student always waits/ never anticipates a correct response?

Try using a longer delay interval.

More potent reinforcement for independent responses only may motivate the student to anticipate the response. Tell the student how to earn the reinforcer (answer without help).

What if the student does not imitate the prompt?

For some students who do not imitate a model, an alternative is to use physical guidance as the controlling prompt.

What if the response requires matching?

Give the student the card to be matched. When prompting, point to the correct answer on the array. The student places the card to indicate the match.

What if the student responds by eye gazing?

The prompt can still be pointing to the correct option. If this is not salient enough, leave your finger on the correct answer until the student selects it.

Can I use CTD with a chained response like calculator use?

Yes. On the first day model each response (each step of the task analysis) and have the student repeat it (e.g., point to the key on the calculator, don't actually press it. Let the student actually press it). Then on the time delay trials, wait the designated number of seconds before prompting each step.

For more ideas, see Additional Resources or consult with an expert in applied behavior analysis.

System of Least Prompts (also known as Least to Most Prompting)

Can be used with a task analysis or a chain of behaviors (e.g., entering a multistep equation into a calculator) or a discrete task (e.g., identifying numbers).

A hierarchy of prompts (with a time delay between each prompt) is used on each step of the task analysis (e.g., verbal, gesture/model, physical) until the student makes the targeted response.

Guidelines for Using System of Least Prompts

1. Select 3 – 4 prompts in the hierarchy (e.g., verbal, gesture/model, physical). Remember these prompts can be adapted for students with a range of sensory impairments.

   Examples:

   - Students with visual impairments: partial verbal, full verbal, physical
   - Students with hearing impairments: sign/gesture, model, physical

2. Provide the task direction/natural cue (e.g., “Use your calculator to solve the equation 8 x 12 = ?”)

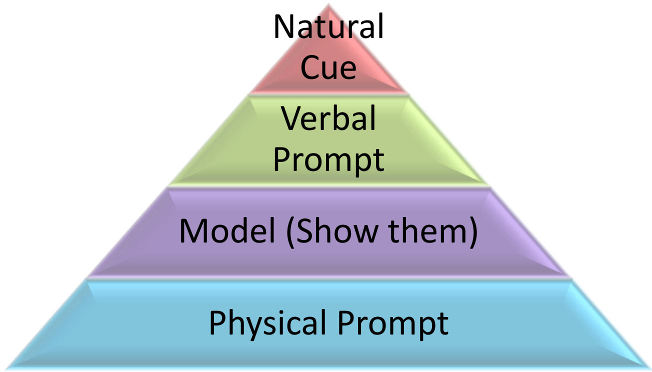
3. Always give the student an opportunity to make the correct response before providing any prompting on each step of the task analysis.

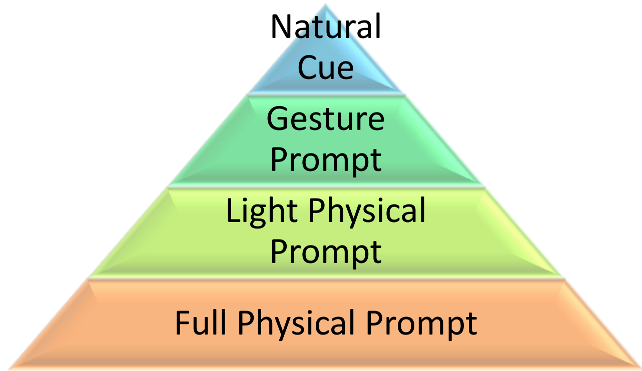
4. Use the least intrusive prompt first and progress to more intrusive prompts until the learner responds correctly (usually 3 to 5 second delay between prompts).

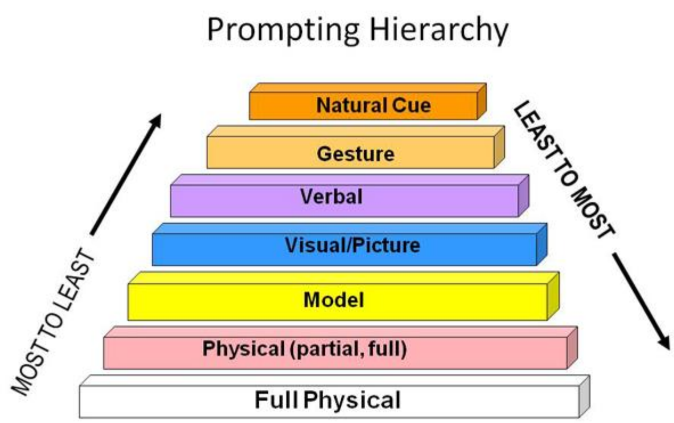
5. If the student makes an error, immediately provide the most intrusive prompt to ensure makes a correct response.

6. Encourage and praise the student after independent, correct responses.

Examples of Prompting Hierarchies







Sample Script for System of Least Prompts (Calculator Use)

Steps/Materials	Teacher Says/Does	Student Response	Teacher Feedback
**Note: In this example, if the student presses the wrong button, the teacher will have to clear the calculator and re-enter the equation up to the step the student was working on when the error occurred.*			
1. Student has worksheet with 8 x 12 = ______ on it, a calculator, and a pencil.	"Use your calculator to solve this equation: Eight times twelve equals?"	Correct: Student enters 8 into calculator.	"Good." Or wait for them to initiate the next step (2).
		Student makes an error.	Provide an immediate physical prompt (take their hand and help them press 8 in the calculator).
		Student does not respond.	Wait 3-5 seconds. Provide a verbal prompt "Push 8."
		No response after being given a verbal prompt.	Wait 3-5 seconds. Provide a gesture prompt (point to the 8 on the calculator).
		No response after being given a gesture prompt.	Wait 3-5 seconds. Provide a physical prompt (take their hand and help them press 8 in the calculator).
2. See above.	N/A (student should start the next step automatically after completing the previous step). Teacher can say "What's next?" or "Keep going."	Correct: Student enters the x into calculator.	"Good." Or wait for them to initiate the next step (3).
		Student makes an error.	Provide an immediate physical prompt (take their hand and help them press x in the calculator).
		Student does not respond.	Wait 3-5 seconds. Provide a verbal prompt "Push the x."
		No response after being given a verbal prompt.	Wait 3-5 seconds. Provide a gesture prompt (point to the x on calculator).
		No response after being given a gesture prompt.	Wait 3-5 seconds. Provide a physical prompt (take their hand and help them press x in the calculator).
3. See above.	N/A (student should start the next step automatically after completing the previous step). Teacher can say "What's next?" or "Keep going."	Correct: Student enters 12 into calculator.	"Good." Or wait for them to initiate the next step (4).
		Student makes an error.	Provide an immediate physical prompt (take their hand and help them press 12 in the calculator).
		Student does not respond.	Wait 3-5 seconds. Provide a verbal prompt "Push 12."
		No response after being given a verbal prompt.	Wait 3-5 seconds. Provide a gesture prompt (point to the 12 on the calculator).
		No response after being given a gesture prompt.	Wait 3-5 seconds. Provide a physical prompt (take their hand and help them press 12 in the calculator).
4. See above.	N/A (student should start the next step	Correct: Student enters = into calculator.	Or wait for them to initiate the next step (5).

Steps/Materials	Teacher Says/Does	Student Response	Teacher Feedback
	automatically after completing the previous step). Teacher can say "What's next?" or "Keep going."	Student makes an error.	Provide an immediate physical prompt (take their hand and help them press = in the calculator).
		Student does not respond.	Wait 3-5 seconds. Provide a verbal prompt "Push =."
		No response after being given a verbal prompt.	Wait 3-5 seconds. Provide a gesture prompt (point to the = on the calculator).
		No response after being given a gesture prompt.	Wait 3-5 seconds. Provide a physical prompt (take their hand and help them press = in the calculator).
5. See above.	"What is eight times twelve?"	Correct: Student writes/stamps/says/selects 96.	"Good work! Eight times twelve equals ninety-six."
		Student makes an error.	Provide an immediate physical prompt (take their hand and help them write/stamp/say/select 96).
		Student does not respond.	Wait 3-5 seconds. Provide a verbal prompt "Look at the calculator."
		No response after being given a verbal prompt.	Wait 3-5 seconds. Provide a gesture prompt (point to the 96 on the calculator).
		No response after being given a gesture prompt.	Wait 3-5 seconds. Provide a physical prompt (take their hand and help them write/stamp/ say/select 96).

Sample Script for System of Least Prompts (Text Based Literal Recall)

<table>
<tr><th>Materials</th><th>Teacher Says/Does</th><th>Student Response</th><th>Teacher Feedback</th></tr>
<tr><td colspan="4">***Note: Student either reads an appropriate leveled text or has the appropriate leveled text read to them prior to asking literal recall questions. For example:
It was early morning when Ben woke up in his racecar bed. He was hungry for breakfast so we walked into the kitchen. Ben’s mom was making pancakes. She put two pancakes with syrup and butter on his plate. Then she said “You better eat quickly, the bus comes at 8:00, and you don’t want to miss it.”
**Note: If needed, students may also have response options provided. Response options should include all types of possible responses (e.g., what, who, where, when, what doing both from the story and non-plausible options).</td></tr>
<tr><td rowspan="5">Student has entire text with adaptations if needed (e.g., Braille, picture symbols, objects, etc.).</td><td rowspan="5">Teacher asks what question: “What was mom cooking?”</td><td>Correct: Student responds “pancakes.”</td><td>“Good. She was making pancakes!”</td></tr>
<tr><td>Student makes an error.</td><td>Provide an immediate verbal model “Pancakes.” Have student repeat the model.</td></tr>
<tr><td>Student does not respond.</td><td>No response after sentence is reread. Wait 3-5 seconds. Remind the student of the rule, “What is a thing. Listen for a thing.” Reread the text to the student and ask the question again.</td></tr>
<tr><td>No response after rereading the text.</td><td>Wait 3-5 seconds. Reread only the sentence with the answer in it.</td></tr>
<tr><td>No response after sentence is reread.</td><td>Wait 3-5 seconds. Provide a verbal model “Pancakes.” Have student repeat the model.</td></tr>
<tr><td rowspan="5">Student has entire text with adaptations if needed (e.g., Braille, picture symbols, objects, etc.).</td><td rowspan="5">Teacher asks who question: “Who woke up in a race car bed?”</td><td>Correct: Student responds “Ben.”</td><td>“Good. Ben woke up in a race car bed!”</td></tr>
<tr><td>Student makes an error.</td><td>Provide an immediate verbal model “Ben.” Have student repeat the model.</td></tr>
<tr><td>Student does not respond.</td><td>Wait 3-5 seconds. Remind the student of the rule, “Who is a person. Listen for a person.” Reread the text to the student and ask the question again.</td></tr>
<tr><td>No response after rereading the text.</td><td>Wait 3-5 seconds. Reread only the sentence with the answer in it.</td></tr>
<tr><td>No response after sentence is reread.</td><td>Wait 3-5 seconds. Provide a verbal model “Ben.” Have student repeat the model.</td></tr>
<tr><td>Student has entire text with adaptations</td><td>Teacher asks where question: “Where</td><td>Correct: Student responds “kitchen.”</td><td>“Good. Mom was in the kitchen!”</td></tr>
</table>

Materials	Teacher Says/Does	Student Response	Teacher Feedback
if needed (e.g., Braille, picture symbols, objects, etc.).	was Mom?"	Student makes an error.	Provide an immediate verbal model "Kitchen." Have student repeat the model.
		Student does not respond. Wait 3-5 seconds. Remind the student of the rule, "Where is a place. Listen for a place." Reread the text to the student and ask the question again.	Wait 3-5 seconds. Reread only the sentence with the answer in it.
		No response after rereading the text.	Wait 3-5 seconds. Reread only the sentence with the answer in it.
		No response after sentence is reread.	Wait 3-5 seconds. Provide a verbal model "Kitchen." Have student repeat the model.
Student has entire text with adaptations if needed (e.g., Braille, picture symbols, objects, etc.).	Teacher asks where question: "What was Mom doing?"	Correct: Student responds "making pancakes."	"Good. She was making pancakes!"
		Student makes an error.	Provide an immediate verbal model "Making pancakes." Have student repeat the model.
		Student does not respond.	No response after sentence is reread. Wait 3-5 seconds. Remind the student of the rule, "What doing is an action. Listen for an action." Reread the text to the student and ask the question again.
		No response after rereading the text.	Wait 3-5 seconds. Reread only the sentence with the answer in it.
		No response after sentence is reread.	Wait 3-5 seconds. Provide a verbal model "Making pancakes." Have student repeat the model.
Student has entire text with adaptations if needed (e.g., Braille, picture symbols, objects, etc.).	Teacher asks when question: "When is the bus coming?"	Correct: Student responds "8:00."	"Good. The bus comes at 8:00!"
		Student makes an error.	Provide an immediate verbal model "8:00." Have student repeat the model.
		Student does not respond.	No response after sentence is reread. Wait 3-5 seconds. Remind the student of the rule, "When is a time. Listen for a time." Reread the text to the student and ask the question again.
		No response after rereading the text.	Wait 3-5 seconds. Reread only the sentence with the answer in it.
		No response after sentence is reread.	Wait 3-5 seconds. Provide a verbal model "8:00." Have student repeat the model.

Model, Lead, Test

Model, lead, test is also known as “I do,” “we do,” “you do.”

It is a form of scaffolding that begins with teacher modeling and guidance to support student learning.

As the student progresses, the teacher should provide less support and helps students gain independence with the skill or task.

Can be especially helpful when teaching students academic skills with multiple steps, such as using the Pythagorean Theorem or completing a graphic organizer.

Steps to Using Model, Lead, Test

1. First (Model or “I do”), the teacher models the skill/task/strategy while students watch.
2. Next (Lead or “we do”), the teacher leads the students to use the skill/task/strategy simultaneously with the teacher.
3. Last (Test or “you do”), the teacher has the students complete the skill/task/strategy independently and observes to see if they responded correctly.

Guidelines for Using Model, Lead, Test

Student(s) must respond with a predetermined level of accuracy during the test phase to consider the skill mastered prior to moving on. For example, 80% accuracy for 2 consecutive sessions.

If students make an error, a correction is provided in the form of modeling the correct response, then having the student correctly perform the step.

Model, Lead, Test is not appropriate for students who are not able to observe someone perform an action and attempt to imitate that action (e.g., students without imitation skills).

You can easily test this by observing the student performing a few behaviors/movements (e.g., raising their hand, clapping their hands, and folding their hands). The point of this step is to ensure the student is physically capable of performing the behavior.

Then, secure the student’s attention and say “do this” while completing the action (e.g., clap your hands). Do NOT say “clap your hands.”

If the student attempts to imitate the action (e.g., claps their hands) then model, lead, test may be an appropriate teaching strategy for that student.

Sample Script for Model, Lead, Test (Measuring Length in Inches with Ruler)

Steps/Materials	Teacher Says/Does	Student Response	Teacher Feedback
Model			
Teacher: 1. Pencil (to measure) 2. Clearly labeled ruler	“We can use a ruler to measure the length of an item. Watch me measure the length of this pencil.” Line up the ruler to the pencil and say, “First, I line up the ruler alongside the pencil, starting at zero.”	Student watches.	“Good watching me.”
See above.	Move your finger to the end of the pencil and point to the corresponding number on the ruler and say, “Then I move my finger to the end of the pencil.”	Student watches.	“Good watching me.”
See above.	“Now I read the number on the ruler that is closest to the end of the pencil. Look this pencil measures seven inches.”	Student watches.	“Good watching me.”
Lead			
Teacher and Student: 1. Marker (to measure) 2. Clearly labeled ruler	“Now, let’s measure the marker. Let’s do it together, watch me and do what I do.” Line up the ruler to the marker and say “First, line up the ruler alongside the marker, starting at zero.”	Student lines up the ruler alongside the marker, starting at zero.	“Good lining up the marker with the zero on your ruler.”
See above.	Move your finger to the end of the marker and point to the corresponding number on the ruler and say, “Then move your finger to the end of the marker.”	Student moves their finger to the end of their marker.	“Good moving your finger to the end of your marker.”
See above.	“Now read the closest number on the ruler. Look this marker measures six inches. How long is your marker?”	Student correctly says/selects/indicates the length of their marker.	“Great work measuring the marker!”
Test			
Students: 1. Spoon (to measure) or other object 2. Clearly labeled ruler	“Ok, now it’s your turn. Measure the spoon.”	Student lines up the spoon alongside the ruler, starting at zero.	“Good lining up the spoon with the zero on your ruler.”
		Student makes an incorrect response or no response.	“Watch me” and model the correct response, then have the student complete it correctly (not scored).
See above.	N/A	Student moves their finger to the end of their spoon.	“Good moving your finger to the end of your spoon.”

Steps/Materials	Teacher Says/Does	Student Response	Teacher Feedback
		Student makes an incorrect response or no response.	“Watch me” and model the correct response, then have the student complete it correctly (not scored).
See above.	N/A	Student correctly says/selects/indicates the length of their spoon.	“Great work measuring the spoon!”
		Student makes an incorrect response or no response.	“Watch me” and model the correct response, then have the student complete it correctly (not scored).

Example/Non-Example Training

Most behaviors need to be performed in response to a variety of different cues, situations, and stimuli. Using example/non-example training is one way to approach teaching students the concepts in a way that will generalize to all of the different cues, situations, and stimuli where they might need it.

Teaching sufficient examples is important when teaching students to respond to all possible demonstrations of a concept.

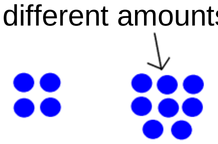
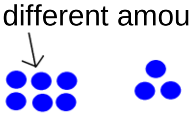
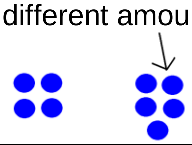
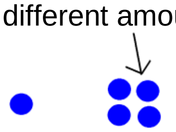
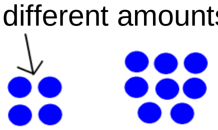
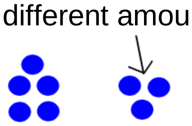
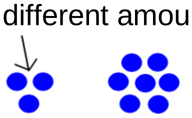
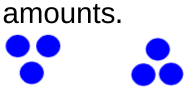
Teaching non-examples is how you teach students when *not* to display the target behavior you are trying to teach. This is important to determine whether or not they truly understand a concept.

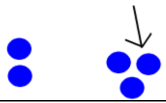
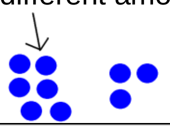
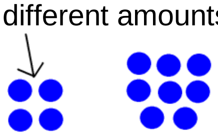
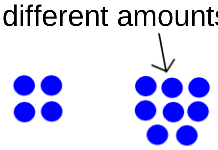
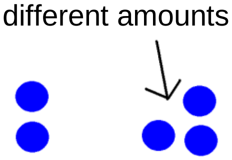
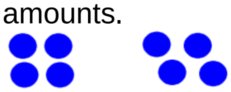
For example: If you teach a student to respond “three” when shown the written number 3, but they also say “three” when shown the numbers 1-9, then they have not mastered the concept of 3.

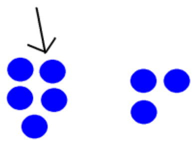
Guidelines for Using Example/Non-Example Training

- Examples and non-examples should be intermixed throughout the teaching process.
- Examples should include a sufficient number of examples that encompass as many possible features of the concept so that students can generalize to untrained examples.
- Generally, generalization of the concept is more likely to successfully occur when more examples and non-examples are used during instruction.
- The actual number of examples and non-examples that need to be included vary according to the skill being taught and the needs of the individual student.
- Non-examples are not always clear enough or occur too infrequently in the natural environment for students to learn when *not* to display the behavior; therefore, explicitly teaching them can be helpful.
- Non-examples should be presented and taught by examining how closely they differ from the example. The most effective non-examples are close-in non-examples that have minimal differences from the actual example; this helps the student discriminate with precision.
- When teaching examples and non-examples, vary only the relevant feature during any particular session. Non-relevant features can be varied from session to session, but not within a session.

Sample Script for Example, Non-Example Training (Teaching Concept of <)

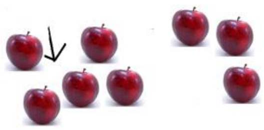
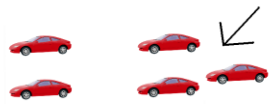
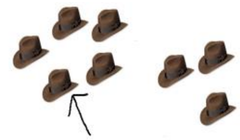
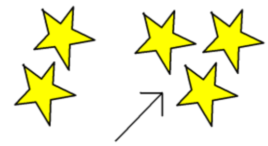
Materials	Teacher Says/Does	Student Response	Teacher Feedback
Examples (Vary only the relevant feature)			
Picture of two different amounts. 	Point to the larger amount and say “This is greater.”	Student watches.	“Good watching.” Or no response.
Picture of two different amounts. 	Point to the larger amount and say “This is greater.”	Student watches.	“Good watching.” Or no response.
Picture of two different amounts. 	Point to the larger amount and say “This is greater.”	Student watches.	“Good watching.” Or no response.
Picture of two different amounts. 	Point to the larger amount and say “This is greater.”	Student watches.	“Good watching.” Or no response.
Interspersed Examples and Non-Examples (Randomize order of trials)			
Picture of two different amounts. 	Point to the smaller amount and say “Not greater.”	Student watches.	“Good watching.” Or no response.
Picture of two different amounts. 	Point to the smaller amount and say “Not greater.”	Student watches.	“Good watching.” Or no response.
Picture of two different amounts. 	Point to the larger amount and say “Not greater.”	Student watches.	“Good watching.” Or no response.
Picture of two same amounts. 	Point to both amounts and say “Not greater.”	Student watches.	“Good watching.” Or no response.

Materials	Teacher Says/Does	Student Response	Teacher Feedback
Picture of two different amounts. 	Point to the larger amount and say “This is greater.”	Student watches.	“Good watching.” Or no response.
Picture of two same amounts. 	Point to both amounts and say “Not greater.”	Student watches.	“Good watching.” Or no response.
Picture of two different amounts. 	Point to the larger amount and say “This is greater.”	Student watches.	“Good watching.” Or no response.
Student Responses (Randomize order of trials)			
Picture of two different amounts. 	Point to the smaller amount and say “Ok, now it’s your turn. Is this a greater or not greater?”	Student responds “not greater” vocally, by using symbols, or an assistive technology device.	“Good, this is not greater.”
		Student makes an incorrect response or no response.	“Not greater. Repeat after me… not greater.” Then repeat 3 trials of you demonstrating greater/not greater before moving to the next trial (not scored).
Picture of two different amounts. 	Point to the larger amount and say “Ok, now it’s your turn. Is this a greater or not greater?”	Student responds “greater” vocally, by using symbols, or an assistive technology device.	“Good, this is greater.”
		Student makes an incorrect response or no response.	“Greater. Repeat after me… greater.” Then repeat 3 trials of you demonstrating greater/not greater before moving to the next trial (not scored).
Picture of two different amounts. 	Point to the larger amount and say “Ok, now it’s your turn. Is this a greater or not greater?”	Student responds “greater” vocally, by using symbols, or an assistive technology device.	“Good, this is greater.”
		Student makes an incorrect response or no response.	“Greater. Repeat after me… greater.” Then repeat 3 trials of you demonstrating greater/not greater before moving to the next trial (not scored).
Picture of two same amounts. 	Point to one amount and say “Ok, now it’s your turn. Is this a greater or not greater?”	Student responds “not greater” vocally, by using symbols, or an assistive technology device.	“Good, this is not greater.”

Materials	Teacher Says/Does	Student Response	Teacher Feedback
		Student makes an incorrect response or no response.	“Not greater. Repeat after me… not greater.” Then repeat 3 trials of you demonstrating greater/not greater before moving to the next trial (not scored).
Picture of two different amounts. 	Point to the larger amount and say “Ok, now it’s your turn. Is this a greater or not greater?”	Student responds “greater” vocally, by using symbols, or an assistive technology device.	“Good, this is greater.”
		Student makes an incorrect response or no response.	“Greater. Repeat after me… greater.” Then repeat 3 trials of you demonstrating greater/not greater before moving to the next trial (not scored).

Generalization When Using Example, Non-Example Training

In order to promote generalization, use different objects/pictures on different days (e.g., on day two use apples, day three use cars, day four use hats, day five use star stickers). Do not vary objects within a session (e.g., if you are using apples, continue to use apples for that entire session). Use the same script as above, simply using the other objects.

Once the student masters greater than in the above format now introduce new formats. These include greater than with volume and greater than with numbers.

5 6

9 2

Once the student masters greater than in the above format, now introduce the symbol (>). Teach students to identify the amount that is greater and turn the opening of the symbol to the greater than amount.

Only after the student has fully mastered the concept of greater, then introduce the concept of less than (e.g., do not teach opposing concepts simultaneously). Use the same procedures as above (less than, not less than) to teach less than; however, if you are showing students a trial of “not less than” you should accept a response of either “not less than” or “greater.”

Sample Script for Example, Non-Example Training (Teaching Setting)

Materials	Teacher Says/Does	Student Response	Teacher Feedback
Examples (Vary only the relevant feature)			
**Note: Student either reads an appropriate leveled text or has the appropriate leveled text read to them prior to teaching setting. For example:* It was early morning when Ben woke up in his racecar bed. He was hungry for breakfast so we walked into the kitchen. Ben's mom was making pancakes. She put two pancakes with syrup and butter on his plate. Then she said "You better eat quickly, the bus comes at 8:00, and you don't want to miss it." Ben ate his pancakes and ran outside. He got on the bus and rode to school. He was excited about school because there was a book fair going on in the library. ***Note: If needed, students may also have response options provided. Response options should include all types of possible responses (e.g., plausible and non-plausible).*			
Picture or symbol for kitchen with the word "kitchen."	Hold up the kitchen visual. "Setting is a place that is in a story. The kitchen is a setting in our story."	Student watches.	"Good watching." Or no response.
Picture or symbol for outside with the word "outside."	Hold up the outside visual. "Outside is a setting in our story."	Student watches.	"Good watching." Or no response.
Picture or symbol for school with the word "school."	Hold up the school visual. "School is a setting in our story."	Student watches.	"Good watching." Or no response.
Picture or symbol for library with the word "library."	Hold up the library visual. "Library is a setting in our story."	Student watches.	"Good watching." Or no response.
Interspersed Examples and Non-Examples (Randomize order of trials)			
Picture or symbol for Ben with the word "Ben."	Hold up the Ben visual. "Ben is NOT a setting in our story."	Student watches.	"Good watching." Or no response.
Picture or symbol for pancakes with the word "pancakes."	Hold up the pancakes visual. "Pancakes are NOT a setting in our story."	Student watches.	"Good watching." Or no response.
Picture or symbol for outside with the word "outside."	Hold up the outside visual. "Outside is a setting in our story."	Student watches.	"Good watching." Or no response.
Picture or symbol for gym with the word "gym."	Hold up the gym visual. "Gym is NOT a setting in our story."	Student watches.	"Good watching." Or no response.
Picture or symbol for kitchen with the word "kitchen."	Hold up the kitchen visual. "The kitchen is a setting in our story."	Student watches.	"Good watching." Or no response.
Picture or symbol for mom with the word "mom."	Hold up the mom visual. "Mom is NOT a setting in our story."	Student watches.	"Good watching." Or no response.

Materials	Teacher Says/Does	Student Response	Teacher Feedback
Picture or symbol for library with the word “library.”	Hold up the library visual. “Library is a setting in our story.”	Student watches.	“Good watching.” Or no response.
Student Responses (Randomize order of trials)			
Picture or symbol for books with the word “books.”	Hold up the books visual. “Okay, now your turn. Are books a setting in our story?”	Student responds “not a setting” vocally, by using symbols, or an assistive technology device.	“Good, books are not a setting.”
		Student makes an incorrect response or no response.	“Not a setting. Books are not a place in our story. Repeat after me… not a setting.” Then repeat 3 trials of you demonstrating setting/not a setting before moving to the next trial (not scored).
Picture or symbol for school with the word “school.”	Hold up the school visual. “Is school a setting in our story?”	Student responds “setting” vocally, by using symbols, or an assistive technology device.	“Good, the school is a setting.”
		Student makes an incorrect response or no response.	“Setting. The school is a place in our story. Repeat after me… setting.” Then repeat 3 trials of you demonstrating setting/not setting before moving to the next trial (not scored).
Picture or symbol for kitchen with the word “kitchen.”	Hold up the kitchen visual. “Is kitchen a setting in our story?”	Student responds “setting” vocally, by using symbols, or an assistive technology device.	“Good, the kitchen is a setting.”
		Student makes an incorrect response or no response.	“Setting. The kitchen is a place in our story. Repeat after me… setting.” Then repeat 3 trials of you demonstrating setting/not setting before moving to the next trial (not scored).
Picture or symbol for park with the word “park.”	Hold up the park visual. “Is park a setting in our story?”	Student responds “not a setting” vocally, by using symbols, or an assistive technology device.	“Good, the park is not a setting.”
		Student makes an incorrect response or no response.	“Not a setting. The park is not a place in our story. Repeat after me… not a setting.” Then repeat 3 trials of you demonstrating setting/not a setting before moving to the next trial (not scored).

Materials	Teacher Says/Does	Student Response	Teacher Feedback
Picture or symbol for library with the word "library."	Hold up the library visual. "Is library a setting in our story?"	Student responds "setting" vocally, by using symbols, or an assistive technology device.	"Good, the library is a setting."
		Student makes an incorrect response or no response.	"Setting. The library is a place in the story. Repeat after me… setting." Then repeat 3 trials of you demonstrating setting/not setting before moving to the next trial (not scored).

References

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National Center and State Collaborative

NCSC Equations Content Module

National Center and State Collaborative

The National Center and State Collaborative (NCSC) is applying the lessons learned from the past decade of research on alternate assessments based on alternate achievement standards (AA-AAS) to develop a multi-state comprehensive assessment system for students with significant cognitive disabilities. The project draws on a strong research base to develop an AA-AAS that is built from the ground up on powerful validity arguments linked to clear learning outcomes and defensible assessment results, to complement the work of the Race to the Top Common State Assessment Program (RTTA) consortia.

Our long-term goal is to ensure that students with significant cognitive disabilities achieve increasingly higher academic outcomes and leave high school ready for post-secondary options. A well-designed summative assessment alone is insufficient to achieve that goal. Thus, NCSC is developing a full system intended to support educators, which includes formative assessment tools and strategies, professional development on appropriate interim uses of data for progress monitoring, and management systems to ease the burdens of administration and documentation. All partners share a commitment to the research-to-practice focus of the project and the development of a comprehensive model of curriculum, instruction, assessment, and supportive professional development. These supports will improve the alignment of the entire system and strengthen the validity of inferences of the system of assessments.

IDEAs that Work
U.S. Office of Special Education Programs

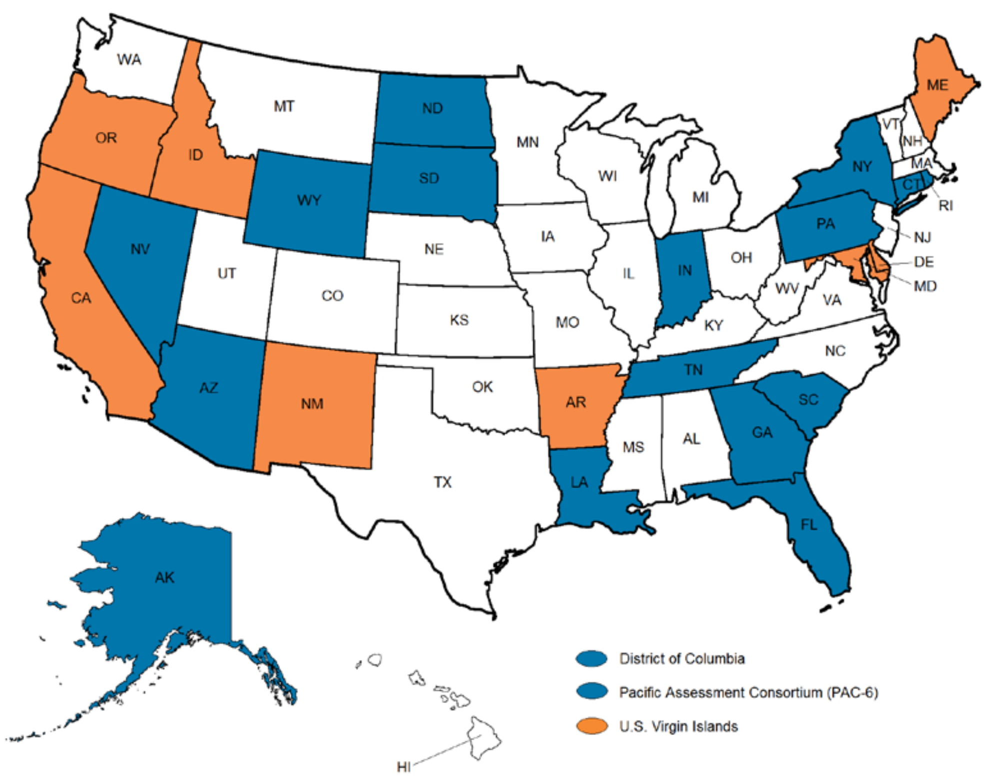
The contents of this resource were developed as part of the National Center and State Collaborative for a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made.

National Center and State Collaborative

NCSC is a collaborative of 18 states and five organizations.

The states include (shown in blue on map): Alaska, Arizona, Connecticut, District of Columbia, Florida, Georgia, Indiana, Louisiana, Nevada, New York, North Dakota, Pacific Assessment Consortium (PAC-6)[1], Pennsylvania, Rhode Island, South Carolina, South Dakota, Tennessee, and Wyoming.

Tier II states are partners in curriculum, instruction, and professional development implementation but are not part of the assessment development work. They are (shown in orange on map): Arkansas, California, Delaware, Idaho, Maine, Maryland, New Mexico, Oregon, and U.S. Virgin Islands.



[1] The Pacific Assessment Consortium (including the entities of American Samoa, Commonwealth of the Northern Mariana Islands, Federated States of Micronesia, Guam, Republic of Palau, and Republic of the Marshall Islands) partner with NCSC as one state, led by the University of Guam Center for Excellence in Developmental Disabilities Education, Research, and Service (CEDDERS).

National Center and State Collaborative

The five partner organizations include: The National Center on Educational Outcomes (NCEO) at the University of Minnesota, The National Center for the Improvement of Educational Assessment (Center for Assessment), The University of North Carolina at Charlotte, The University of Kentucky, and edCount, LLC.

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NCSC Equations Content Module

January 2013

Reposted April 26, 2013

Table of Contents

Plot the course

The rationale

Everyday people use linear equations to problems solve in their life. For example, ever wondered how many miles you could drive on a certain number of gallons of gas when planning a road trip? In addition to everyday activities, there are many jobs that require a firm understanding of linear equations such as jobs in construction.

Module Goal

The goal of this module is to provide detailed instruction on the more difficult concepts within linear equations to teachers of students with disabilities at the middle and high school level. This module promotes a mathematical understanding of these concepts so that a teacher can begin to plan how to teach the concepts to students. Additionally, this module will provide instructors with potential adaptations and modifications to consider when designing materials and instruction for students with severe disabilities.

Module Objectives

After viewing the content module, teachers will:

1. Identify the type of slope (positive, negative, zero, or undefined) when provided a graph of a linear equation.
2. Find the slope of a line when given the coordinates for two points on that line.
3. Determine the equation for a line when given the slope of a line and one point on the line.

Note: Throughout this module you will see the following icons

 3rd

 4th

 5th

 6th

 7th

 8th

 High School

These icons highlight Core Content Connectors that are addressed in a Simple Activities with Scripted Systematic Instruction (SASSI) lesson plan.

Time for take off

Understanding the vocabulary used within linear equations is important for both teachers and students in planning and implementing math lessons. As a teacher, knowing and using the mathematical terms not only ensures your instruction stays true to the math content, but will also help with collaborating with other math teachers or content experts. When choosing which vocabulary to teach, it is most important that the teacher selects the most salient, important, or most frequently used vocabulary for each lesson.

Below you will find a list of vocabulary included within this module. It may or may not be necessary to provide instruction for all terms as students may have learned them previously. Linear equations are mostly covered in middle school so vocabulary for this content module has been combined. If you are a high school teacher and are not confident your students know some of these vocabulary terms, you may want to review and teach some unknown terms in the focus and review part of your lesson plan.

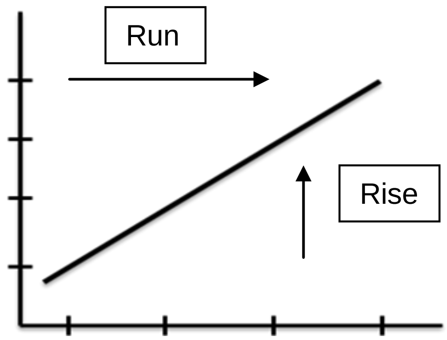
While providing vocabulary instruction, you may consider including pictures or objects to make the instruction more concrete for students with disabilities (See Ideas to support vocabulary learning below).

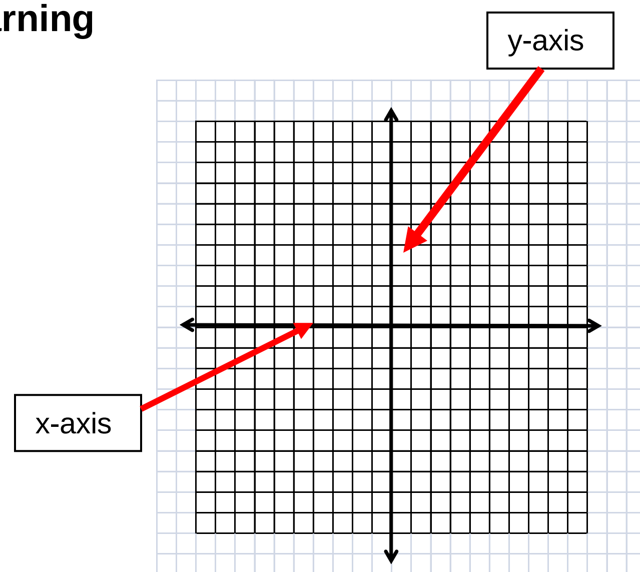
Vocabulary

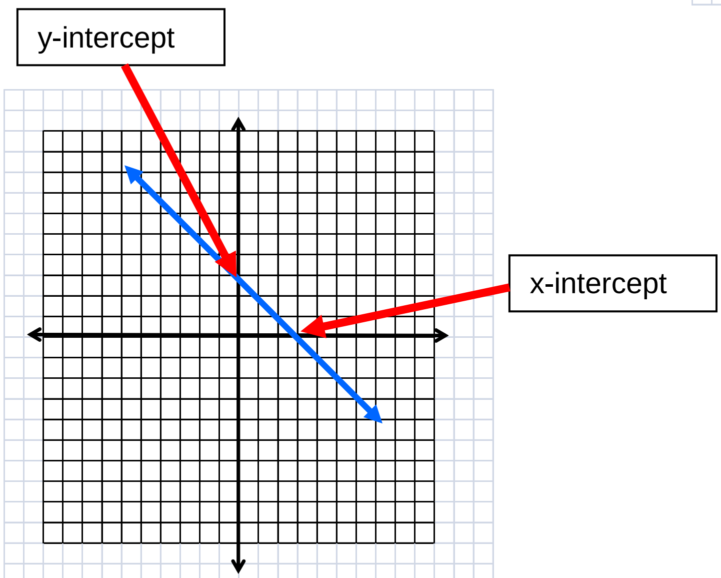
- Variable- a letter that represents a value
- Terms- expressions that are separated by a plus or minus sign (e.g., 2t-3b=)
  - Like terms- terms that have the same variable (e.g., 2t and 3t are like terms)
- Linear equation- an equation whose solution falls on a line when graphed
- Rate of change- a ratio that compares change in a dependent variable in relation to a change in the independent variable
- Slope- the slope of a line is the ratio of rise over run for any two points on that line
- Rise- difference between two y-values on a line
- Run- difference between two x-values on a line
- X-intercept- the point where a line crosses the x axis
- Y-intercept- the point where a line crosses the y-axis

Ideas to support vocabulary learning

- Include pictorial representations







Floating on Air

Before you can begin teaching linear equations, you need a deep understanding of these mathematical concepts. Some of these concepts may be familiar to you. Below is a list of skills that should be covered at each grade level. For concepts that you need more information about, please view the accompanying PowerPoint presentations that will walk you through an example as well as make some suggestions for instruction.

NOTE: Icons designate Core Content Connectors addressed within a developed SASSI

Middle and High School

In middle school skills include:

- 6.PRF.2a4 Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation
-  6.PRF.1d1 Solve real-world single step linear equations

(Insert solving linear equations PowerPoint presentation here)

- 6.PRF.2b2 Using provided table with numerical patterns, form ordered pairs
- 7-8.NO.3c5 Explain each step to solve a problem (e.g., explain how to solve-multi-step equation)
-  7.PRF.1g2 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities
- 8.PRF.2c1 Given two graphs, describe the function as linear and not linear

(Insert interpreting graphs PowerPoint presentation here)

- 8.PRF.1f2 Describe or select the relationship between the two quantities given a line graph of a situation
- 8.PRF.1g3 Solve linear equations with 1 variable
- 8.PRF.2e2 Identify the rate of change (slope) and initial value (y-intercept) from graphs

In high school skills include:

- H.PRF.1a1 Interpret the rate of change using graphical representations
- H.PRF.2a1 Translate an algebraic expression into a word problem

(Insert linear equations and word problems PowerPoint here)

-  H.PRF.1b1 In a linear situation using graphs or numbers, predicts the change in rate based on a given change in one variable
- H.PRF.2b1 Translate a real-world problem into a one variable linear equation
- H.PRF.1c1 Select the appropriate graphical representation of linear model based on real-world events
- H.PRF.2b2 Solve equations with one or two variables using equations or graphs

Great! Now that you have viewed the PowerPoint presentations most useful to you, the next section will provide some ideas to consider when planning for universal design for learning.

Slope-Intercept and Point-Slope form

The contents of this content module were developed by special educator Bethany Smith, MEd and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

Different forms of linear equations

- At times, linear equations may be given or a problem may ask you to rewrite an equation in two different forms

  - Slope-intercept form
    - You can find the slope of a line when given the coordinates of two points on the line
  - Point-intercept form
    - You can find the equation of a line when given a point on the line and its slope

Slope intercept form

- Slope intercept form

y-intercept

$$y = mx + b$$

slope

So, for the equation $y=3x+2$

- The slope is $\frac{3}{1}$
  - That means as y increases by 3; x increases by 1
- The y-intercept is 2
  - That means if you extend the line, it will cross the y-axis at 2

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Rewriting to slope-intercept form

- Rewrite the equation to slope-intercept form

8x= 5y

Step 1: Re-write so y is on the left

$$5y = 8x$$

$$\frac{5y}{5y} = \frac{8x}{5y}$$

Step 2: Divide both sides by 5y to isolate the variable

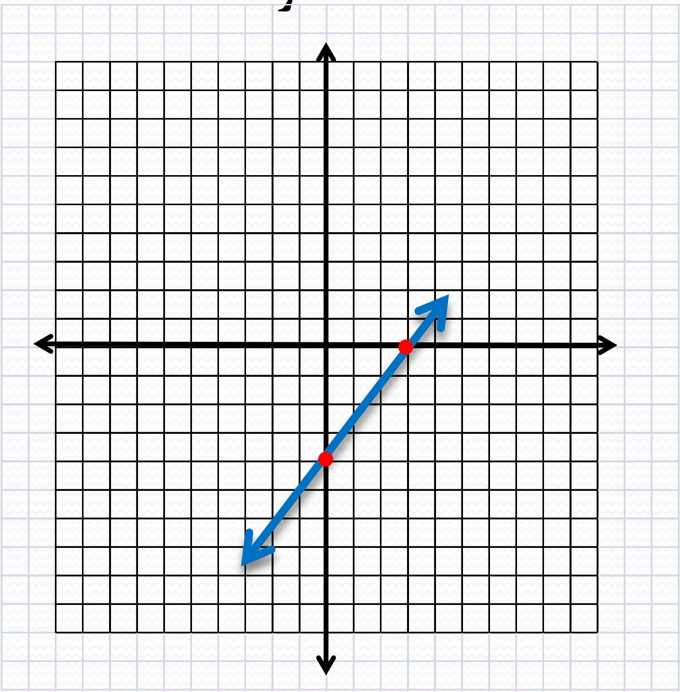
$$y = \frac{8}{5}x + 0$$

Step 3: Add y-intercept of 0 because none was provided

Intercepts

- X-intercept is where the line crosses the x-axis
- Y-intercept is where the line crosses the y-axis

In this example the x- intercept is 3 and the y-intercept is -4



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Finding intercepts: an example

- Problem: Find the x intercept if y=0 for the following equation

$$3x + 4y = 12$$

$$3x + 4(0) = 12$$

Step 1: Fill in the given y value

$$3x = 12$$

Step 2: isolate the variable by dividing both sides of the equation by 3

$$\frac{3x}{3} = \frac{12}{3}$$

$$x = 4$$

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Point-slope form

slope

$$y-y_1=m(x-x_1)$$

Point on the line

For the equation $(y-9)=3(x+8)$

- The slope of the line is $\frac{3}{1}$
- There is a point on the line with the coordinates (8,9)

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Finding point-slope form: An example

- Write the point-slope form of the line described below
  - The line with slope 4 that passes through (-2,3)

Point-slope form: $y-3=4(x+2)$

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Ideas for application

- If the floor in your classroom is made of 1ft by 1ft tiles, create a graph on your floor. Allow students to explore ordered pairs, finding intercepts, and graphing equations by physically moving around

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Making connections

- Slope-intercept form and point-slope form addresses the middle and high school Core Content Connectors of
  - 6.PRF.1d1 Solve real-world single step linear equations
  - 7.PRF.1g2 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities
  - 8.PRF.1g3 Solve linear equations with 1 variable
  - H.PRF.1a Interpret the rate of change using graphical representations
  - H.PRF.2a Translate an algebraic expression into a word problem
  - H.PRF.2b1 Translate a real-world problem into a one variable linear equation
  - H.PRF.2b2 Solve equations with one or two variables using equations or graphs

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An Overview of Linear Equations

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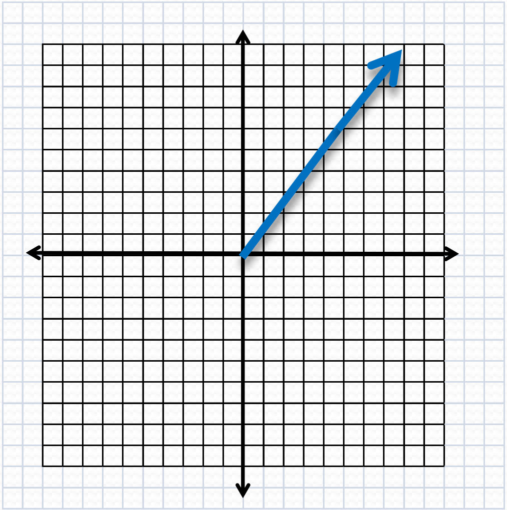
What is a linear equation

- If an equation is linear, then a change in x-values correspond to a change in y-values
  - In this example, when the x-values increases by 1, so does the y-values

Notice that the table satisfy a linear equation only if the differences are constant

Ordered pairs

X-values	Y-values
0	0
1	1
2	2
3	3



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Rate of change and Slope

- Rate of change is the ratio that compares the amount of change in a dependent variable in comparison to the change in the independent variable

- In a linear equation, this rate of change is constant and called slope
  - Slope is calculated by finding the ratio of the rise to run $\frac{rise}{run}$
    - Rise- difference in the y-values (vertical)
    - Run- difference in the x-values (horizontal)

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Is it linear?

One way to tell if an equation is linear is to determine whether or not the rate of change is constant. Below are two tables, one shows a constant rate of change and one does not

$y=3x-4$

+1 +1 +1

x	0	1	2	3
y	-4	-1	2	5

+3 +3 +3

Linear

$y=-x^2$

+1 +1 +1

x	**-2**	**-1**	**0**	**1**
	-4	-1	0	-1

+3 +1 +3

NOT linear

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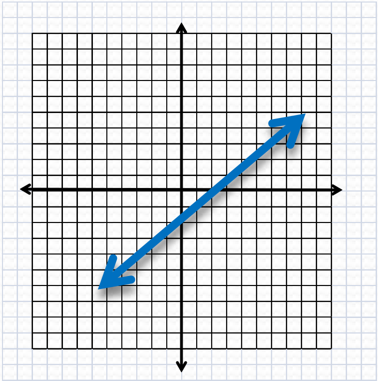
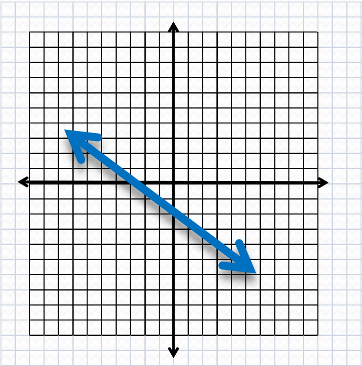
Slopes

- The slope of a line can be positive, negative, zero, or undefined

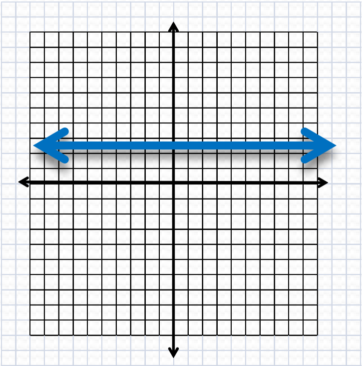
positive



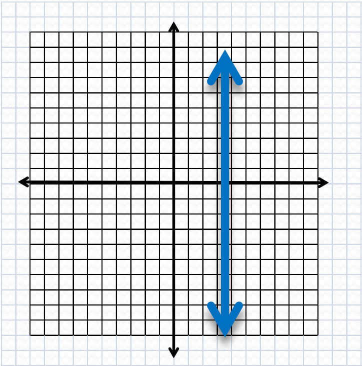
negative



zero



undefined



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Linear Equations and Word Problems

Problem: The table shows the units of electricity used at the school over a certain period of time. Use the data to make a graph and find the slope of the line.

Electricity used at Bethany Elementary	
Weeks	Units of electricity (Watts)
5	75,000
10	150,000
15	225,000
20	300,000

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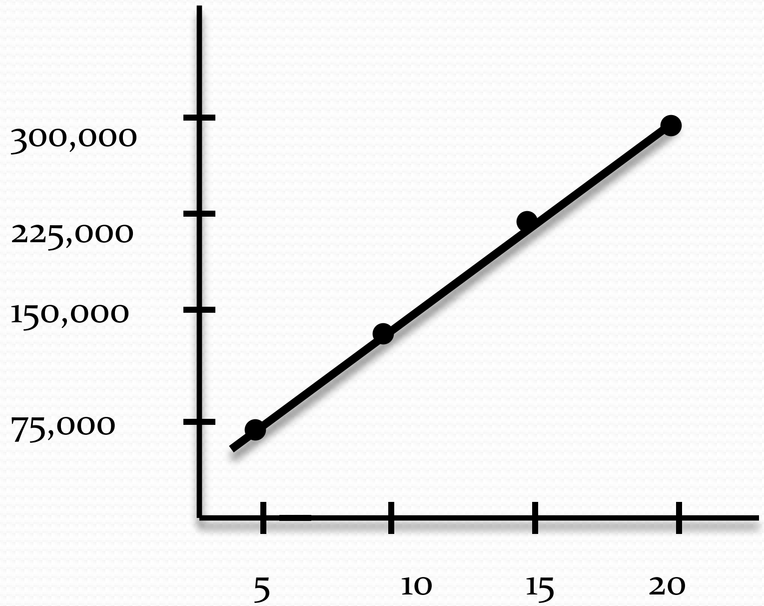
Word problem cont.

- To solve the problem, graph the ordered pairs from the table provided

Graph of ordered pairs



- Find the slope by subtracting the x and y values of two points on the line

$$\frac{150{,}000 - 75{,}000}{10 - 5}$$

$$\frac{75{,}000}{5}$$

Slope=15,000

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Ideas for application

- Use manipulatives to create concrete examples of linear equations (use material with texture for students with visual impairments like the soft side of velcro
- Create personally-relevant word problems

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Making connections

- Linear equations addresses the middle and high school Core Content Connectors of
  - 6.PRF.1d1 Solve real-world single step linear equations
  - 6.PRF.2b2 Using provided table with numerical patterns, form ordered pairs
  - 8.PRF.1g3 Solve linear equations with 1 variable
  - 8.PRF.2e2 Identify the rate of change (slope) and initial value (y-intercept) from graphs
  - H.PRF.1a Interpret the rate of change using graphical representations
  - H.PRF.2a Translate an algebraic expression into a word problem
  - H.PRF.2b1 Translate a real-world problem into a one variable linear equation

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Sharing the Sky

UNIVERSAL DESIGN FOR LEARNING

(UDL in planning, and provide for additional differentiated instruction when teaching equations.)

	Sensory Differences such as Blindness, Visual Impairment, Deafness, or Deaf/Blindness	Physical Disability or Motor Differences (such as weakness or motor planning difficulty)	Extremely limited evidence of experience/ skill or motivation/attention.	Lack of or extremely limited use of speech.
Representation	Use a graphing calculator so students can just plug in the equation; raise the lines of the graphed linear equation or of the grid when graphing; add corresponding textures (e.g., Velcro) to equations and calculators; have students scan raised lines with hands to discriminate between the different kinds of slope (i.e., negative, positive, zero, and undefined)	Student scans an array of possible options and use a switch to select the appropriate slope or ordered pair; graph linear equations on the computer that can be manipulated with switch	Create personally-relevant word problems; use graphing calculators or computer software to find slopes or graph equations; Have student use graphing calculator; color code equations and corresponding parts of calculator to support students correctly entering equations; use conversion tables with pictures or objects to points on a line	Provide customized display of information Consistent model by utilizing modes of communication used by students (point to symbols representing concepts, operations) Teacher model competent use of AAC during instruction
Expression	Student states answer or scans raised numbers to select correct answer; use voice output devices for student to select the correct answer	Use a switch to indicate correct answers; use an eye gaze board to select answer; use a blink response to complete a table to find points on a line given the equation; phrase questions so that they require a “yes/no” response, these can easily be answered using an eye gaze, head turn, two switches, etc	Student selects numbers versus writing them; selection of correct answer is done after a model; student points to each part of a the equation when asked to touch the slope or the y-intercept Have students graph linear equations using high interest manipulatives (e.g., computer software, on an iPad, using	Provide options for modes of communication: Incorporate responses into student’s AAC device or eye gaze array Phrase questions so that they require a “yes/no” response, these can easily be answered using an eye gaze, head turn, two switches, etc Choose response by pointing

			their favorite color)	to or selecting object or item Use a blink response to count tiles or select answer; count tiles/cubes out loud having student move in some voluntary way (e.g., nod head, tap hand, tap foot) to count along
Engagement	Teach students to use their hands to scan the raised lines or grid of a graph; use graphing calculator and add texture to support entering linear equations	Use a computer with AT where the student can click to answer; use manipulatives that are large and easily manipulated; pair student with another student without a physical impairment and have them work together to solve linear equations or complete tables	Student uses graphing calculator, limit ordered pairs to numerals less than 10, use bright colors to represent the ordered pairs that are the coordinates of a point on the line. Find dilations of local buildings and use those dilations to make a scale model of the neighborhood; create personally-relevant word problems about highly preferable activities (e.g., going to the mall; food)	Recruit interest with modes of communication: Allow students to choose items or subjects that are relevant to them via AAC devices, symbols, or eye gaze array

Prepare for landing

Below you will find ideas for linking graphing and transformations in the coordinate plane to real-world applications, the college and career readiness skills addressed by teaching these concepts, module assessments for elementary, middle school, and high school teachers, sample general education lesson plans incorporating universal design for learning framework, blog for teachers to share their ideas, and a place to upload and share lesson plans from teachers who completed this module.

Teaching a variety of strategies for using the coordinate plane may seem like a lot of work and developing creative, yet concrete demonstrations can be difficult. One way to help assist in a special educator's development within this curricular area is through collaboration with other teachers in your building. Often these skills are practiced outside of a math classroom in other curricular areas like art. Some activities with real world connection include:

- Make a snowflake reflection
- Look at fabrics such as Kuba cloth made by tribes from the Congo (Zaire) region of Africa. Look for examples of slides and flips.
- Take students outside and allow them to trace reflections of themselves using sidewalk chalk
- Using construction and tissue paper, make a mock quilt using reflections, rotations, and transformation of different shapes (have a different quilts for different polygons)
- Use amalgamations to make an art project
- Cut a picture of a preferable object in half. Use the second half to demonstrate a reflection (putting the two sides together) and a rotation (put the pictures together with one side upside down)
- Use examples which incorporate home décor. For example, a student might have to use a reflection to show where the next picture should be hung on the wall to complete a grouping of pictures. Or, students may use the vocabulary terms like "rotate" to describe where to put furniture in a home decorating layout.
- Use photo program and have students orient the pictures correctly

In addition to the real-world applications of these concepts, skills taught within this content module also promote the following college and career readiness skills.

<u>Communicative competence:</u>
Students will increase their vocabulary to include concepts related to "coordinate plane, rotations, reflections, and translations" In addition, they will be learning concepts such as: "up", "down", "left", "right", "positive", and "negative".

<u>Fluency in reading, writing, and math</u>
Students will have an opportunity to increase their numeracy and sight word fluency while participating in problem solving related to the "coordinate plane" such as number recognition, counting, and one-to-one correspondence.

Age appropriate social skills
Students will engage in peer groups to solve problems related to the coordinate plane that will provide practice on increasing reciprocal communication and age appropriate social interactions. For example, students might work together with their peers to find ordered pairs to graph the translation of a quadrilateral.

Independent work behaviors
By solving real life problems related to the coordinate plane students will improve work behaviors that could lead to employment such as locating items on a map.

In addition to collaborating with other educational professionals in your building, the following list of resources may also help provide special educators with ideas for activities or support a more thorough understanding of the mathematical concepts presented in this content module

Additional Resources

- http://www.ncpublicschools.org/acre/standards/common-core-tools/ - this website provides an "unpacking document" for the Mathematics Common Core Standards that helps teachers identify what is most important and the essential skills for each standard
- www.teachertube.com - Youtube for teachers! Simply search for your content area and this websites provides a variety of videos including videos of math experts working through math problems step by step (**free registration required)**
- http://www.k8accesscenter.org/training_resources/math.asp#webinar - this website not only provides some ideas and activities to use in your classroom, but also includes presentations and webinars from the North Carolina Department of Public Instruction about research-based strategies that have proven effective in teaching math for students with varying level of disability.
- http://exchange.smarttech.com/search.html?subject=Mathematics - this SMART board exchange has developed lessons by classroom teachers differentiated by grade level. You can also search by skill and/or state standards.
- http://www.google.com/url?q=http://www.ksde.org/LinkClick.aspx%3Ffileticket%3DVq9AjrFFWzE%253D%26tabid%3D3763%26mid%3D11170&sa=U&ei=8lB3Try4CJOltwfmq5DfDA&ved=0CBIQFjAA&usg=AFQjCNE_DzuxI_rhYkU0H1qpjuqmM9sjng - this website provides a webinar about how to adapt materials for students who have visual impairments
- http://www.teachforever.com/2007/09/lesson-plan-graphing-on-coordinate.html - website with a lesson plan for using battleship to practice finding ordered pairs in the coordinate plane
- http://funbasedlearning.com/algebra/graphing/default.htm - website provides tutorials for graphing that range from easy to hard
- http://commoncoretools.wordpress.com - website provides explanation of common core connectors
- http://www.uen.org/Lessonplan/preview.cgi?LPid=15432 - lesson plan on graphing translations and reflections on a coordinate plane

Module Assessments

Insert elementary assessment

Insert middle and high school assessment

Sample General Education lesson plans

Insert developed lesson plans here

Have an idea: Upload the lesson plans you've created here

Insert link for teachers to upload lesson plans

Teacher's Corner: Blog with other teachers

Insert forum or blog for teachers to share ideas

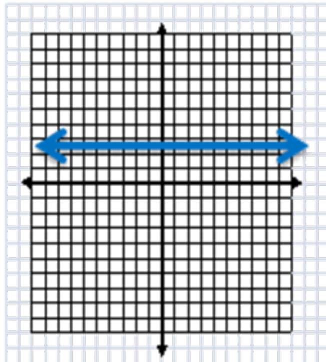
Up for a Challenge???

Adapt the following general education lesson plan, adapt, and upload. These lesson plans may be shared with higher education professionals developing strategies to provide meaningful academic instruction in mathematics to students with severe disabilities.

Insert blank lesson plan form with UDL chart here

Insert link for teachers to upload lesson plans

Linear equations



1. What kind of slope is this?

   a. positive
   b. negative
   c. zero
   d. undefined

Correct feedback: Yes, the answer is zero

Incorrect feedback: Sorry, the answer is zero. Please review the linear equations PowerPoint.

2. What is the y-intercept in the equation y= x-6 ?

   a. 6
   b. -6
   c. $\frac{1}{6}$
   d. $-\frac{1}{6}$

Correct feedback: Yes, the answer is-6.

Incorrect feedback: Sorry, the answer is-6. Please review the slope-intercept and point-slope PowerPoint.

3. What is the slope of a line that contains the points with the coordinates (2,5) and (2,6)?

   a. 4
   b. 3
   c. 2
   d. 1

Correct feedback: Yes, the answer is 1

Incorrect feedback: Sorry, the answer is 1. Please review the slope intercept and point-slope PowerPoint.

4. What is the y-intercepts for the following equation? 8x=5y

   a. $\frac{8}{5}$
   b. 0
   c. -1
   d. $\frac{5}{8}$

Correct feedback: Yes, the answer is 0

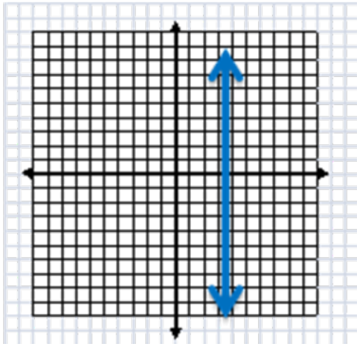
Incorrect feedback: Sorry, the answer is 0. Please review the slope intercept and point-slope PowerPoint.

5. What is the point-slope form of the equation for a line with a slope of 5 passing through (0,6)?

   a. $7x - 5y = 18$
   b. $y - 7 = 5x$
   c. $y - 6 = 5x$
   d. $y - 7 = 0x$

Correct feedback: Yes, the answer is $y - 6 = 5x$

Incorrect feedback: Sorry, the answer is $y - 6 = 5x$. Please review the slope intercept and point-slope PowerPoint.



6. What kind of slope is this?

   a. positive
   b. negative
   c. zero
   d. undefined

Correct feedback: That's right, the slope of this line is undefined

Incorrect feedback: Sorry, the slope of this line is undefined. Please review the linear equations PowerPoint.

7. What is the point-slope form of a line with slope $\frac{3}{4}x$ that passes through the point (-16, 5)?

   a. $y - 5 = \frac{3}{4}(x - 16)$
   b. $y - 5 = \frac{4}{3}(x + 16)$
   c. $y - 5 = \frac{3}{4}(x + 16)$
   d. $y - 5 = -\frac{4}{3}(x - 16)$

Correct feedback: The answer is $y - 5 = \frac{3}{4}(x + 16)$

Incorrect feedback. The answer is $y - 5 = \frac{3}{4}(x + 16)$. Please review the slope intercept and point-slope PowerPoint.

8. Below is a table of values. What is the rate of change?

x	-3	-2	-1	0
y	8	6	4	2

a. -1
b. -2
c. 1
d. 2

Correct feedback, Great! The answer is -2

Incorrect feedback: Sorry, the answer is -2. Please review the linear equations PowerPoint.

9. What is the y-intercept for the following equation? $y = x - 6$

a. -6
b. 6
c. $\frac{1}{6}$
d. 0.6

Correct feedback: That's correct, the answer is -6

Incorrect feedback. Sorry, the answer is -6. Please review the slope intercept and point-slope PowerPoint.

10. What is the equation in slope-intercept form for a line that passes through points (1,6) and (-1, -2)?

a. $y = 2x + 4$
b. $y = -3x + 6$
c. $y = 4x - 2$
d. $y = 4x + 2$

Correct feedback: That's right, the answer is $y = 4x + 2$

Incorrect feedback: Sorry, the answer is $y = 4x + 2$. Please review the slope intercept and point-slope PowerPoint.

General Education Math Lesson Plan

Linear Equations: Slope of a Line

Source: Bennett, J.M., Burger, E. B., Chard, D. J., Hall, E., Kennedy, P. A…Waits, B. W. (2011). *Mathematics.* Austin, TX: Holt McDougal

Standard: 8.PRF.2e2 Identify the rate of change (slope) and initial value (y-intercept) from graphs

Materials:

Activities:

- Focus and Review: Ask students if they've ever been skiing or seen it on TV. Talk about the difference between bunny slope and the expert trails and how the different trails vary in steepness.
- Lecture: Teacher works through a variety of problems work finding and graphing different slopes.
- Guided Practice: Give students graphs of four lines (one with positive slope, one with negative slope, one with zero slope, and one with undefined slope) and five pairs of points (one for each line and one extra pair). Students use the slope formula to find the slope between each pair and match those to the appropriate graph.
- Independent Practice: Students work 5 word problems identifying slope of provided lines and finding slope of ordered pairs.

Activity: Create a universally designed version of the above lesson

UDL Planning	My ideas
Representation- adaptations in materials (e.g., adapt for sensory impairments)	Use visual models of lines; raise lines for students with VI; raise lines of graph for students to find slope; allow students to plug equations into graphing calculator to find lines
Expression- how will student show learning (e.g., use of assistive technology; alternative project)	Use a calculator or manipulatives; student uses AT to scan possible answers; student indicates type of slope (positive, negative, zero, or undefined), but is not responsible for completing the calculations to find the slope
Engagement- how will student participate in the activity	Student can work in a pair during independent practice; alter word problems to make personally relevant (e.g., add student's name, change the context to be something familiar)

The High School Math Project —Focus on Algebra

http://www.pbs.org/mathline Page 1

The Yo-Yo Problem

(Solving Linear Equations)

Students explore linear patterns, write a pattern in symbolic form, and solve linear equations using algebra tiles, symbolic manipulation, and the graphing calculator.

The lesson starts with the presentation of the yo-yo problem. Students then complete a hands-on activity involving a design created with pennies that allows them to explore a linear pattern and express that pattern in symbolic form. Algebra tiles are introduced as the students practice solving linear equations. Working from the concrete to the abstract is especially important for students who have difficulty with mathematics, and algebra tiles help students make this transition. In addition to using algebra tiles, students also use symbolic manipulation and the graphing calculator. Finally, the students return to solve the yo-yo problem. A very special feature of this lesson is the effective use of peer tutors in this inclusion classroom.

- graphing calculator and overhead unit
- overhead projector
- overhead algebra tiles

For each group:

- 31 pennies
- set of algebra tiles
- graphing calculators

PBS MATHLINE®

HSMP — The Yo-Yo Problem Lesson Guide • http://www.pbs.org/mathline Page 2

1. **Introduction of the Problem**: Explain the details of *The Yo-Yo Problem* to the class.

   The Yo-Yo Problem
   Andy wants to buy a very special yo-yo. He is hoping to be able to save enough money to buy it in time to take a class in which he will learn how to do many fancy tricks. The 5-ounce aluminum yo-yo costs $89.99 plus 6% sales tax. Andy has already saved $17.25, and he is earning $7.20 a week by doing odd jobs and chores. How many weeks will it take him to save enough money for the yo-yo?

   Have the students calculate the total amount of money he will have to save by determining the sales tax and adding that amount to the price of the yo-yo. Before students begin to solve the problem, you should review linear patterns and have students practice solving linear equations.

2. **Penny Pattern Exploration**: For the second part of the lesson, students create a design in stages. The first stage is one penny surrounded by six pennies. For each successive stage, six more pennies are added to the outside of the pattern. Have students continue to make several more stages of this design with their groups. They should create a table of values using n for the stage number and p for the number of pennies used. Finally, have each group determine an algebraic rule representing the relationship between the stage number and the number of pennies used. Then have each group share its rule with the entire class.

3. **Solving Equations Using Algebra Tiles**: After distributing the algebra tiles, write the equation to be solved, $4x + 3 = 3x – 1$, on the board. Demonstrate the use of the algebra tiles while the students work the problem with their groups. Ask questions to help draw out the strategies for solving the equation. Students should remember that the goal is to isolate the variable on one side of the equation. Students accomplish this by performing the same operations on both sides of the equation. Have students check their solutions using symbol manipulation.

   The second problem to give the students to solve, $3(x + 1) – 1 = 2(2x – 3)$, involves the distributive property. Have one pair of students solve the problem at the overhead using the algebra tiles, have another pair of students work at the board using symbol manipulation, and have the rest of the class work in groups at their seats. In each group working at their seats, make sure that some students use algebra tiles, and have the others use symbol manipulation.

4. **Solving Equations Using the Graphing Calculator**: After the students have completed solving the equation, $3(x + 1) – 1 = 2(2x – 3)$, using algebra tiles and symbol manipulation, model solving the equation using the graphing calculator. Have a student, along with a peer tutor, come to the overhead and follow your directions to solve the equation. Enter the left-hand side of the equation into Y1 and the right-hand side into Y2. After students have graphed both equations, have them look to see where the lines intersect. You should adjust the viewing so that the intersection of the two lines is visible. After changing the viewing window, students can use the trace key to find the intersection point, and they should also examine the values in the table to determine what the x-value is when Y1 and Y2 are the same. The students should see that the function values are the same when $x = 8$. This answer agrees with the solutions determined using the algebra tiles and symbol manipulation.

5. **Back to The Yo-Yo Problem**: Review the basic facts of *The Yo-Yo Problem* for the students and direct them to work with their groups to solve the problem. Have students use various methods to determine the solution, including writing a symbolic equation and solving it, using the trial and error method, and using simple arithmetic. Have students go the board and present their solutions to the class.

   This lesson offers many opportunities for ongoing assessment. As students work in groups and as they make presentations to the class, you can evaluate their mathematical understanding. This lesson also gives students many opportunities to connect ideas from the various activities and use those ideas as they work to solve *The Yo-Yo Problem*. Did they see *The Yo-Yo Problem* as another linear pattern? What was the initial value, and what was the rate of change? Could they express this symbolically? Could they solve the symbolic representation? Do they have generally good problem solving ideas?

Teachers have the very difficult task of analyzing such situations and using their observations to shape future activities for students. Any one lesson is just a small part of the mathematical journey on which students are traveling. What is the destination? Teachers need to have a clear vision of what the goal is. They need to have a firm understanding of what it is they are trying to achieve in order to be able to effectively assess whether they are successful.

- You could have students write problems that are similar to *The Yo-Yo Problem*. They could share their problems with the entire class. You could assign some of these problems as class work or homework. You could also display the problems on a bulletin board in the classroom.

- Have students bring in examples of linear models. Ask them to explain why the relationship is a linear model. Also have the students bring in examples that are not linear and explain why they are not linear.

- As a follow-up activity, ask the students to write an equation for the relationship between the pattern number, n, and the number of pennies required to make the pattern. Also, have them draw the graph and make a table of values. Have them do the same thing for the relationship between the number of weeks, w, and the amount of money saved in *The Yo-Yo Problem*. Then ask the students to compare the equations, graphs, and tables and describe how they are different and how they are the same.

Linear functions have been and will continue to be a fundamental part of the study of algebra as mathematics education develops to meet the needs of the ever changing technological world of today. These functions help us to better understand the world around us, and to answer problems and explain phenomena from many diverse fields, including physics, biology, and economics.

In the traditional approach to algebra, students generally learn how to solve linear equations before they study linear models. Traditionally, solving equations was devoid of any real-world context until after the symbol manipulations were mastered. However, some of the newer materials emphasize understanding the mathematical model as it is related to a real-world application, and then building on that conceptual understanding in order to help students solve equations.

In this lesson, students work with linear situations involving a penny pattern and the total amount of money saved over a period of time. Helping students understand the basic patterns involved in linear models—in their symbolic form, graph form, and table form—gives them a firm foundation for solving problems involving linear situations. It also gives students more ways to solve these types of problems. They are not limited to using symbol manipulation. They can solve an equation using a graph or a table of values in addition to using symbol manipulation.

One of the major goals of mathematics education is developing good problem solving skills. Students need a solid understanding of multiple representations of various functions, including linear functions, in order to be considered mathematically literate. Teaching from this perspective, and making multiple opportunities available to help students make connections and develop these skills is very important in today's mathematics classroom.

The Yo-Yo Problem

Andy wants to buy a very special yo-yo. He is hoping to be able to save enough money to buy it in time to take a class in which he would learn how to do many fancy tricks. The 5-ounce aluminum yo-yo costs $89.99 plus 6% sales tax. Andy has already saved $17.25, and he is earning $7.20 a week by doing odd jobs and chores. How many weeks will it take him to save enough money for the yo-yo?

Part I

1. How much sales tax will Andy have to pay?

2. What will be the total cost of the yo-yo, including tax?

Part II

3. Let *w* be the number of weeks that it will take Andy to save enough money to buy the yo-yo. Write an algebraic equation that will help you solve the problem.

4. Solve your equation for *w*, and check your answer. Be prepared to present your solution to the class.

The Penny Pattern

Exploring Linear Models

1. Create a pattern using pennies. Stage one of the pattern is shown next to the title above—one penny surrounded by six additional pennies. To create each additional stage of the design, place more pennies extending out from the six that surround the center penny. Continue making this design until you have used up all of your pennies. On the back of this sheet, sketch the first four stages of the pattern.

2. Using your penny pattern or the sketches of your penny pattern, create a table of values.

Stage number, n	1	2	3	4	5
Number of pennies required, p					

3. How many pennies are needed to make stage 6, stage 7, and stage 8 of the penny pattern? How did you determine your answer?

4. Write an algebraic model that expresses the relationship between the stage number, *n*, and the number of pennies required to make that design, *p*.

5. Use your model to determine how many pennies are needed to make stage 80, stage 95, and stage 100 of the penny pattern.

6. Using your graphing calculator, make a scatter plot of the table of values from problem 2. Graph your model from problem 4 to determine if it is correct, and then use the graphing calculator to create a table of values to check your answers to problems 3 and 5.

7. If you use 127 pennies to make the penny pattern, how many pennies will be in each spoke coming out from the center penny? Can you find this answer three different ways?

Activity: Create a universally designed version of the above lesson

UDL Planning	My ideas
Representation- adaptations in materials (e.g., adapt for sensory impairments)	
Expression- how will student show learning (e.g., use of assistive technology; alternative project)	
Engagement- how will student participate in the activity	

National Center and State Collaborative

Mathematics Instructional Families – Patterns, Relations and Functions

Reposted for NCSC state use on March 27, 2013. All materials in this version have been approved for public distribution with all necessary permissions. Selected excerpts are accompanied by annotated links to related media freely available online at the time of the publication of this document.

ncsc

National Center and State Collaborative

The National Center and State Collaborative (NCSC) is applying the lessons learned from the past decade of research on alternate assessments based on alternate achievement standards (AA-AAS) to develop a multi-state comprehensive assessment system for students with significant cognitive disabilities. The project draws on a strong research base to develop an AA-AAS that is built from the ground up on powerful validity arguments linked to clear learning outcomes and defensible assessment results, to complement the work of the Race to the Top Common State Assessment Program (RTTA) consortia.

Our long-term goal is to ensure that students with significant cognitive disabilities achieve increasingly higher academic outcomes and leave high school ready for post-secondary options. A well-designed summative assessment alone is insufficient to achieve that goal. Thus, NCSC is developing a full system intended to support educators, which includes formative assessment tools and strategies, professional development on appropriate interim uses of data for progress monitoring, and management systems to ease the burdens of administration and documentation. All partners share a commitment to the research-to-practice focus of the project and the development of a comprehensive model of curriculum, instruction, assessment, and supportive professional development. These supports will improve the alignment of the entire system and strengthen the validity of inferences of the system of assessments.

IDEAs that Work
U.S. Office of Special Education Programs

This work was developed as part of the National Center and State Collaborative and supported by a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). The contents do not necessarily represent the policy of the U.S. Department of Education, and no assumption of endorsement by the Federal government should be made.

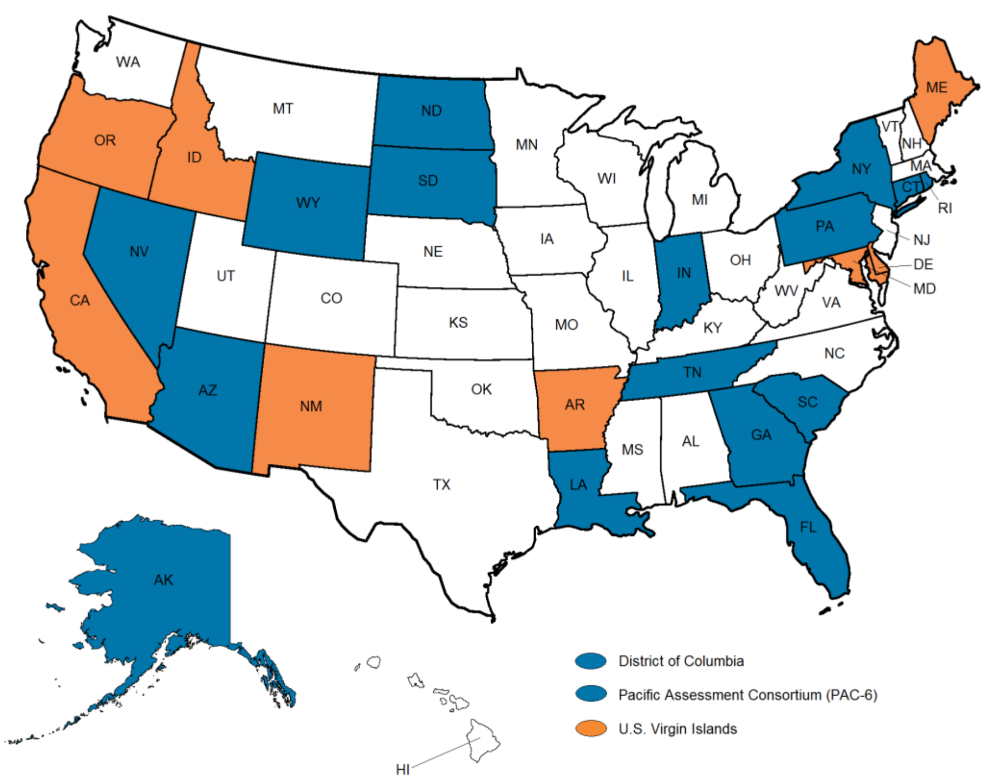
ncsc

National Center and State Collaborative

NCSC is a collaborative of 18 states and five organizations.

The states include (shown in blue on map): Alaska, Arizona, Connecticut, District of Columbia, Florida, Georgia, Indiana, Louisiana, Nevada, New York, North Dakota, Pacific Assessment Consortium (PAC-6)[1], Pennsylvania, Rhode Island, South Carolina, South Dakota, Tennessee, and Wyoming.

Tier II states are partners in curriculum, instruction, and professional development implementation but are not part of the assessment development work. They are (shown in orange on map): Arkansas, California, Delaware, Idaho, Maine, Maryland, New Mexico, Oregon, and U.S. Virgin Islands.



[1] The Pacific Assessment Consortium (including the entities of American Samoa, Commonwealth of the Northern Mariana Islands, Federated States of Micronesia, Guam, Republic of Palau, and Republic of the Marshall Islands) partner with NCSC as one state, led by the University of Guam Center for Excellence in Developmental Disabilities Education, Research, and Service (CEDDERS).

The five partner organizations include: The National Center on Educational Outcomes (NCEO) at the University of Minnesota, The National Center for the Improvement of Educational Assessment (Center for Assessment), The University of North Carolina at Charlotte, The University of Kentucky, and edCount, LLC.

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Mathematics Instructional Families – Patterns, Relations and Functions

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NCSC Partner States

January 2013

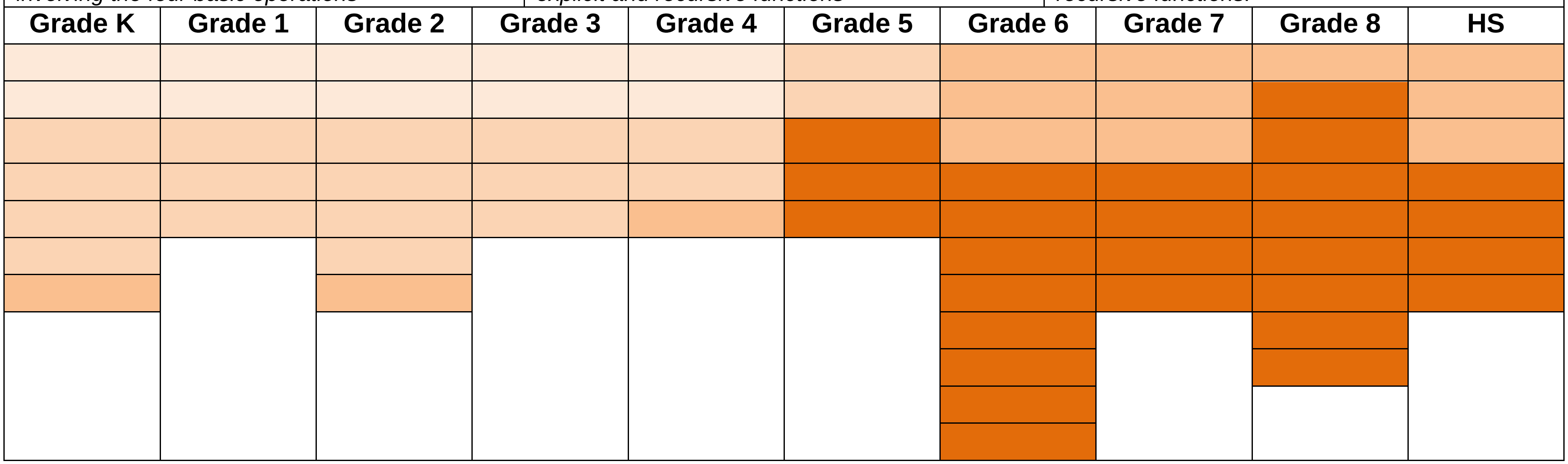
Reposted April 3, 2013

Table of Contents

View of Learning Targets and Families across Grades

<u>Distribution of Instructional Families</u>: Patterns, Relations and Functions

(K-4) Elementary School Learning Targets	(5-8) Middle School Learning Targets	(9-12) High School Learning Targets
E.PRF-1 *Use concrete, pictorial, and symbolic representations to identify, describe, compare, and model situations that involve change.*	***M.PRF-1*** *Describe and compare situations that involve change and use the information to draw conclusions:* • *Model contextual situations using multiple representations;* • *Calculate rates of change for real-world situations (constant)*	***H.PRF-1*** *Approximate, calculate, model, and interpret change:* • *Use graphical and numerical data resulting from complex situations;* • *Model complex real-world phenomena to make predictions and provide explanations*
E.PRF-2 *Give examples, interpret, and analyze repeating and growing patterns and functions involving the four basic operations*	***M.PRF-2*** *Give examples, interpret, and analyze a variety of mathematical patterns, relations, and explicit and recursive functions*	***H.PRF-2*** *Use trends and analyze a variety of mathematical patterns, relations, and explicit and recursive functions.*



Representing and Modeling Problems	**Describing and Extending Patterns**	**Problem Solving and Using Variables**	**Proportional Relationships and Graphing**

View of Learning Targets, Families, and CCCs by Grade-band

Overview of CCCs: Patterns, Relations and Functions

Representing and Modeling Problems		Describing and Extending Patterns		Problem Solving and Using Variables
(K-4) Elementary School Learning Targets				
E.PRF-1 *Use concrete, pictorial, and symbolic representations to identify, describe, compare, and model situations that involve change.*				
E.PRF-2 *Give examples, interpret, and analyze repeating and growing patterns and functions involving the four basic operations*				
Grade K	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**
K.PRF.1b1 Using objects or pictures respond appropriately to “add __” and “take away ___” *K.OA.1*	1.PRF.1b3 Using objects or pictures respond appropriately to “add __” and “take away ___” *1.OA.1*	2.PRF.1c3 Solve one or two step addition and subtraction problems, and add and subtract within 100, using objects, drawings, pictures *2.OA.1*	3.PRF.1d1 Use objects to model multiplication and division situations involving up to 5 groups with up to 5 objects in each group and interpret the results *3.OA.1* *3.OA.2*	4.PRF.1d2 Use objects to model multiplication and division situations involving up to 10 groups with up to 5 objects in each group and interpret the results *3.OA.1*
K.PRF.1c1 Solve one step addition and subtraction word problems, and add and subtract within 10 using objects, drawings, pictures *K.OA.2*	1.PRF.1c2 Solve one step addition and subtraction word problems where the change or result is unknown (4+_=7) or (4 + 3 = __), within 20 using objects, drawings, pictures *1.OA.1*	2.PRF.1c4 Use pictures, drawings or objects to represent the steps of a problem *2.OA.1*	3.PRF.2d2 Apply properties of operations as strategies to multiply and divide *3.OA.5*	4.PRF.1f3 Apply the distributive property to solve problems with models *3.MD.7c*
K.PRF.2a1 Describe or select the repeating pattern using objects or pictures (AB or ABC) *No CCSS linked*	1.PRF.2a4 Use a number line to extend the numerical patterns that grow at a constant rate (2,4,6,8) *No CCSS linked*	2.PRF.2a6 Use a number line to extend the numerical patterns that grow at a constant rate (2,4,6,8) *No CCSS linked*	3.PRF.1e1 Describe the rule for a numerical pattern (e.g., increase by 2, 5 or 10) *3.OA.9*	4.PRF.2d3 Generate a pattern when given a rule and word problem *4.OA.5*
K.PRF.2a2 Extend a repeating pattern using objects or pictures (AB or ABC) *No CCSS linked*	1.PRF.2b2 Create a growing pattern using numbers or objects *No CCSS linked*	2.PRF.2b3 Use a number line to extend arithmetic patterns that are decreasing	3.PRF.1e2 Select or name the 3 next terms in a numerical pattern where numbers increase by 2, 5 or 10 *3.OA.9*	4.PRF.2e1 Extend a numerical pattern when the rule is provided *4.OA.5*
K.PRF.2a3 Extend a repeating numerical AB pattern *No CCSS linked*	1.PRF.2c1 Identify the rule of a given arithmetic pattern *No CCSS linked*	2.PRF.2c2 Identify the rule of arithmetic patterns that are increasing *No CCSS linked*	3.PRF.2d1 Identify multiplication patterns in a real-world setting *3.OA.9*	4.PRF.1e3 Solve multiplicative comparisons with an unknown using up to 2-digit numbers with

				information presented in a graph or word problem *4.OA.2*
K.PRF.2b1 Create a repeating pattern using objects, pictures, or numbers *No CCSS linked*		2.PRF.2c3 Identify the rule of arithmetic patterns that are decreasing *No CCSS linked*		
K.PRF.1b2 Communicate answer after adding or taking away *K.OA.1*		2.PRF.1c5 Write or select an equation representing the problem and its solution *2.OA.1*		

Overview of CCCs: Patterns, Relations and Functions

Describing and Extending Patterns		Problem Solving and Using Variables		Proportional Relationships and Graphing	
(5-8) Middle School Learning Targets					
M.PRF-1 *Describe and compare situations that involve change and use the information to draw conclusions:* • *Model contextual situations using multiple representations;* • *Calculate rates of change for real-world situations (constant)*					
M.PRF-2 *Give examples, interpret, and analyze a variety of mathematical patterns, relations, and explicit and recursive functions*					

Grade 5	Grade 6	Grade 7	Grade 8
5.PRF.1b1 Given 2 patterns involving the same context (e.g., collecting marbles) determine the 1^{st} 5 terms and compare the values *5.OA.3*	6.PRF.1d1 Solve real-world single step linear equations *6.EE.7*	7.PRF.1g1 Solve real-world multi step problems using whole numbers *7.EE.3*	8.PRF.1g3 Solve linear equations with 1 variable *8.EE.7*
5.PRF.2a1 Generate a pattern that follows the provided rule *4.OA.5*	6.PRF.2a2 Use variable to represent numbers and write expressions when solving real-world problems *6.EE.6*	7.PRF.1g2 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities *7.EE.4*	8.PRF.1e2 Represent proportional relationships on a line graph *8.EE.5*
5.PRF.1b2 When given a line graph representing two arithmetic patterns, identify the relationship between the two *5.OA.3*	6.PRF.2a3 Use variables to represent two quantities in a real-world problem that change in relationship to one another *6.EE.9*	7.PRF.2d1 Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers *7.EE 4b*	8.PRF.1f2 Describe or select the relationship between the two quantities given a line graph of a situation *8.EE.5*
5.PRF.2b1 Generate or select a comparison between two graphs from a similar situation *5.OA-3*	6.PRF.1a2 Determine whether or not the quotient will increase or decrease based on the divisor *5.NF.5*	7.PRF.1e2 Represent proportional relationships on a line graph *7.RP.2b*	8.PRF.2c1 Given two graphs, describe the function as linear and not linear *8.F.3* *8.F.5*
5.PRF.1a1 Determine whether the product will increase or decrease based on the multiplier *5.NF.5*	6.PRF.1c1 Describe the ratio relationship between two quantities for a given situation *6.RP.1*	7.PRF.1f1 Use proportional relationships to solve multi step percent problems in real-world situations. *7.RP.3*	8.PRF.2e1 Distinguish between functions and non-functions, using equations, graphs or tables *No CCSS linked*

Describing and Extending Patterns	Problem Solving and Using Variables	Proportional Relationships and Graphing

(5-8) Middle School Learning Targets

M.PRF-1 *Describe and compare situations that involve change and use the information to draw conclusions:*

- *Model contextual situations using multiple representations;*
- *Calculate rates of change for real-world situations (constant)*

M.PRF-2 *Give examples, interpret, and analyze a variety of mathematical patterns, relations, and explicit and recursive functions*

Grade 5	Grade 6	Grade 7	Grade 8
	6.PRF.1c2 Represent proportional relationships on a line graph *6.RP.2*	7.PRF.2a5 Use variables to represent two quantities in a real-world problem that change in relationship to one another *6.EE.9*	8.PRF.2e2 Identify the rate of change (slope) and initial value (y-intercept) from graphs *8.F.4*
	6.PRF.2a4 Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation *6.EE.9*	7.NO.2f4 Use a rate of change or proportional relationship to determine the points on a coordinate plane *7.RP.2d*	8.PRF.2e3 Given a verbal description of a situation, create or identify a graph to model the situation *8.F.5*
	6.PRF.2b2 Using provided table with numerical patterns, form ordered pairs *5.OA.3*		8.PRF.2e4 Given a graph of a situation, generate a description of the situation *8.F.5*
	6.PRF.2b3 Complete a statement that describes the ratio relationship between two quantities *6.RP.1*		8.DPS.2g2 Interpret the slope and the y-intercept of a line in the context of a problem *8.SP.3*
	6.PRF.2b4 Determine the unit rate in a variety of contextual situations *6.RP.2*		
	6.PRF.2b5 Use ratios and reasoning to solve real-world mathematical problems *6.RP.3*		

<u>Overview of CCCs</u>: Patterns, Relations and Functions

Problem Solving and Using Variables	Proportional Relationships and Graphing
(9-12) High School Learning Targets	
H.PRF-1 *Approximate, calculate, model, and interpret change:* • *Use graphical and numerical data resulting from complex situations;* • *Model complex real-world phenomena to make predictions and provide explanations*	
H.PRF-2 *Use trends and analyze a variety of mathematical patterns, relations, and explicit and recursive functions.*	
HS	
H.PRF.2a1 Translate an algebraic expression into a word problem *A.SSE.1*	
H.PRF.2b1 Translate a real-world problem into a one variable equation *A.CED.1*	
H.PRF.2b2 Solve equations with one or two variables using equations or graphs *A.REI.1* *A.REI.3* *A.CED.2*	
H.PRF.1a1 Interpret the rate of change using graphical representations *S.ID.7*	
H.PRF.1b1 In a linear situation using graphs or numbers, predicts the change in rate based on a given change in one variable *F.LE.1b*	
H.PRF.1c1 Select the appropriate graphical representation of a linear model based on real-world events *F.LE.1*	
H.PRF. 2c1 Make predictions based on a given model *F.LE.3*	

View by Instructional Families, LPF, and CCSS Domains

<u>Instructional Families within LPF Strand</u>: Patterns, Relations and Functions

CCSS Domain: Operations and Algebraic Thinking; Measurement and Data	**CCSS Domain:** Operations and Algebraic Thinking	**CCSS Domain:** Operations and Algebraic Thinking; Expressions and Equations; Seeing Structure in Expressions; Creating Equations; Reasoning with Equations and Inequalities	**CCSS Domain:** Operations and Algebraic Thinking; Number Operations-Fractions; Ratios and Proportional Relationships; Expressions and Equations; Functions; Interpreting Categorical and Quantitative Data; Linear, Quadratic and Exponential Models
Representing and Modeling Problems	**Describing and Extending Patterns**	**Problem Solving and Using Variables**	**Proportional Relationships and Graphing**
K.PRF.1b1 Using objects or pictures respond appropriately to “add __” and “take away ___” *K.OA.1*	K.PRF.2a1 Describe or select the repeating pattern using objects or pictures (AB or ABC) *No CCSS linked*	K.PRF.1b2 Communicate answer after adding or taking away *K.OA.1*	5.PRF.1b2 When given a line graph representing two arithmetic patterns, identify the relationship between the two *5.OA.3*
K.PRF.1c1 Solve one step addition and subtraction word problems, and add and subtract within 10 using objects, drawings, pictures *K.OA.2*	K.PRF.2a2 Extend a repeating pattern using objects or pictures (AB or ABC) *No CCSS linked*	2.PRF.1c5 Write or select an equation representing the problem and its solution *2.OA.1*	5.PRF.2b1 Generate or select a comparison between two graphs from a similar situation *5.OA-3*
1.PRF.1b3 Using objects or pictures to respond appropriately to “add _” and “take away __” *1.OA.1*	K.PRF.2a3 Extend a repeating numerical AB pattern *No CCSS linked*	4.PRF.1e3 Solve multiplicative comparisons with an unknown using up to 2-digit numbers with information presented in a graph or word problem (e.g., an orange hat cost $3. A purple hat cost 2 times as much. How much does the purple hat cost? [3 x 2 = p]) *4.OA.2*	5.PRF.1a1 Determine whether the product will increase or decrease based on the multiplier *5.NF.5*
1.PRF.1c2 Solve one step addition and subtraction word problems where the change or result is unknown (4+_=7) or (4 + 3 = __), within 20 using objects, drawings, pictures *1.OA.1*	K.PRF.2b1 Create a repeating pattern using objects, pictures, or numbers *No CCSS linked*	6.PRF.1d1 Solve real world single step linear equations *6.EE.7*	6.PRF.1a2 Determine whether or not the quotient will increase or decrease based on the divisor *5.NF.5*

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Representing and Modeling Problems	**Describing and Extending Patterns**	**Problem Solving and Using Variables**	**Proportional Relationships and Graphing**
2.PRF.1c3 Solve one or two step addition and subtraction problems, and add and subtract within 100, using objects, drawings, pictures *2.OA.1*	1.PRF.2a4 Use a number line to extend the numerical patterns that grow at a constant rate (2,4,6,8) *No CCSS linked*	6.PRF.2a2 Use variable to represent numbers and write expressions when solving real world problems *6.EE.6*	6.PRF.1c1 Describe the ratio relationship between two quantities for a given situation *6.RP.1*
2.PRF.1c4 Use pictures, drawings or objects to represent the steps of a problem *2.OA.1*	1.PRF.2b2 Create a growing pattern using numbers or objects *No CCSS linked*	6.PRF.2a3 Use variables to represent two quantities in a real-world problem that change in relationship to one another *6.EE.9*	6.PRF.1c2 Represent proportional relationships on a line graph *6.RP.2*
3.PRF.1d1 Use objects to model multiplication and division situations involving up to 5 groups with up to 5 objects in each group and interpret the results *3.OA.1* *3.OA.2*	1.PRF.2c1 Identify the rule of a given arithmetic pattern *No CCSS linked*	7.PRF.1g1 Solve real world multi step problems using whole numbers *7.EE.3*	6.PRF.2a4 Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation *6.EE.9*
3.PRF.2d2 Apply properties of operations as strategies to multiply and divide *3.OA.5*	2.PRF.2a6 Use a number line to extend the numerical patterns that grow at a constant rate (2,4,6,8) *No CCSS linked*	7.PRF.1g2 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities *7.EE.4*	6.PRF.2b2 Using provided table with numerical patterns, form ordered pairs *5.OA.3*
4.PRF.1d2 Use objects to model multiplication and division situations involving up to 10 groups with up to 5 objects in each group and	2.PRF.2b3 Use a number line to extend arithmetic patterns that are decreasing *No CCSS linked*	7.PRF.2d1 Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers	6.PRF.2b3 Complete a statement that describes the ratio relationship between two quantities *6.RP.1*

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Representing and Modeling Problems	**Describing and Extending Patterns**	**Problem Solving and Using Variables**	**Proportional Relationships and Graphing**
interpret the results *3.OA.1*		*7.EE 4b*	
4.PRF.1f3 Apply the distributive property to solve problems with models *3.MD.7c*	2.PRF.2c2 Identify the rule of arithmetic patterns that are increasing *No CCSS linked*	8.PRF.1g3 Solve linear equations with 1 variable *8.EE.7*	6.PRF.2b4 Determine the unit rate in a variety of contextual situations *6.RP.2*
	2.PRF.2c3 Identify the rule of arithmetic patterns that are decreasing *No CCSS linked*	H.PRF.2a1 Translate an algebraic expression into a word problem *A.SSE.1*	6.PRF.2b5 Use ratios and reasoning to solve real-world mathematical problems (e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations) *6.RP.3*
	3.PRF.1e1 Describe the rule for a numerical pattern (e.g., increase by 2, 5 or 10) *3.OA.9*	H.PRF.2b1 Translate a real-world problem into a one variable equation *A.CED.1*	7.PRF.1e2 Represent proportional relationships on a line graph *7.RP.2b*
	3.PRF.1e2 Select or name the 3 next terms in a numerical pattern where numbers increase by 2, 5 or 10 *3.OA.9*	H.PRF.2b2 Solve equations with one or two variables using equations or graphs *A.REI.1* *A.REI.3* *A.CED.2*	7.PRF.1f1 Use proportional relationships to solve multi step percent problems in real world situations. *7.RP.3*

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Representing and Modeling Problems	**Describing and Extending Patterns**	**Problem Solving and Using Variables**	**Proportional Relationships and Graphing**
	3.PRF.2d1 Identify multiplication patterns in a real-world setting *3.OA.9*		7.PRF.2a5 Use variables to represent two quantities in a real-world problem that change in relationship to one another *6.EE.9*
	4.PRF.2d3 Generate a pattern when given a rule and word problem (I run 3 miles every day, how many miles have I run in 3 days) *4.OA.5*		7.NO.2f4 Use a rate of change or proportional relationship to determine the points on a coordinate plane *7.RP.2d*
	4.PRF.2e1 Extend a numerical pattern when the rule is provided *4.OA.5*		8.PRF.1e2 Represent proportional relationships on a line graph *8.EE.5*
	5.PRF.1b1 Given 2 patterns involving the same context (e.g., collecting marbles) determine the 1st 5 terms and compare the values *5.OA.3*		8.PRF.1f2 Describe or select the relationship between the two quantities given a line graph of a situation *8.EE.5*
			8.PRF.2c1 Given two graphs, describe the function as linear and not linear *8.F.3* *8.F.5*
	5.PRF.2a1 Generate a pattern that follows the provided rule *4.OA.5*		8.PRF.2e1 Distinguish between functions and non-functions, using equations, graphs or tables *No CCSS linked*

CCSS Domain: Operations and Algebraic Thinking; Measurement and Data	**CCSS Domain:** Operations and Algebraic Thinking	**CCSS Domain:** Operations and Algebraic Thinking; Expressions and Equations; Seeing Structure in Expressions; Creating Equations; Reasoning with Equations and Inequalities	**CCSS Domain:** Operations and Algebraic Thinking; Number Operations-Fractions; Ratios and Proportional Relationships; Expressions and Equations; Functions; Interpreting Categorical and Quantitative Data; Linear, Quadratic and Exponential Models
Representing and Modeling Problems	**Describing and Extending Patterns**	**Problem Solving and Using Variables**	**Proportional Relationships and Graphing**
			8.PRF.2e2 Identify the rate of change (slope) and initial value (y-intercept) from graphs *8.F.4*
			8.PRF.2e3 Given a verbal description of a situation, create or identify a graph to model the situation *8.F.5*
			8.PRF.2e4 Given a graph of a situation, generate a description of the situation *8.F.5*
			8.DPS.2g2 Interpret the slope and the y-intercept of a line in the context of a problem *8.SP.3*
			H.PRF.1a1 Interpret the rate of change using graphical representations *S.ID.7*

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Representing and Modeling Problems	**Describing and Extending Patterns**	**Problem Solving and Using Variables**	**Proportional Relationships and Graphing**
			H.PRF.1b1 In a linear situation using graphs or numbers, predicts the change in rate based on a given change in one variable (e.g., I have been adding sugar at a rate of 1T per cup of water. What happens to my rate if I switch to 2T of sugar for every cup of water?) *F.LE.1b*
			H.PRF.1c1 Select the appropriate graphical representation of a linear model based on real world events *F.LE.1*
			H.PRF. 2c1 Make predictions based on a given model (for example, a weather model, data for athletes over years) *F.LE.3*

Elements of Instructional Families: Patterns, Relations and Functions

ncsc

National Center and State Collaborative

The National Center and State Collaborative (NCSC) is applying the lessons learned from the past decade of research on alternate assessments based on alternate achievement standards (AA-AAS) to develop a multi-state comprehensive assessment system for students with significant cognitive disabilities. The project draws on a strong research base to develop an AA-AAS that is built from the ground up on powerful validity arguments linked to clear learning outcomes and defensible assessment results, to complement the work of the Race to the Top Common State Assessment Program (RTTA) consortia.

Our long-term goal is to ensure that students with significant cognitive disabilities achieve increasingly higher academic outcomes and leave high school ready for post-secondary options. A well-designed summative assessment alone is insufficient to achieve that goal. Thus, NCSC is developing a full system intended to support educators, which includes formative assessment tools and strategies, professional development on appropriate interim uses of data for progress monitoring, and management systems to ease the burdens of administration and documentation. All partners share a commitment to the research-to-practice focus of the project and the development of a comprehensive model of curriculum, instruction, assessment, and supportive professional development. These supports will improve the alignment of the entire system and strengthen the validity of inferences of the system of assessments.

IDEAs that Work

U.S. Office of Special Education Programs

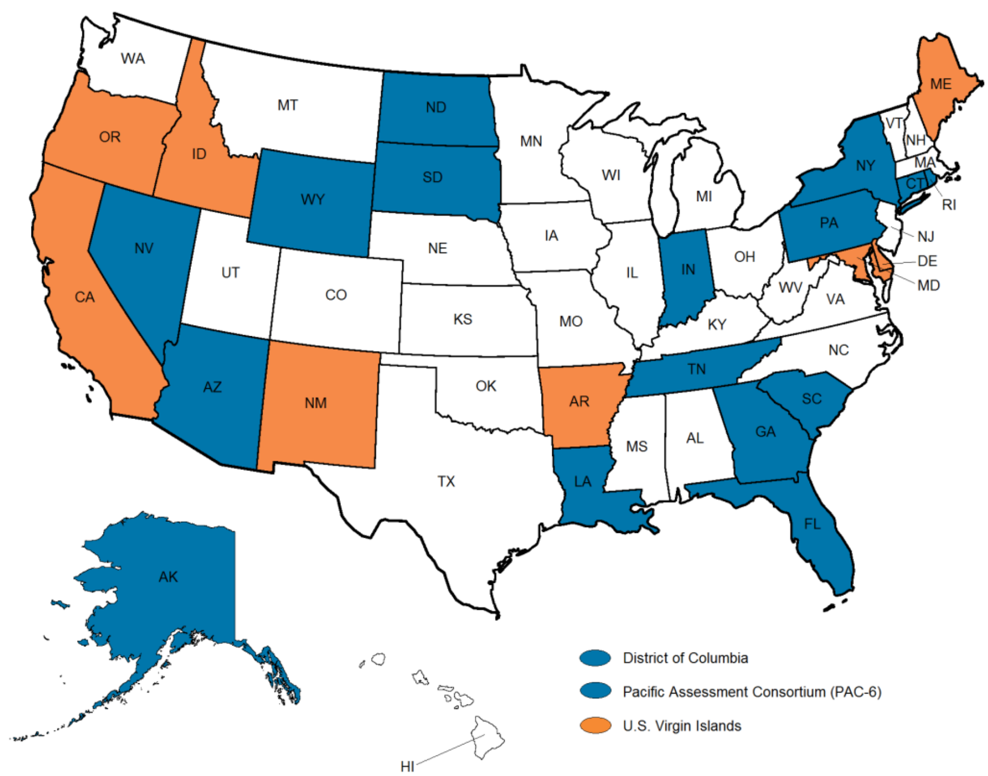
The contents of this instructional resource were developed as part of the National Center and State Collaborative for a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made.

National Center and State Collaborative

NCSC is a collaborative of 18 states and five organizations.

The states include (shown in blue on map): Alaska, Arizona, Connecticut, District of Columbia, Florida, Georgia, Indiana, Louisiana, Nevada, New York, North Dakota, Pacific Assessment Consortium (PAC-6)1, Pennsylvania, Rhode Island, South Carolina, South Dakota, Tennessee, and Wyoming.

Tier II states are partners in curriculum, instruction, and professional development implementation but are not part of the assessment development work. They are (shown in orange on map): Arkansas, California, Delaware, Idaho, Maine, Maryland, New Mexico, Oregon, and U.S. Virgin Islands.



[1] The Pacific Assessment Consortium (including the entities of American Samoa, Commonwealth of the Northern Mariana Islands, Federated States of Micronesia, Guam, Republic of Palau, and Republic of the Marshall Islands) partner with NCSC as one state, led by the University of Guam Center for Excellence in Developmental Disabilities Education, Research, and Service (CEDDERS).

National Center and State Collaborative

The five partner organizations include: The National Center on Educational Outcomes (NCEO) at the University of Minnesota, The National Center for the Improvement of Educational Assessment (Center for Assessment), The University of North Carolina at Charlotte, The University of Kentucky, and edCount, LLC.

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Elements of Instructional Families: Patterns, Relations and Functions

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CCC Mathematics | Patterns, Relations, and Functions

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<tr><td colspan="4">Progress Indicator: E.PRF.1c modeling problem solving situations that involve addition and subtraction of whole numbers using objects, diagrams, and symbols</td></tr>
<tr><td>Essential Understandings</td><td colspan="2">Concrete Understandings:
• Create an array (e.g., show me 1 group/row of 3).</td><td>Representation:
• Select a numeral to place under each representation in a modeled equation.
• Select a pictorial representation of an array that matches the addition or subtraction problem.
• Vocabulary: add, subtract</td></tr>
<tr><td colspan="4">Suggested Instructional Strategies:
• Teach explicitly how to count objects in a set and that the last number name said tells the number of counted objects
    ○ Present a set of objects for the student to count.
    ○ Rearrange the objects and ask the student how many objects there are (the student understands cardinality of numbers if s/he states the same number without recounting the objects)
• Teach explicitly how to create a group/row/set of objects for a given number or for a number provided in a simple word problem
• Multiple Exemplar Training
    ○ An array/row: “This is a group/row of three apples. This is another group/row of three apples. This is another group/row of three apples. This is one apple. Show me a group/row of three apples.”
• Example/Non-example
    ○ Present a row of objects (≤ 10). Present a second row of objects that has a different number of objects. Ask the student to select the row with a specified number of objects.
    ○ Present three rows of objects (≤ 10), two that are equal and one that is not equal. Ask the student to match the two rows that both include the same number of specified objects (e.g., a row of three hats, a row of three hats, a row of 5 shoes).
• Use System of Least Prompts to form an array (group/row) given a number:
    ○ “Make a row/group of three pencils.” The student responds correctly. “Good work. You made a row/group of three pencils.” OR
    ○ The student doesn’t respond. Wait 3-5 seconds and provide a gesture prompt by</td></tr>
</table>

    pointing to the pencils. OR
    - The student doesn't respond. Wait 3-5 seconds and provide a verbal prompt. "Pick up three pencils. Make a group of three pencils." OR
    - The student makes an error; provide a physical prompt. Take the student's hand and give him or her three pencils and help him or her make a row of pencils.
- Model, lead, test ("Watch me make a row of four books. Let's make a row of four books. Now you try to make a row of four books.")
- Model, lead, test ("Here is a story problem. It says there are seven dogs. Watch me make a set of seven dogs to match the story problem. Let's make a set of seven dogs together. Now you try to make a set of seven dogs."); repeat with the other number of objects in the story problem.
- Task Analysis: Use two rows of pictures or objects to model one step addition and subtraction problems
    - Present a simple one step addition problem (e.g., The boys have four backpacks. The girls have two backpacks. How many backpacks do the boys and girls have?)
    - Present a set of objects () and a problem solving template:
    - Template: ___ ___ ___ ___ + ___ ___ = ___ ___ ___ ___ ___ ___
    - Show me a row of four backpacks. Put the backpacks in the equation.
    - Show me a row of two backpacks. Put the backpacks in the equation.
    - Combine the backpacks; fill in the equation with the total number of backpacks the boys and girls have.
- Backward chaining: Model setting up a one-step addition word problem using two arrays and ask the student to complete the last step by combining the arrays and/or counting the number of objects in the combined arrays.
- Forward chaining: Present a one step addition or subtraction word problem and ask the student to complete the first step (e.g., show me a row of four backpacks.) Then complete the steps to solve the equation.

Supports and Scaffolds:

- Counters
- 2D and 3D shapes or objects, pictures
- Counters
- Number lines
- Egg cartons or muffin tins to illustrate/create arrays
- Ones blocks to form different rectangles (rows and columns)
- Manipulatives, visuals and Wiki Sticks to illustrate/define arrays
- Raised grid (to keep structure of array) or graph paper
- Highlighted text is present simple word problem
- Use PPT and shape tool to create arrays to match a provided problem
- Interactive whiteboard or other technology to create arrays

CCSS: NA

CCC:	K.PRF.2a3	Extend a repeating numerical AB pattern

Strand: Patterns, Relations and Functions	**Family:** Describing and Extending Patterns

Progress Indicator: ***E.PRF.2a*** *recognizing, describing, and extending simple repeating (ABAB) and growing (A+1, A+2, A+3) patterns (e.g., colors, sounds, words, shapes, numeric – counting, odd, even)*

Essential Understandings

Concrete Understandings:

- Recognize/identify a pattern as something that repeats over and over.

Representation:

- Recognize patterns and use words to describe the patterns they see.
- Use colors, shapes, symbols or objects to represent an AB pattern.
- Vocabulary: pattern

Suggested Instructional Strategies:

- Multiple Exemplar Training or Example/Non-Example Training:
  - AB Pattern: "Here is a pattern (). Here is the same pattern. Here is the same pattern. This is not the same pattern. Show me a pattern that is the same as this pattern (point to the first pattern)."
- Teach explicitly how an AB pattern has the same pattern over and over and never changes using colors, shapes, or objects.
- Model, Lead, Test
  - Model an AB growing pattern using pictures, 2D shapes or 3D objects (e.g., "Watch me show a pattern…Let's make a pattern together…You try making a pattern.")
- Task Analysis (Backward Chaining)
  - Provide an "**AB"** pattern. Add on to the pattern by adding "**A"** to the pattern. Ask the student to add the next element/what comes next ("**B"**) in the pattern.
  - Continue to extend the pattern.

Supports and Scaffolds:

- Examples of AB patterns in real world setting (e.g., in the environment and art)
- Use of graphic organizers to illustrate an AB pattern in which the student places pictures, 2D or 3D shapes or colors
- Counters
- 2D and 3D shapes or objects, pictures
- Interactive whiteboard or other technology to model AB patterns
- Use of graphic organizers to illustrate a pattern of sets in which the student places 2D or 3D shapes or colors showing an AB pattern

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CCSS: 1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

CCC:	1.PRF.1b3	Using objects or pictures respond appropriately to "add __" and "take away ___"

Strand: Patterns, Relations and Functions	**Family:** Representing and Modeling Problems

Progress Indicator: ***E.PRF.1b*** *exploring and describing how addition or subtraction changes a quantity*

Essential Understandings	**Concrete Understandings:** • Show addition with objects to an existing set. • Recognize whole numbers.	**Representation:** • Select a pictorial representation that matches an addition or subtraction problem • Symbols and vocabulary: =, +, - , take away, add

Suggested Instructional Strategies:

- Teach explicitly the meaning of "add" and "take away" by connecting the vocabulary to known language (e.g., "add" means plus, more, join; "take away" means less, fewer);
- Teach/model "adding to" a set of object results in a larger set; teach "take away" from a set of objects results in a smaller set;
- Compare two sets of unequal number of objects and ask the student which set has been added to (larger set) OR which set has objects taken away (smaller set);
- Model, Lead, Test:
  - Model "adding to" and "taking away" using objects. (e.g., "Watch me add to this group of objects…Let's add to this group of objects together…You try adding to this group of objects. Good, this group is bigger because we added to it.")
  - Indicate that the new group of objects is larger if adding to and smaller if taking away.
- Use counting strategies; and
- Modeling problem solving with supports.

Supports and Scaffolds:

- Counters (chips)
- Picture and objects
- Number line
- Graphic organizers

CCSS: NA

CCC:	1.PRF.2b2	Create a growing pattern using numbers or objects

Strand: Patterns, Relationships and Functions	**Family:** Describing and Extending Patterns

Progress Indicator: ***E.PRF.2b*** *creating and explaining repeating and growing patterns using objects or numbers*

Essential Understandings	**Concrete Understandings:**	**Representation:**
	• Recognize/identify an AB pattern as a pattern that has the same pattern over and over and never changing using colors, shapes, or objects	• Match a pattern using symbols or objects to represent a provided growing addition pattern.

Suggested Instructional Strategies:

- Multiple Exemplar Training or Example/Non-Example Training
  - Growing Pattern: "Here is a pattern that grows by 1 ( , ). Here is that pattern that grows by one (). Here is that pattern that grows by one (). This pattern does not grow by one (). Show me a pattern that grows by one."
- Teach explicitly how a growing pattern increases/changes by the same number (+1 or +2) pattern using colors, shapes, or objects.
- Teach/model a growing addition pattern using 2D shapes or 3D objects as a pattern that increases by the same number in each row of the pattern (e.g., a pattern that grows by +2 would have 1 in the first row, 3 in the second row, 5 in the third row, and 7 in the fourth row).
- Model, Lead, Test
  - Model making a growing pattern use objects. (e.g., "Watch me add two to this pattern to make it grow by 2. …Let's add two more objects to this pattern together to make it grow by two…You try adding two more objects to this pattern to make it grow by two.")
- Task Analysis (Backward Chaining)
  - Provide the first two rows of a growing addition pattern and ask the student to create the third row.
  - Using a T-chart, provide the first two of the growing pattern and ask the student to create the third part of the pattern.

Supports and Scaffolds:

- Unit blocks of ones
- Colored tiles
- Counters
- Graphic organizer (tables with two columns)
- Colored 2D figures and 3D objects

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- Examples of growing patterns in real world setting (e.g., in the environment and art)
- T-Charts for growing patterns
- Use of graphic organizers to illustrate a pattern of sets in which the student places 2D or 3D shapes or colors using addition: e.g., +2 growing pattern:

  (□)
  (□□□)
  (□□□□□)
- Interactive whiteboard or other technology to model growing patterns

CCSS: NA		
CCC:	2.PRF.2c2	Identify the rule of arithmetic patterns that are increasing

Strand: Patterns, Relationships and Functions	**Family:** Describing and Extending Patterns

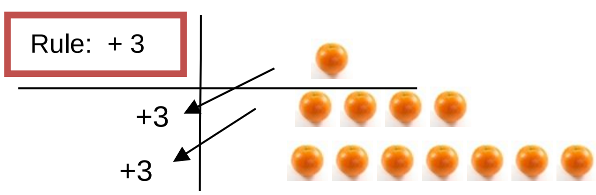
Progress Indicator: ***E.PRF.2c*** *extending and analyzing simple numeric patterns with rules that involve addition and subtraction*

Essential Understandings

Concrete Understandings:	**Representation:**
• Recognize/identify a growing pattern with a single element (i.e., apple) that has one more member at each successive step using colors, shapes, or objects	• Match a pattern using symbols or objects to represent a provided growing addition pattern. • Recognize patterns and describe the patterns • Understand concepts and vocabulary: growing pattern, level, increasing/increases, objects or shapes

Suggested Instructional Strategies:

- Multiple Exemplar Training or Example/Non-Example Training
  - Growing Pattern: “Here is a pattern that grows by two. Here is that pattern growing by two. Here is that pattern growing by two more. This pattern does not grow by two. Show me a pattern that grows by two.”
- Teach explicitly how a growing pattern increases/changes by the same number (+1 or +2) pattern using colors, shapes, or objects.
- Teach/model a growing addition pattern using 2D shapes or 3D objects as a pattern that increases by the same number in each row of the pattern (e.g., a pattern that grows by +3 would have 1 in the first row, 4 in the second row, 7 in the third row).
- Model a growing pattern using a T-Chart:



- Using a T-chart, provide the first two rows of the growing pattern and ask the student to create the third row of the growing pattern.
- Model, Lead, Test
  - Model making a growing pattern using objects. (e.g., “Watch me add two to this pattern to make it grow by 2. …Let’s add two more objects to this pattern together to make it grow by two…You try adding two more objects to this pattern to make it grow by two.”) Ask, what number did we add each time? What is the rule for this growing pattern?
- Task Analysis (Backward Chaining)
  - Provide the first two rows of a growing addition pattern and ask the student to create

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the third row.

- Ask students to determine if the rule for a growing pattern exists for a provided pattern OR to identify a rule for a growing pattern. (A pattern follows a predictable sequence (+1, +2, +3) OR there is no predictable sequence in this example, i.e., no rule can be stated.)

Supports and Scaffolds:

- Counters, shapes and objects
- Examples of growing patterns in real world setting (e.g., in the environment and art)
- Use of graphic organizers to illustrate a growing pattern in which the student places pictures, 2D or 3D shapes or colors
- 2D and 3D shapes or objects, pictures
- Interactive whiteboard or other technology to model AB patterns
- Use of graphic organizers to illustrate a pattern of sets in which the student places 2D or 3D shapes or colors showing a growing pattern

CCSS: 2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

CCC:	2.PRF.1c5	Write or select an equation representing the problem and its solution

Strand: Patterns, Relationships and Functions

Family: Problem Solving and Using Variables

Progress Indicator: ***E.PRF.1c*** *modeling problem solving situations that involve addition and subtraction of whole numbers using objects, diagrams, and symbols*

Essential Understandings	Concrete Understandings:	Representation:
	• Match the action of combining with vocabulary (i.e., in all; altogether) or the action of decomposing with vocabulary (i.e., have left; take away) in a word problem	• Identify a representation of an array that matches the problem • State what the numbers represent • Understand concepts and vocabulary: adding to, take away, equation

Suggested Instructional Strategies:

- Teach explicitly the meaning of "add" and "take away" by connecting the vocabulary to known language (e.g., "add" means plus, more, join; "take away" means less, fewer);
- Teach/model "adding to" a set of object results in a larger set; teach "take away" from a set of objects results in a smaller set;
- Compare two sets of unequal number of objects and ask the student which set has been added to (larger set) OR which set has objects taken away (smaller set).
- Model, Lead, Test:
  - Model "adding to" and "taking away" using objects. (e.g., "Watch me add to this group of objects. …Let's add to this group of objects together…You try adding to this group of objects.")
  - Indicate that the new group of objects is larger if adding to and smaller if taking away.
- Teach explicitly how to create a group/row/set/array of objects for a given number or for a number provided in a simple word problem
- Example/Non-example
  - Present a row of objects (≤ 10). Present a second row of objects that has a different number of objects. Ask the student to select the row with a specified number of objects.
  - Present three rows of objects (≤ 10), two that are equal and one that is not equal. Ask the student to match the two rows that both include the same number of specified objects (e.g., a row of three hats, a row of three hats, a row of 5 shoes).
- Present a simple word problem and a set of arrays to match specified numbers of objects in the problem. Ask the student to identify which array matches a specific number of objects.
- Task Analysis: Provide an equation for a simple addition or subtraction problem and have the student identify the numbers and a corresponding group of objects indicated in the

problem (provide cards with numbers and sets of objects related to the problem).
    - Present a simple subtraction problem (e.g., There are six eggs in the carton. The cook took away three eggs to make breakfast. How many eggs are left in the carton?)
    - State a number of objects in the problem (e.g., six eggs)
    - Show me the number six in the problem.
    - Six tells how many what in the problem?
    - Make a set of six eggs.
    - Show me the number three in the problem.
    - Three tells how many what in the problem?
    - Take away three eggs from six eggs.
    - How many eggs are left in the carton?
    - Show me three in the equation.
- Task Analysis: Provide an equation template for a simple addition or subtraction problem and have the student identify the numbers and a corresponding group of objects indicated in the problem to complete the equation on the template.
- Task Analysis: Use two rows of pictures or objects to model one step addition and subtraction problems
    - Present a simple one step addition problem and sets of object (e.g., Three girls are in the choir. Five boys are in the choir. How many girls and boys are there altogether in the choir?)
    - Here is a group/array of three girls. Here is a group/array of five girls. Show me the array that matches the number of girls in the problem.
    - Here is a group/array of five boys. Here is a group/array of two boys. Show me the array that matches the number of boys in the problem.
    - Provide two problem solving templates with corresponding arrays (let the student manipulate the objects to test out the solutions):
        - Here is a group/array of six girls and boys. ___ ___ ___ ___ + ___ ___ = ___ ___ ___ ___ ___ ___
        - Here is a group/array of eight girls and boys: ___ ___ ___ + ___ ___ ___ ___ ___ = ___ ___ ___ ___ ___ ___ ___ ___
        - Show me the array that matches how many girls and boys are in the choir.
- Provide two or three equations and ask the student to match the equation to a presented simple word problem.

Supports and Scaffolds:

- Pictures and manipulatives
- Template for solving an equation
- Number line
- Calculator

CCSS: 3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

CCC:	3.PRF.2d1	Identify multiplication patterns in a real world setting

Strand: Patterns, Relations, and Functions	**Family:** Describing and Extending Patterns

Progress Indicator: ***E.PRF.2d*** *representing and analyzing patterns and rules (e.g., doubling, adding 3) using words, tables, graphs, and models*

Essential Understandings

Concrete Understandings:	**Representation:**
• Recognize/identify an AB pattern as a pattern that has the same pattern over and over and never changing using colors, shapes, symbols or objects	• Match a pattern using symbols or objects to represent a provided growing multiplication pattern in a real world setting • Recognize patterns and use words to describe the patterns they see. • Understand concepts and vocabulary: growing pattern, multiplication, level, increasing/increases, decreasing/decreases, objects or shapes

Suggested Instructional Strategies:

- Multiple Exemplar Training or Example/Non-Example Training
  - Growing Pattern: "Here is a growing pattern. Here is a growing pattern. Here is growing pattern. This not a growing pattern. Show me a growing pattern."
- Ask students to determine if a rule exists for a provided pattern. (A pattern follows a predictable sequence OR there is no predictable sequence in this example, i.e., no rule can be stated.)
- Model, Lead, Test
  - Teach/model a growing addition pattern using 2D shapes or 3D objects as a pattern that increases by the same number in each row of the pattern (e.g., a pattern that grows by +2 would have 1 in the first row, 3 in the second row, 5 in the third row, and 7 in the fourth row)
  - Teach/model a growing multiplication problem using pictures (1 flower, 2 bees; 2 flowers : 4 bees; 3 flowers : 6 bees)
- Task Analysis (Backward Chaining)
  - Provide the first three rows of a growing addition pattern and ask the student to create the fourth row.
  - Using a T-chart, provide the first three parts of the growing pattern and ask the student to create the fourth part of the pattern.

Supports and Scaffolds:

- Examples of repeating patterns in real world setting (e.g., in the environment and art)
- T-Charts for growing patterns
- Use of graphic organizers to illustrate a pattern of sets in which the student places 2D or 3D shapes or colors using addition or multiplication: e.g., X3 growing pattern

  (□ □ □)
  (□ □ □) (□ □ □)
  (□□ □) (□ □ □) (□ □ □)
- Counters
- 2D and 3D shapes or objects, pictures
- Interactive whiteboard or other technology to model growing patterns

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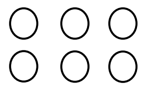
CCSS: 3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*		
CCC:	4.PRF.1d2	Use objects to model multiplication and division situations involving up to 10 groups with up to 5 objects in each group and interpret the results
Strand: Patterns, Relations and Functions	**Family:** Representing and Modeling Problems	
Progress Indicator: ***E.PRF.1d*** *describing and modeling how addition, subtraction, multiplication, or division changes a quantity, including with fractions*		
Essential Understandings	**Concrete Understandings:** • Create an array (e.g., show me 2 groups/rows of 3; or 2X3)	**Representation:** • Use an array to represent a multiplication or division problem • Select a numeral to place under each representation in the modeled equation • Select a pictorial representation of an array that matches the multiplication or division problem • Understand concepts, vocabulary and symbols: =, X, ÷ , groups, objects, set, equal groups, combination, comparison, multiplication, division, array, row, column, equation

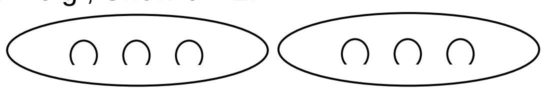
Suggested Instructional Strategies:

- Multiple Exemplar Training
  - Equal sets: “This is a set. This is an equal set. This is an equal set. This is not an equal set. Show me an equal set.”
- Teach multiple ways of describing multiplication (e.g., 2 x 2 = 4; 2 times 2 = 4; a 2 by 2 array is 4).
- Task Analysis for solving simple multiplication and division problems using arrays:
  - **When multiplying**, teach that “X” means multiply and to read the multiplication symbol as “rows of.” For example, read the problem 2 x 3 as "two rows of three."
    - Teach that the first number indicates the number of rows and the second number indicates the number of shapes/objects in each row.
    - Using grid paper or other graphic organizer, draw the first row of the array (e.g., one row of three).
    - Draw the second row of the array (e.g., second row of three).
    - Count the symbols, shapes, pictures or objects in both rows to solve.
    - To interpret the results, state the answer (total number of symbols, shapes, pictures or objects).
  - **When dividing**, solve the problem using arrays. For example, solve 20 ÷ 4:
    - Create a set of 20 shapes/objects based on the first number (the dividend);
    - Divide/separate the set into 4 equal sets (given the second number (4) in the

division problem); and
    - Count the number of equal sets to solve the problem.
- Use arrays to model multiplication and division problems
  - Show (2 X 3); two (number of groups/rows) times three (counters in each group); using a rectangle, the height is the number of rows and the base is the number of units in each row:



  - e.g., Show 6 ÷ 2:



- Trial and error to form equal sets of objects to make the arrays
- Multiple exemplars for equal and not equal
- Model, lead, test

Supports and Scaffolds:
- Counters
- Number lines
- Egg cartons or muffin tins to illustrate/create arrays
- Ones and tens blocks to form different rectangles (rows and columns)
- Manipulatives, visuals and Wiki Sticks to illustrate/define arrays
- T-Chart with two columns showing the multiplier in the first column and the number of units in the second column to represent a growing pattern
- Raised grid (to keep structure of array) or graph paper
- Use PPT and shape tool to create arrays to match a provided problem
- Interactive whiteboard or other technology to create arrays

CCSS: 5.NF.5 Interpret multiplication as scaling (resizing), by:

a) Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

CCC:	5.PRF.1a	Determine whether the product will increase or decrease based on the multiplier

Strand: Patterns, Relations and Functions	**Family:** Proportional Relationships and Graphing

Progress Indicator: ***M.PRF.1a*** *describing how multiplication or division changes a quantity, including with fractions or decimals*

Essential Understandings

Concrete Understandings:

- Show what happens to set when multiplied by 1(1X) or some other whole number (2X)
  - Example: I have 3 chips; which is one set of 3 chips…Now show me 2 sets of 3 chips. …Which is more?
- Use whole numbers.

Representation:

- Select a pictorial representation that matches the multiplication problem
- Recognize when a number is multiplied by a fraction or decimal, the product will decrease
- Recognize when a number is multiplied by a number greater than one, the product will increase
- Vocabulary and symbols: =, X, >, <, proportion, product, multiplier, factor

Suggested Instructional Strategies:

- Explicitly teach that a multiplicand multiplied by a whole number multiplier increases the product and a fraction/decimal multiplier decreases the product; strategy for self-checking answer
- Task Analysis example:
  - State the problem using a whole number multiplier.
  - Predict if the product will increase or decrease
  - Show me one set of (X) chips. Count the chips. How many?
  - Now show me 2 sets of (X) chips. Count the chips. How many?
  - State the total number of chips.
  - Student states if the product is greater or less than the multiplicand.
- Task Analysis example:
  - State the problem using a fraction multiplier.
  - Predict if the product will increase or decrease
  - Provide a set of total number of chips to be divided by a fraction.
  - Model the number of created sets.
  - Student states if the product is greater or less than the multiplicand.

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• Use counting strategies • Use number patterns (i.e., skip counting) • Modeling problem solving with supports • Show multiplication as repeated addition (write 3 X 3 as 3 + 3 + 3)
Supports and Scaffolds: • Counters (chips) • Picture and objects • Number line • Fraction strips and tables • Decimal tables • Multiplication table or calculator to self-check answers • Graphic organizers (rows and columns)

CCSS: 6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

CCC:	6.PRF.1c1	Describe the ratio relationship between two quantities for a given situation

Strand: Patterns, Relationships and Functions

Family: Proportional Relationships and Graphing

Progress Indicator: ***M.PRF.1c*** *comparing two rates and evaluating them for a given situation (e.g., best value)*

Essential Understandings

Concrete Understandings:

- Match/identify a simple ratio (1: X) to the relationship between two quantities.

Representation:

- Recognize the meaning of the placement of numbers in a ratio for a given situation
- Show a ratio in three ways: number to number (1 to 2) expressed as a fraction (1/2) or using a colon 1:2
- Represent the ratio of objects (e.g., red hats) to the total number of objects (red and green hats) Part-to-whole.
- Represent the ratio of the number of one object (red hats) to the number of other objects (green hats) from a set of objects (red and green hats) Part-to-part.
- Understand concept and vocabulary: ratio, rate, proportion, prices, portions per person

Suggested Instructional Strategies:

- Multiple Exemplar Training
    - Example for equal sets: "This is a proportional relationship. This is a proportional relationship. This is a proportional relationship. This is not a proportional relationship. Show me a proportional relationship."
    - Example for representing ratios: There are three chairs for one/each table. The ratio is **3 to 1**. The ratio is **3:1**. The ratio is **3/1.** The ratio is not **1 to 3.** Show me the ratio for three chairs for one table.
- Explicit teaching of three ways to represent a ratio
- Task Analysis example:
    - Read the story problem/situation: "In one (1) day, Jack reads two (2) books. How many books will Jack read in 4 days?"
    - Use the information to fill in a table. Find the word/picture that follows the number. Write the first word/picture in the first row/column (point to the row/column).
    - Write the second word/picture in the second row/first column (point to the row/column).

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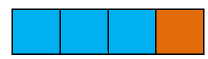
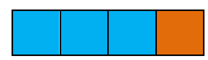
- Use the information in the problem/situation to fill in the number of days.
- Use the information in the problem/situation to fill in the number of books.

Day	1	2	3	4
Total Books Read	2			

- Here is a way to show the ratio/compare the two numbers. (___: ___)
- Put the number of days here. Put the number of books here.
- You showed the ratio of days to books. Show/tell me the ratio.

- Teach what “twice as many” (2 times the original) or “three times as many” (3 times the original) means
- Multiple Exemplar Training example:
  - Ratio: Here is the ratio 3:1. This picture shows the ratio. 
  - This picture shows the ratio. 
  - This does not show the ratio. 
  - Show me a picture that shows the ratio 3:1.
- Provide a ratio and ask the student to use unit blocks to show the ratio (e.g., the ratio of girls to boys in our class is 3:2. Use the unit blocks to show the ratio of girls to boys.)
- Provide unit blocks showing a ratio and ask the student to define the proportional relationship/rule.
- Provide a ratio based on standard measurement and ask the student to complete a table for increasing quantities (e.g., 1 foot = 12 inches; 3 feet = 1 yard; 1 dime = 2 nickels or four quarters = one dollar)

Supports and Scaffolds:

- Unit blocks of ones
- Colored tiles
- Graphic organizer (tables with two columns)
- Grid paper or raised grid paper
- Colored 2D figures and 3D objects
- Interactive whiteboard or other technology to manipulate representations of ratios

<table>
<tr><td colspan="4">CCSS: 6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.</td></tr>
<tr><td>CCC:</td><td>6.PRF.1d1</td><td colspan="2">Solve real world single step linear equation</td></tr>
<tr><td colspan="2">Strand: Patterns, Relationships and Functions</td><td colspan="2">Family: Problem Solving and Using Variables</td></tr>
<tr><td colspan="4">Progress Indicator: M.PRF.1d using symbolic equations to summarize how the quantity of something changes</td></tr>
<tr><td>Essential Understandings</td><td colspan="2">Concrete Understandings:
• Recognize the intended outcome of a word problem based on a linear equation</td><td>Representation:
• Match a representation of an equation with a variable to a real world problem
• Set up an equation in which both sides are equal (adding or subtracting the same number/value from both sides of the equation)
• Understands vocabulary and symbols: +, -, X, ÷, =
• Understands concepts and vocabulary: variable, solution, equation</td></tr>
<tr><td colspan="4">Suggested Instructional Strategies:
• Explicitly teach strategies for determining the operation required to solve a single step real world problem.
• Task Analysis
   o Read the story problem.
   o Identify what question is being asked (define “x”).
   o Identify the facts and the operation (+, - , x, ÷) in the story to write an equation.
   o Add or subtract the number/value from both sides of the equation.
   o Solve the equation for “x”.
   o Show the answer as “x” = ___
• Adding and subtracting strategies</td></tr>
<tr><td colspan="4">Supports and Scaffolds:
• Pictures and manipulatives
• Template for solving an equation
• Number line
• Calculator</td></tr>
</table>

CCSS: 7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error*		
CCC:	7.PRF.1f	Use proportional relationships to solve multistep percent problems
Strand: Patterns, Relationships and Functions	**Family:** Proportional Relationships and Graphing	
Progress Indicator: ***M.PRF.1f*** *identifying essential quantitative relationships in a situation and using symbolic expressions to represent it and draw reasonable conclusions from it*		

Essential Understandings	**Concrete Understandings:**	**Representation:**
	• Identify how one variable changes in relation to another variable in a directly proportional relationship ○ *a/b* = *c/d* (If *a* increases, what will happen to *c*?)	• Use a proportion method to solve (part/whole = number/100) • Set up a proportion to solve for an unknown value • Follow a sequence of steps to solve a problem • Understands vocabulary and symbol: % (percent), proportion

Suggested Instructional Strategies:

- Task Analysis example: Read the story problem/situation: *If 3 out of 5 animals are dogs, what percent of the animals is made up of dogs?*
    1. Fill in the proportion using the provided information in the story problem to record what is known and what is unknown/represented by "**x**" and for which to solve.
    2. Say, "In this problem, you are being asked **3 is what percent** of **5**. You are given two numbers (3 and 5) and asked to find the third (**x**) in this problem."
    3. The **3** is the part. Write **3** in the proportion.
    4. The **5** is the whole. Write **5** in the proportion.
    5. The percent is unknown or **x**. Write **x** in the proportion.

$$\frac{\boxed{\text{part } (\mathbf{3})}}{\boxed{\text{whole } (\mathbf{5})}} = \frac{\boxed{\text{percent } (\mathbf{x})}}{\boxed{100}}$$

    6. Fill in the numbers on each side. $\boxed{\text{whole}}\ \boxed{\text{percent } (\mathbf{x})} = \boxed{\text{Part}}\ \boxed{100}$
    7. Use the calculator to multiply the numbers on each side of the equation.
    8. Use the calculator to divide each side of the equation by 5.
    9. What percent is **x**? Write that number (3 is ___% of 5).
- Explicit instruction on using ratio tables to find a percent of a quantity.
- Explicit instruction on cross multiplying.
- Explicit instruction on solving for "**x**" (dividing both sides).

Supports and Scaffolds:

- Highlight text that provides important information/vocabulary

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- Counters
- Multiplication table
- Calculator
- Table showing fractions and percentages (3/5 = 60%)
- Graphic organizers
- Interactive whiteboard or other technology to manipulate representations

<table>
<tr><td colspan="4">CCSS: 7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
a) Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?
b) Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.</td></tr>
<tr><td>CCC:</td><td>7.PRF.1g2</td><td colspan="2">Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities</td></tr>
<tr><td colspan="2">Strand: Patterns, Relationships and Functions</td><td colspan="2">Family: Problem Solving and Using Variables</td></tr>
<tr><td colspan="4">Progress Indicator: M.PRF.1g modeling, solving, and explaining contextualized problems using various representations such as graphs, tables, functions, and equations</td></tr>
<tr><td>Essential Understandings</td><td colspan="2">Concrete Understandings:
• Record/replace a variable in an equation with a fact from a story on a graphic organizer.</td><td>Representation:
• Create a pictorial array of a simple equation to translate wording
• Understand concepts, vocabulary and symbols +, -, X, ÷, =, ≠, $<$, $>$, equation, equal, inequality</td></tr>
<tr><td colspan="4">Suggested Instructional Strategies:
• Explicitly teach equals vs. inequality.
• Explicitly teach strategies for determining the operation required to solve a single step real world problem.
• Task Analysis
  ○ Read a story problem that is personally relevant to the student.
  ○ Identify what question is being asked (define “x”).
  ○ Identify the facts and the operation (+, - x, ÷) in the story.
  ○ Provide graphic organizer or template to organize the facts and write.
  ○ Write an equation to solve for “x”.
  ○ Add, subtract, multiply or divide the number/value to both sides of the equation.
  ○ Solve the equation for “x”
  ○ Show the answer as “x” = ___
• Explicit teaching of how to identify what question is being asked (i.e., what “x” represents in</td></tr>
</table>

the story problem). • Provide an equation for which the student will determine a story problem. • Create a personally relevant story; Provide graphic organizers as a means for organizing student's work; Task analytic instruction to break down skills and chain them in order to isolate each step in solving the math task • Adding and subtracting strategies • Multiplying and dividing strategies
Supports and Scaffolds: • Counters • Number lines • Multiplication tables • Calculator

CCSS: 8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*		
CCC:	8.PRF.1e2	Represent proportional relationships on a line graph
Strand: Patterns, Relationships and Functions	**Family:** Proportional Relationships and Graphing	
Progress Indicator: ***M.PRF.1e*** *representing and computing unit rates associated with ratios of lengths, areas, and other quantities measured in like or different units*		
Essential Understandings	**Concrete Understandings:** • Recognize a positive relationship between two variables.	**Representation:** • Graph a series of coordinates on a graph • Identify given coordinates (x,y) as a point on a graph • Identify the intercept(s) on a graph • Understand concepts, vocabulary and symbols: coordinates, ordered pairs (x,y), intercept, grid, axis, point, proportion, line, slope

Suggested Instructional Strategies:

- Teach explicitly that a coordinate grid has two perpendicular lines, or axes, labeled like number lines.
- Teach explicitly how to recognize the relationship between y and x using the coordinates of several points (e.g., y increases as **x** increases; the ratio is the same for all values if they are directly proportional).
- Provide multiple examples of line graphs with different, labeled coordinates and slopes.
- Teach explicitly how to plot coordinates on a grid and draw the line.
- Teach explicitly how to define a line provided on a grid by multiple coordinates.
- Teach explicitly simple distance/time problems that illustrate how the rates of two objects can be represented, analyzed and described graphically.
- Task Analysis
    - Provide a series of proportional coordinates
    - Present a labeled graph
    - Identify the **x** coordinate and **y** coordinate and plot each point
    - List coordinates on a T-chart, (**x** in one column and **y** in the other) for each set of coordinates
    - State the proportional relations; _ : _

Supports and Scaffolds:

- Grid paper with raised perpendicular lines (horizontal and vertical lines) and points
- Models
- T-Chart, graphic organizer

The Element Cards for the prioritized mathematics CCCs are being reviewed by NCSC Partner States to improve and expand their content. Additionally, new Element Cards are being developed to address the remaining CCCs. As the Element Cards are updated and created, they will be posted on SharePoint and the Wiki for teacher use.

- Rulers, straightedge
- Calculator

<table>
<tr><td colspan="4">CCSS: 8.EE.7 Solve linear equations in one variable.
a) Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different)
b) Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.</td></tr>
<tr><td>CCC:</td><td>8.PRF.1g3</td><td colspan="2">Solve linear equations with 1 variable</td></tr>
<tr><td colspan="2">Strand: Patterns, Relationships and Functions</td><td colspan="2">Family: Problem Solving and Using Variables</td></tr>
<tr><td colspan="4">Progress Indicator: M.PRF.1g modeling, solving, and explaining contextualized problems using various representations such as graphs, tables, functions, and equations</td></tr>
<tr><td>Essential Understandings</td><td colspan="2">Concrete Understandings:
• Use manipulatives or graphic organizer to solve a problem.</td><td>Representation:
• Create a pictorial array of a simple equation to translate wording to solve for x or y
• Understand concepts, vocabulary and symbols: +, -, X, ÷, =, variable, equation</td></tr>
<tr><td colspan="4">Suggested Instructional Strategies:
• Explicit strategy to solve equation by dividing both sides of the equation by the value in front of the variable and then simplify
• Use of trial and error to determine the value of x or y (is the product too low, too high)
• Use arrays (e.g., 3y = 12; a total of 12 counters), divided into three equal sets, how many tokens in each set (= “y”)
• Task Analysis
o Read the story problem.
o Identify what question is being asked / x represents (define “x”).
o Identify the facts and the operation (+, - x, ÷) in a story to write an equation.
o Solve the equation for “x”.
o Show the answer as “x” = ___</td></tr>
<tr><td colspan="4">Supports and Scaffolds:
• Counters
• Grids or graphic organizers to create arrays
• Multiplication chart
• Calculator</td></tr>
</table>

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CCSS: 8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x, y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.			
CCC:	8.PRF.2e2	Identify the rate of change (slope) and initial value (y-intercept) from graphs	
Strand: Patterns, Relationships and Functions		**Family:** Proportional Relationships and Graphing	
Progress Indicator: ***M.PRF.2e*** *using functions to describe quantitative relationships*			
Essential Understandings	**Concrete Understandings:** • Indicate the point on a line that crosses the y-axis.		**Representation:** • Interpret/define a line graph with coordinates for multiple points • Identify coordinates (points) on a graph • Understand concepts, vocabulary: x axis, y axis, x intercept, y intercept, line, rise, fall, slope, rate of change

Suggested Instructional Strategies:

- Explicit teaching of axis (x-axis is the horizontal axis and the y-axis is the vertical axis) and coordinates for points
- Explicit teaching of identifying x,y coordinates for points on a graph
- Explicit teaching of counting distances between points on each axis
- Explicitly teach that when x = 0, you are on the y axis (the y intercept); the initial value is the "starting point" when the line only passes through the y axis once.
- Models of line graphs (positive: rises from left to right and negative: falls from left to right); and coordinates of varying slope; match coordinates to graphs
- Task analysis for rate of change/slope:
    - Present a line graph showing the unit rate as the slope for a series of proportional coordinates.
    - Present a formula template: slope = rise/run
    - Teach slope as the steepness, or slant, of a line is called the slope (e.g., a steep mountain)
    - Identify two points on the line
    - Label the p points 1 and 2
    - Using two different colored pencils, mark the rise (red) and run (blue)
    - Count the rise (How many units the student counts **up** (positive) or **down** (negative) to get from one point to the next? Record this number (change in value) as your numerator.)
    - Count the run (How many units do you count **left** (down/negative) or right to get to the point? Record this number (change in value) as your denominator.)
    - Simplify the fraction if possible.
- Give students opportunities to gather their own data or graphs in familiar contexts

- Task analysis for initial value (y intercept)
    - Provide a template for recording the y intercept: y intercept/starting point = (0, **(y)**)
    - Find the y axis.
    - Highlight the y axis.
    - Look at the graph and identify and circle the point at which the line passes through the **y** axis.
    - Fill in the value of y in the template.

Supports and Scaffolds:

- Grid paper with raised perpendicular lines (horizontal and vertical lines) and points
- Models
- T-Chart, graphic organizer
- Rulers, straightedge
- Colored pencils/markers

<table>
<tr><td colspan="4">CCSS: F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.</td></tr>
<tr><td>CCC:</td><td>H.PRF.1c</td><td colspan="2">Select the appropriate graphical representation of a linear model based on real world events</td></tr>
<tr><td colspan="2">Strand: Patterns, Relationships and Functions</td><td colspan="2">Family: Proportional Relationships and Graphing</td></tr>
<tr><td colspan="4">Progress Indicator: H.PRF.1c creating mathematical models, using rules and relationships to describe and predict objects and events in the real world</td></tr>
<tr><td>Essential Understandings</td><td colspan="2">Concrete Understandings:
Match a point not on a line as not being part of a data set for a given line</td><td>Representation:
Identify coordinates (points) on a graph and in a data table
Select a graph that represents a simple linear equation
Match or plot the points from a data table on a graph
Understand concepts and vocabulary: x axis, y axis, x intercept, y intercept, line, slope</td></tr>
<tr><td colspan="4">Suggested Instructional Strategies:
Models of line graphs and coordinates of varying slopes; match coordinates to graphs
Teach explicitly the relationship between positive slope and a line that slopes up left to right and negative slope and a line that goes down left to right
Task Analysis:
Present a story problem and a simple equation (e.g., y = 5x).
Create a formula template and substitute x for at least three values to determine y.
Create a table (T-Chart) listing coordinates (x,y).
Plot points on a coordinate grid; connect the points.
Identify the coordinates on the line graph.
(Reverse the steps and begin with a line graph; identify the coordinates of at least three points, create a table listing the x and y coordinates; write a simple linear equation to represent the line graph.)</td></tr>
<tr><td colspan="4">Supports and Scaffolds:
Grid paper with raised perpendicular lines (horizontal and vertical lines) and points
Models
T-Chart, graphic organizer
Rulers, straightedge</td></tr>
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<table>
<tr><td colspan="4">CCSS: A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions</td></tr>
<tr><td>CCC:</td><td>H.PRF.2b1</td><td colspan="2">Translate a real-world problem into a one variable linear equation</td></tr>
<tr><td colspan="2">Strand: Patterns, Relationships and Functions</td><td colspan="2">Family: Problem Solving and Using Variables</td></tr>
<tr><td colspan="4">Progress Indicator: H.PRF.2b creating equations and inequalities (in one or two variables) and use them to solve problems and graph solutions</td></tr>
<tr><td>Essential Understandings</td><td colspan="2">Concrete Understandings:
• Match an equation with one variable to the real world context.</td><td>Representation:
• Create a pictorial array of a simple equation to translate wording
• Symbols: +, -, X, ÷, =</td></tr>
<tr><td colspan="4">Suggested Instructional Strategies:
• Task Analysis
o Present the story problem based on real-world, relevant context and provide a template for recording facts/operations to solve the real-world problem.
o Highlight key information in the problem; strike through irrelevant information.
o Identify what question is being asked (define x).
o Identify the facts.
o Fill in the facts in the order presented in the story problem on the template.
o Determine the operation(s) (+, - X, ÷)
o Identify what operation should be completed first.
o Fill in the operation.
o State the equation.
o Solve for x.
o Answer the problem statement.</td></tr>
<tr><td colspan="4">Supports and Scaffolds:
• Counters
• Multiplication chart
• Calculator</td></tr>
</table>

NCSC Curriculum Resource to Prepare Students for AA-AAS

Mathematics Content: Equations

National Center and State Collaborative

The National Center and State Collaborative (NCSC) is applying the lessons learned from the past decade of research on alternate assessments based on alternate achievement standards (AA-AAS) to develop a multi-state comprehensive assessment system for students with significant cognitive disabilities. The project draws on a strong research base to develop an AA-AAS that is built from the ground up on powerful validity arguments linked to clear learning outcomes and defensible assessment results, to complement the work of the Race to the Top Common State Assessment Program (RTTA) consortia.

Our long-term goal is to ensure that students with significant cognitive disabilities achieve increasingly higher academic outcomes and leave high school ready for post-secondary options. A well-designed summative assessment alone is insufficient to achieve that goal. Thus, NCSC is developing a full system intended to support educators, which includes formative assessment tools and strategies, professional development on appropriate interim uses of data for progress monitoring, and management systems to ease the burdens of administration and documentation. All partners share a commitment to the research-to-practice focus of the project and the development of a comprehensive model of curriculum, instruction, assessment, and supportive professional development. These supports will improve the alignment of the entire system and strengthen the validity of inferences of the system of assessments.

IDEAs that Work

U.S. Office of Special Education Programs

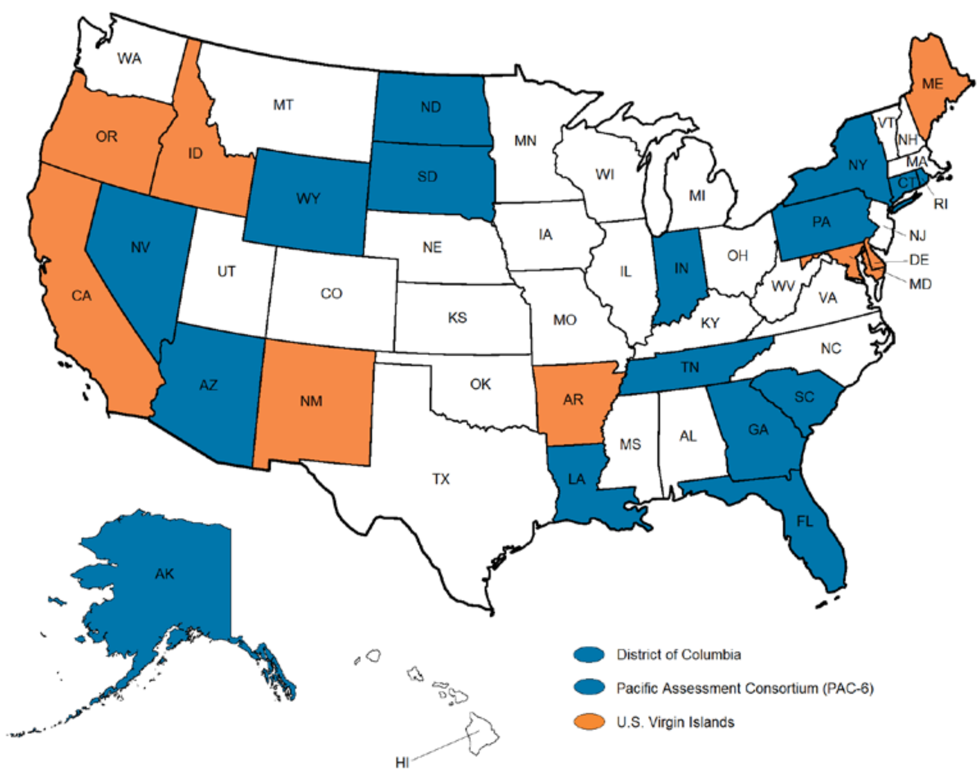
The contents of this Resource Guide were developed as part of the National Center and State Collaborative by Julie Thompson and Diane Browder at the University of North Carolina at Charlotte, and verified by mathematics content experts Amy Lehew, Charlotte-Mecklenburg Schools, and William Kliche, South Dakota Department of Education, for a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made.

National Center and State Collaborative

NCSC is a collaborative of 18 states and five organizations.

The states include (shown in blue on map): Alaska, Arizona, Connecticut, District of Columbia, Florida, Georgia, Indiana, Louisiana, Nevada, New York, North Dakota, Pacific Assessment Consortium (PAC-6)[1], Pennsylvania, Rhode Island, South Carolina, South Dakota, Tennessee, and Wyoming.

Tier II states are partners in curriculum, instruction, and professional development implementation but are not part of the assessment development work. They are (shown in orange on map): Arkansas, California, Delaware, Idaho, Maine, Maryland, New Mexico, Oregon, and U.S. Virgin Islands.



[1] The Pacific Assessment Consortium (including the entities of American Samoa, Commonwealth of the Northern Mariana Islands, Federated States of Micronesia, Guam, Republic of Palau, and Republic of the Marshall Islands) partner with NCSC as one state, led by the University of Guam Center for Excellence in Developmental Disabilities Education, Research, and Service (CEDDERS).

ncsc

National Center and State Collaborative

The five partner organizations include: The National Center on Educational Outcomes (NCEO) at the University of Minnesota, The National Center for the Improvement of Educational Assessment (Center for Assessment), The University of North Carolina at Charlotte, The University of Kentucky, and edCount, LLC.

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National Center and State Collaborative

NCSC Curriculum Resource to Prepare Students for AA-AAS

Mathematics Content: Equations

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Diane Browder
Amy Lehew
William Kliche

January 2013

Table of Contents

Curriculum Resource to Prepare Students for AA-AAS
Mathematics Content: Equations

The purposes of the Curriculum Resource Guides Are:
• To provide guidance for teaching the Common Core State Standards (CCSS) to students with Significant Cognitive Disabilities (SWSCD) that both aligns with these standards and provides differentiation for individual student needs • To provide examples for differentiating instruction for a wide range of SWSCD. These examples can be used in planning specific lessons, alternate assessment items, and professional development. • To serve as a companion document to the Progress Indicators for the CCSS found in the NCSC Learning Progressions • To help educators build knowledge of the essential content reflected in these Progress Indicators of the CCSS • To delineate the necessary skills and knowledge students need to acquire to master these indicators

1. What are “equations” and how are they taught in general education settings?

1a.1 The essential knowledge in this content area

Equations are a statement that the vales of two mathematical expressions are equal. Expressions can be thought of as a phrase while an equation is a complete sentence.

Comparison of Terms[2]	
Expression	**Equation**
mathematical phrase: x + 3	mathematical sentence: x + 3 = 9
word phrase: a number plus three	word sentence: a number plus three is nine
number, operation, variable	number, operation, variable, equal sign
evaluate: 1. substitute 2. simplify	solve for/isolate the variable
	one solution

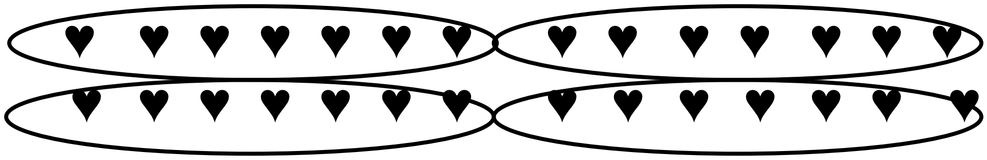
$$\underbrace{\underbrace{4 \times 7}_{\text{expression}} = 28}_{\text{equation}}$$

$$\underbrace{\underbrace{4 \times 7}_{\text{expression}} = \underbrace{2 \times 14}_{\text{expression}}}_{\text{equation}}$$

[2] Commonwealth of Pennsylvania, 2012. Retrieved from http://www.pdesas.org/module/content/resources/6028/view.ashx.

Students can use models (objects or drawings) to represent expressions or equations.

4 x 7 →



Multiplication and Division Equations

The standards require students to understand that “x” or “times” means “groups of” and 4 x 7 means 4 groups of 7. Also, “÷” or “divided by” means “how many in each group” or “how many groups can you make?” Example: The children rode in 4 cars to the museum. There were 3 children in each car. How many children went to the museum? 4 groups of 3, 4 x 3 = 12.

When given word problems, students will need to be able to identify the key word in order to determine what operation is required to represent and solve the problem. Key words for multiplication include: product, of, multiplied, times, as much, by, and twice. Key words for division include: divide evenly, cut, split, each, every, average, equal pieces, out of, ratio, share, and quotient. Students need practice reading word problems to identify the key words and match the correct operation to the problem.

Examples: Circle the key word and write the correct operation.

Jon gets a $12 allowance per month. How much allowance does he get each week? (4 weeks in a month)
Operation: ÷

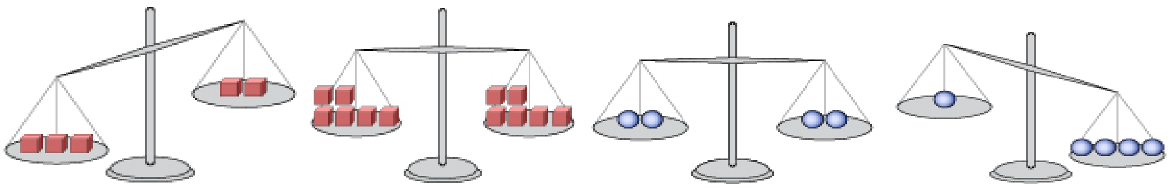
Nasir wants to play cars with his friends. He has 9 cars that he wants to share with his 3 friends. How many cars will each friend receive?
Operation: ÷

Esteban finished 4 math problems. Cecily finished 2 times as much. How many math problems did Cecily finish?
Operation: x

Identify when two expressions are equivalent

The expressions on either side of the equal sign must represent the same quantity. Students can first be taught this as a rule (e.g., “You must have the same amount on both sides of the equal symbol.”) Then provide students with practice in determining whether sets are equal. Another way to describe equality is that there must be “fair shares” on either side of the equals sign. One way to teach equality is to use a balance. For example, provide the following chart then use a balance and weights to represent the amounts. Then discuss whether the numbers are equal (fair shares) or not equal and fill in the third column.

Left side	Right side	Equation
3	2	**3 ≠ 2**
6	6	**6 = 6**
2	2	**2 = 2**
1	4	**1 ≠ 4**



Once students are firm on their understanding of equality using single whole numbers, then they can better determine whether expressions are equivalent. Students can do this by simplifying or solving the expression.

For example:

Are the expressions equivalent?

8 – 4 and 3 + 2

Expression	Simplifying Expression
8 – 4	4
3 + 2	5

4 ≠ 5

No, the expressions are not equivalent.

Solving for Variables in Equations

In equations, variables are often used as placeholders for unknown quantities. When given a word problem or real life situation, students can be taught to assign a variable to the unknown quantity. In an equation, this variable represents a specific value. For example:

Shelby wrote some thank you letters in the afternoon. She wrote 3 more that night. She wrote 10 thank you letters in all. How many letters did Shelby write that afternoon?

Students can use “n” to represent the unknown quantity of letters written in the afternoon. “In all” is a key word that indicates this is an addition problem.

$$n + 3 = 10$$

Using inverse operations to solve equations

In order to solve an equation by determining the value of the variable, students must learn to isolate the variable, or work the problem so that the variable is “alone” on one side of the equal symbol. To isolate a variable, students need to use inverse operations. Inverse operations can be thought of as “opposite” operations. The inverse operation of addition is subtraction and the inverse operation of multiplication is division. For example:

$$\begin{array}{rr} & n + 3 = 10 \\ \text{inverse operation} \longrightarrow & -3 \quad -3 \\ & n = 7 \end{array}$$

In the example above, the inverse operation of minus 3 was used on both sides of the equation. Students must be taught the rule that “what you do on one side of the equals symbol, you must do on the other side.” To help students comprehend this, you can use the scale example again. Teachers can set up a scale with equal amounts of items so that the scale is balanced. Have students take away a few items from one side and observe that the scale becomes unbalanced (i.e., the sides are unequal). Then have the students take away the same amount from the other side and observe that the scale is balanced again (i.e., the side are equal).

1a.2 Common misunderstandings in this content area

Students often think that the equals sign means “get an answer.” It is important to provide repeated opportunities to demonstrate equality (as described above) to ensure they comprehend the meaning of equality.

1a.3 Prior knowledge/skills needed (can be taught concurrently)

- Performing basic operations (addition, subtraction, multiplication, and division)
- Number and symbol identification

2. What are some of the types of activities general educators will use to teach this skill?

2.1 Activities from General Education Resources

- **2** **2** “Story translations”[3] – students are given a math story and asked to write an equation that means the same thing.
- **4** **2** “Names for numbers”[2] – students are given a number and asked to write several expressions to represent the number (e.g., given number 10, student writes the following expressions: 2+8, 20÷2, 17-7)
- **3** **1** “Tilt or balance”[2] – draw a balance on the board and write an expression above each pan and ask students whether it will tilt or balance. (e.g., left pan says 2 x 7 and right pan says 6 + 6; students call out “tilt”)

[3] Walle. J. A. V. de, & Lovin, L. A. H. (2005a). *Teaching Student-Centered Mathematics: Grades K-3* (1st ed.). Allyn & Bacon.
Walle. J. A. V. de, & Lovin, L. A. H. (2005b). *Teaching Student-Centered Mathematics: Grades 3-5 Volume* (1st ed.). Allyn & Bacon.
Walle. J. A. V. de, & Lovin, L. A. H. (2005c). *Teaching Student-Centered Mathematics: Grades 5-8* (1st ed.). Allyn & Bacon.

- **2** **1** Given a story problem and a list of items/concepts related to the story problem, the student is asked to identify which items are variables (unknown quantities). For example, if the story problem says "Nina worked for thirty minutes, and then she took a break. Then she worked for another 45 minutes. She was at work a total of one and a half hours." Circle the variable: first work session, break, or second work session.
- **5** **3** Given a formula and batting statistics for a player, the student is asked to substitute the variables for the correct values and then solve the problem to determine a player's batting average.

Links Across Content Areas

- Science – use formulas, substitute values for variables, and solve equation to determine: acceleration, mass, volume, friction, orbit, etc.

3. What Connectors to the Common Core Standards Are Addressed in Teaching "Equations"?

Grade Differentiation	Core Content Connectors	Common Core State Standards
3rd grade	3.NO.2b Use the relationships between addition and subtraction to solve problems	3.NBT.2
	3.NO.2c1 Solve multi-step addition and subtraction problems up to 100	3.NBT.2
	3.NO.2d1 Find the total number of objects when given the number of identical groups and the number of objects in each group neither number larger than 5	2.OA.4 3.OA.1
	3.NO.2d2 Find total number inside an array with neither number in the columns or rows larger than 5	2.OA.4 3.OA.1
	3.NO.2d3 Solve multiplication problems with neither number greater than 5	3.OA.1
	3.NO.2d4 Determine how many objects go into each group when given the total number of objects and the number of groups where the number in each group or number of groups is not greater than 5	3.OA.2
	3.NO.2d5 Determine the number of groups given the number of total number of objects and the number of objects in each group where the number in each group and the number of groups is not greater than 5	3.OA.2
	3.NO.2e1 Solve and check one or two-step word problems requiring addition, subtraction, or multiplication with answers up to 100.	3.OA.8
4th grade	4.NO.2c2 Solve multi digit addition and subtraction problems up to 1000	3.NBT.2
	4.NO.2d6 Find total number inside an array with neither number in the columns or rows larger than 10	3.OA.1
	4.NO.2d7 Determine how many objects go into each group when given the total number of objects and the number of groups where the number in each group or number of groups is not greater than 10	3.OA.2
	4.NO.2d8: Match an accurate addition and multiplication equation to a representation	3.OA.1
	4.NO.2e2 Solve or solve and check one or two step word problems requiring addition, subtraction or multiplication with answers up to 100	4.OA.3
	4.PRF.1d2 Use objects to model multiplication and division situations involving up to 10 groups with up to 5 objects in each group and interpret the results	3.OA.1
	4.PRF.1e3 SOLVE multiplicative comparisons with an unknown using up to 2-digit numbers with information presented in a graph or word problem (e.g., an orange hat cost $3. A purple hat cost 2 times as much. How much does the purple hat cost? [3 x 2 = p])	4.OA.2
5th grade	5.SE.1a1 Given a real world problem, write an equation using 1 set of parentheses	5.OA.1
	5.SE.1b Evaluate whether or not both sides of an equation are equal	6.EE.4
	5.NO.2a1 Solve problems or word problems using up to three digit numbers and addition or subtraction	4.OA.3
	5.NO.2a5 Solve word problems that require multiplication or division	5.NBT.6
6th grade	6.SE.1a2 <u>Repeated</u> Given a real world problem, write an equation using 1 set of parentheses	6.EE.2c 6.EE.6
	6.SE.1a3 Write expressions for real-world problems involving one unknown number	
	6.PRF.2a2 Use variable to represent numbers and write expressions when solving real world problems	6.EE.6
	6.PRF.2a3 Use variables to represent two quantities in a real-world problem that change in relationship to one another	6.EE.9

Grade Differentiation	Core Content Connectors	Common Core State Standards
	6.PRF.1d1 Solve real world single step linear equations	6.EE.7
7th grade	7.SE.1f1 Set up equations with 1 variable based on real world problems	7.EE.4
	7.SE.1f2 Solve equations with 1 variable based on real world problems	7.EE.4
	7.PRF.1g2 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities	7.EE.4
	7.PRF.2a5 Repeated Use variables to represent two quantities in a real-world problem that change in relationship to one another	6.EE.9
	7.PRF.2d Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers	7.EE 4b
8th grade	8.PRF.1f2 Describe OR SELECT THE relationship between the two quantities Given a line graph of a situation	8.EE.5
	8.PRF.1g3 Solve linear equations with 1 variable	8.EE.7
Grades 9-12	HS.NO.1a1 Simplify expressions that include exponents	N.RN.2 A.SSE.3
	H.NO.2a Solve simple equations using rational numbers with one or more variables	A.REI.2
	H.NO.3a2 Rewrite mathematical statements (e.g., an expression) in multiple forms	
	H.PRF.2b1 Translate a real-world problem into a one variable linear equation	A.CED.1
	H.PRF.2b2 Solve equations with one or two variables using equations or graphs	A.REI.1 A.REI.3 A.CED.2

Performance Examples for Priority CCCs			
Grade 3			
CCC	**Performance Example**	**Essential Understandings: Concrete Understandings and Representations**	
Numbers: 3.NO.2d3 Solve multiplication problems with neither number greater than 5. (3.OA.1)	Student solves simple multiplication. "This says four times five. What is four times five?" 4 x 5	**Concrete Understandings:** • Create an array of sets (e.g., 3 rows of 2 objects)	**Representation:** • Identify or draw pictorial representation of an array that matches the multiplication problem • State what the numbers represent (ex. first number is number of sets, second number is number within each set)
Numbers: 3.NO.2e1 Solve and check one or two-step word problems requiring addition, subtraction, or multiplication with answers up to 100. (3.OA.8)	Student selects expression that matches word problem, solves problem, then selects equivalent expression that can be used to check work. Kunius had 3 weeks to sell cookies for school. He sold 6 boxes each week. Which of these will show how many boxes Kunius sold? 3 + 6 3 x 6 3 – 6 How many boxes did Kunius sell? Which one of these can be used to check your work? 3 + 3 + 3 18 – 3 6 + 6 + 6	**Concrete Understandings:** • Combine (+), decompose (-), and multiply (x) with concrete objects; use counting to get the answers • Match the action of combining with vocabulary (i.e., in all; altogether) or the action of decomposing with vocabulary (i.e., have left; take away; the difference) in a word problem	**Representation:** • Draw or use a representation of the word problem • Understand symbols: +, =, -, x • Add on or count back depending upon the words in the problem

Grade 4			
CCC	**Performance Example**	**Essential Understandings: Concrete Understandings and Representations**	
Patterns: 4.PRF.1d2: Use objects to model multiplication and division situations involving up to 10 groups with up to 5 objects in each group and interpret the results (3.OA.1)	Present a paper with the following printed on it and read it aloud: "Ms. Smith is an art teacher. She is preparing to teach an art lesson to five students. Each student will need four markers to complete the art activity. You need to find out how many markers Ms. Smith will need all together." Give the student 24 markers. "Use these markers to show me how five students would each get four markers. You may not use all the markers." If the student makes an error, model the correct answer and say "There should be five groups of four markers, like this." "How many markers does the teacher need all together?"	**Concrete Understandings:** • Create an array (e.g., show me 2 groups/rows of 3; or 2X3)	**Representation:** • Use an array to represent a multiplication or division problem • Select a numeral to place under each representation in the modeled equation • Select a pictorial representation of an array that matches the multiplication or division problem • Understand concepts, vocabulary and symbols: =, X, ÷ , groups, objects, set, equal groups, combination, comparison, multiplication, division, array, row, column, equation

Numbers: 4.NO.2d7: Determine how many objects go into each group when given the total number of objects and the number of groups where in each group or number of groups is not greater than 10 (3.OA.2)	"Bethany and her friends decided to start a dog walking business after school to earn some spending money. "Bethany has three friends, for a total of four people who want to walk dogs." Point to the picture of Bethany and her friends. "There are eight dogs that need to be walked." Point to the picture of the dogs. "Each person will walk an equal number of dogs. How many dogs will each person walk? You can use these blocks to help solve the problem."	**Concrete Understandings:** • Create an array of objects given a specific number of rows and the total number, place one object in each group/row at a time	**Representation:** • Draw an array using the given information • Understand symbols: ÷, =
Numbers: 4.NO.2d8: Match an accurate addition and multiplication equation to a representation (3.OA.1)	Present the student with the football pictures and say "Here is a picture of a bunch of footballs. When things are lined up like this it helps us think about addition and multiplication." ♪ ♪ ♪ ♪ ♪ ♪ ♪ ♪ ♪ ♪ ♪ ♪ ♪ ♪ ♪ "Now here are three addition equations. Which equation shows what you see in this picture?" 3+3+3+3=12 5+5+5=15 5+5+3=13 "Now here are three multiplication equations. Which equation shows what you see in this picture?" 1x3=3 3x4=12 3x5=15	**Concrete Understandings:** • Select the representation of manipulatives on a graphic organizer to show addition/multiplication equation. • Match to same for representations of equations (may be different objects but same configuration)	**Representation:** • Select a representation to place under each numeral in addition equation • State what the numbers represent in multiplication equation (ex. first number is number of sets, second number is number within each set) • Select a pictorial representation of an array that matches the multiplication problem • Understand symbols +, x, =

Grade 5			
CCC	**Performance Example**	**Essential Understandings: Concrete Understandings and Representations**	
Symbols 5.SE.1b Evaluate whether both sides of an equation are equal. (6.EE.4)	Student indicates whether an equation is true. I am going to show you some equations. Tell me if each equation is true. An equation is true if the expressions on both sides of the equation are equal. Here is the first equation. Is this equation true? Is 3 + 8 equal to 10 + 1? 3 + 8 = 10 + 1 Is 12 – 3 equal to 5 + 4? 12 – 3 = 5 + 4 Is 18 ÷ 6 equal to 2 x 3? 18 ÷ 6 = 2 x 3	**Concrete Understandings:** • Determine if sets are equal/ not equal • Model an equation with objects	**Representation:** • Understand symbols for =, ≠, +, -, ÷, ×
Numbers: 5.NO.2a1: Solve problems or word problems using up to three digit numbers and addition or subtraction. (4.OA.3)	Present the note card with the following word problem on it and read aloud to the student “The baseball team has decided to sell candles to raise money to go on a trip to play baseball at a tournament. They have 450 candles to sell. After one day, they had sold 324 candles. How many candles are left to sell? Show me your work and how you would solve this equation.” If the student does not generate a correct equation show them an already written equation: 450 – 324 = ______ “Solve the equation, how many candles do they have left to sell?”	**Concrete Understandings:** • Combine (+) or decompose (-) with concrete objects; use counting to get the answers • Match the action of combining with vocabulary (i.e., in all; altogether) or the action of decomposing with vocabulary (i.e., have left; take away, difference) in a word problem	**Representation:** • Draw or use a representation of the word problem • Understand symbols +, =, -, • Add on or count back depending upon the words in the problem • Translate wording into numeric equation

Numbers: 5.NO.2a5: Solve word problems that require multiplication or division. (5.NBT.6)	Problem 1: Present the note card with the following word problem on it and read aloud to the student "Ms. Wood's class is going on a field trip to the zoo. There are 9 people going to the zoo. It costs $11 per person for admission into the zoo. How much money does the class need in all? Show me your work as you solve the problem." If the student does not generate a correct equation show them an already written equation: 9 x 11 = ______ "Solve the equation, how many much money do they need?" Problem 2: Present the note card with the following word problem on it and read aloud to the student "While at the zoo, they decide to feed the donkey. The zoo sells bags of 36 carrots to feed to the donkeys. Remember, there are nine people on the trip. If they split it up evenly, how many carrots does each person get to feed to the donkey? Show me your work as you solve the problem." If the student does not generate a correct equation show them an already written equation: $\frac{36}{9}$ = ______ "Solve the equation, how many carrots does each person get to feed to the donkey?"	**Concrete Understandings:** • Combine (x) or decompose (÷) with concrete objects; use counting to get the answers • Match the action of combining with vocabulary (i.e., in all; altogether) or the action of decomposing with vocabulary (i.e., have left; take away) in a word problem • Understand concept of division: Sharing or grouping numbers into equal parts • Understand concept of multiplication: the result of making some number of copies of the original	**Representation:** • Draw or use a representation of the word problem • Symbols ÷, =, x • Identifying purpose to either find a total (multiplication) or one component (number of sets or number within each set for division) depending upon the problem • Translate wording into numeric equation

Grade 6			
CCC	**Performance Example**	**Essential Understandings: Concrete Understandings and Representations**	
Patterns: 6.PRF.1d1: Solve real world single step linear equations (6.EE.7)	Show students the following word problem and read it aloud: "Hans has $10 to spend while playing mini-golf. He spent $7 on the ticket and spent the rest on candy. Write an equation to show how much Hans spent on snacks. Use the letter S to represent the amount he spent on snacks." Student writes an appropriate equations (e.g., 10-s=7, 10-7=s, 10=7+s etc.). After student write the equation (if student gets this portion wrong, write an appropriate equation for the student) ask: "Solve this equation to see how much money Hans spent on snacks. Show your work."	**Concrete Understandings:** • Recognize the intended outcome of a word problem based on a linear equation	**Representation:** • Match a representation of an equation with a variable to a real world problem • Set up an equation in which both sides are equal (adding or subtracting the same number/value from both sides of the equation) • Understands vocabulary and symbols: +, -, X, ÷, = • Understands concepts and vocabulary: variable, solution, equation

Grade 7			
CCC	**Performance Example**	**Essential Understandings: Concrete Understandings and Representations**	
Patterns: 7.PRF.1g2: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities (7.EE.4)	Show the students the following word problem and read it aloud: Barney wants to buy a new video game. He has 24 dollars. He needs 50 dollars to buy the new video game. Present the student with the equation template and cut out response cards (below) and say: "Use these cards to make the equation. Use the letter d to represent how much more money he needs because that's an unknown variable." $\boxed{24}$ $\boxed{50}$ $\boxed{D}$ $\boxed{} + \boxed{} = \boxed{}$ Say: "Use the equation to solve for how much more money Barney needs to buy the video game."	**Concrete Understandings:** • Record/replace a variable in an equation with a fact from a story on a graphic organizer.	**Representation:** • Create a pictorial array of a simple equation to translate wording • Understand concepts, vocabulary and symbols: +, -, X, ÷, =, ≠, <, >, equation, equal, inequality

Grade 8			
CCC	**Performance Example**	**Essential Understandings: Concrete Understandings and Representations**	
Patterns: 8.PRF.1g3: Solve linear equations with 1 variable (8.EE.7)	Present the equation to the student and say "You are going to solve a problem using a variable. Remember, a variable is a letter that represents an unknown number." Read the equation to the student and have them solve it. $54 = 9x$	**Concrete Understandings:** • Use manipulatives or graphic organizer to solve a problem.	**Representation:** • Create a pictorial array of a simple equation to translate wording to solve for x or y • Understand concepts, vocabulary and symbols: +, -, X, ÷, =, variable, equation

High School			
CCC	**Performance Example**	**Essential Understandings: Concrete Understandings and Representations**	
Numbers: H.NO.3a2 Rewrite mathematical statements (e.g., an expression in multiple forms) (No CCC listed)	Show students the following word problem and read it aloud "Rita went to buy some snack foods for her friends. She is buying packages of Skittles and bags of pretzels. She doesn't know how many Skittles or pretzels are in each bag, but it is always the same amount. She asked Francis and Dan how many they wanted. Francis wants 3 bags of Skittles and 2 bags of pretzels. Dan wants 2 bags of Skittles and 5 bags of pretzels." Which expression shows how many things Rita bought all together? 3s + 2p + 2s + 5p / 6s + 6p + 1s + 9p / 3s + 2p Once student makes a selection, remove the incorrect responses, give students paper to write on, and say "Simplify the expression here."		
Patterns: H.PRF.2b1: Translate a real-world problem into a one-variable equation (A.CED.1)	Show the student the following word problem and read it aloud: Omar picked 7 baskets of apples. He gave 20 apples to his teacher, 40 apples to his the debate team, and 80 apples to the football team. Write an equation to show how many apples were in each basket. Use the letter "a" to represent the unknown variable. You do not need to solve the equations, just write it.	**Concrete Understandings:** • Match an equation with one variable to the real world context.	**Representation:** • Create a pictorial array of a simple equation to translate wording • Symbols: +, -, X, ÷, =

4. What are Some Additional Activities That Can Promote Use of this Academic Concept in Real World Contexts?

- 5 4 Use a formula and solve the equation to determine the cost of additional text message charges.
- 5 4 Solve an equation to determine amount of paint needed to paint a room.
- 5 4 Solve an equation to determine how much a sale item will cost.
- 5 4 Solve an equation to determine how much mulch is needed to cover a section of lawn.
- 5 4 Given a recipe, determine how long a roast must be cooked based on the weight of the meat and the time per pound that is required to cook the meat to a safe temperature.

5. How Can I Further Promote College and Career Readiness when Teaching “Equations”?

Ideas for Promoting Career/ College Ready Outcomes

<u>Communicative competence</u>
Students will increase their vocabulary to include concepts related to “equations.” In addition, they will be learning concepts such as: “equal”, “multiply”, “divide”, “add”, “subtract”, “balance”, “same”, “each”, “times”, “more”, and “take away.”

<u>Fluency in reading, writing, and math</u>
Students will have an opportunity to increase their numeracy and sight word fluency while participating in problem solving related to “equations” such as number recognition, symbol identification, reading comprehension, composing equations, and identifying key words.

<u>Age appropriate social skills</u>
Students will engage in peer groups to solve problems related to “equations” that will provide practice on increasing reciprocal communication and age appropriate social interactions. For example, students might work together with their peers to develop equations based on story problems and substitute values for variables when given a science formula and values.

<u>Independent work behaviors</u>
By solving real life problems related to “equations” students will improve work behaviors that could lead to employment such as landscaping, working as a cashier, stocking shelves, or a chef. When providing opportunities for real life problems, leave some materials out and prompt/teach the students to determine who they should ask and what they should ask for to be able to solve the problem.

<u>Skills in accessing support systems</u>
At times, students will need to ask for assistance to complete activities related to “equations” which will give them practice in accessing supports. Students will gain practice asking for tools such as talking calculators, number lines, graphic organizers, and formulas. They can ask a peer to complete the physical movements of the tasks they are not able to do themselves. Be sure to teach students to ask versus having items or supports automatically given to them.

6. How Do I Make Instruction on “Equations” Accessible to ALL the Students I Teach?

6.1 <u>Teach Prerequisites and BasicEquation Skills Concurrently</u>: Remember that students can continue to learn basic numeracy skills in the context of this grade level content.

Basic numeracy skills that can be worked on as a part of a lesson relating to equations:

- Number identification
- Equal and/or same
- Symbol identification (+, -, =, x, ÷)
- Addition and subtraction
- Creating sets

6.2 Incorporate Universal Design for Learning (UDL in planning, and provide for additional differentiated instruction when teaching equations.)

<table>
<tr><th colspan="5">Some examples of options for teaching equations to students who may present instructional challenges due to:</th></tr>
<tr><th></th><th>Sensory Differences such as Blindness, Visual Impairment, Deafness, or Deaf/Blindness</th><th>Physical Disability or Motor Differences (such as weakness or motor planning difficulty)</th><th>Extremely limited evidence of experience/ skill or motivation/attention.</th><th>Lack of or extremely limited use of speech.</th></tr>
<tr><td>Options for Representation</td>
<td>Provide auditory options
– Talking calculator when solving equations
– Text-to-speech software or voice recordings to read aloud story problems
– Single message sequence voice–output devices to count aloud
– Captioning software that presents auditory information visually

Provide tactile options:
– Object cues, using miniature objects or other tangible symbols to assist with problem comprehension and operations
– Tactile equation mat
– Create numbers and symbols out of tactile materials such as sandpaper or wiki stix

Provide visual and manipulative options to scaffold representation of concepts:
– Color code equations and corresponding parts of calculator to support students correctly entering equations
– Provide manipulatives for quantities, such as Cuisenaire rods.</td>
<td>Reduce Physical Effort
– When reading word problems, student can scan array of key math operation words and select correct key word and operation for equation
o Place equations and graphic organizers on slant board or eye gaze board
– Display flip chart, interactive white board or other teaching materials at student eye level
– Utilize a switch instead of a computer mouse or software that allows the mouse to be controlled with the students’ head rather than their hands</td>
<td>Illustrate through multiple media
– Utilize interactive whiteboard
– Incorporate interactive websites that provide nonlinguistic tools for exploring math concepts:

Illuminations
http://illuminations.nctm.org/ActivitySearch.aspx

Math Open Reference
http://www.mathopenref.com/

There are many resources listed here:
http://www.udlcenter.org/implementation/examples

– Use virtual manipulatives and technology to show equations
– Incorporate computer representations, videos, and animations</td>
<td>Provide customized display of information
– Consistent model by utilizing modes of communication used by students (point to symbols representing concepts, operations)
– Teacher model competent use of AAC during instruction</td></tr>
</table>

	Sensory Differences such as Blindness, Visual Impairment, Deafness, or Deaf/Blindness	Physical Disability or Motor Differences (such as weakness or motor planning difficulty)	Extremely limited evidence of experience/ skill or motivation/attention.	Lack of or extremely limited use of speech.
Options for Expression	**Vary the methods for response by:** – Student states answer or scans raised numbers to select correct answer; use voice output devices for student to select the correct answer o Provide manipulatives for student to respond or contribute to interaction – Student states answer by selecting picture or symbol – Allow students who are deaf to videotape their answers/ process descriptions.	**Provide options for responses/expression:** – Student selects numbers versus writing them; matches numerals and operation symbols to equation – Choose response by pointing to, eye gazing, or selecting object or item – Place operations and symbols and/or equations on electronic whiteboard and have student use switch to select correct answer or create equation **Optimize access to tools/ alternatives for responding:** – Provide symbols, objects, manipulatives, and pictures for matching/ student responses	**Provide multimedia options for responses/expression:** – Allow the student to make selections by pointing to, gazing at, or selecting answers on the interactive white board – Utilize a switch or adapted computer mouse	**Provide options for modes of communication:** – Incorporate responses into student's AAC device or eye gaze array – Phrase questions so that they require a "yes/no" response, these can easily be answered using an eye gaze, head turn, two switches, etc – Choose response by pointing to or selecting object or item – Use a blink response to count tiles or select answer; count tiles/cubes out loud having student move in some voluntary way (e.g., nod head, tap hand, tap foot) to count along

	Sensory Differences such as Blindness, Visual Impairment, Deafness, or Deaf/Blindness	Physical Disability or Motor Differences (such as weakness or motor planning difficulty)	Extremely limited evidence of experience/ skill or motivation/attention.	Lack of or extremely limited use of speech.
Options for Engagement	**Recruit interest by providing choices:** – Digital/talking representations, videos, interactive websites **Increase personal relevance:** – Use items that are familiar and reinforcing to students. – Incorporate high preference items into story problems, as well as student names	**Recruit interest by increasing personal relevance:** – Ensure that engaging and high preference content is visible and accessible to student – Highlight key words in story problems – When creating response options make them large enough and separate them far enough so that student can make clear eye gaze or head nod to make intentional selection – Provide opportunities to work with typically developing peer on items (teach peer how to interpret student's responses)	**Recruit interest by providing choices:** – Digital/talking representations, videos, talking calculators – Use of computer representations, videos – Provide manipulatives that may be of high interest to the student and use high interest scenarios in word problems **Provide options for sustaining effort and persistence:** – Break tasks down to maximize student attention – Token economy system that embeds equations (You have 2 Justin Bieber tokens. You need 5 total. How many more do you need to earn before you can listen to a song?) – Vary demands and materials to maintain interest – Follow equation unit with a community-based instruction field trip which require the skills learned to be used	**Recruit interest with modes of communication:** – Allow students to choose items or subjects that are relevant to them via AAC devices, symbols, or eye gaze array

Promoting Career and College Readiness	Standards for Mathematical Practice
1 Communicative Competence	1 Make sense of problems and persevere in solving them.
2 Fluency in reading, writing, and math	2 Reason abstractly and quantitatively.
3 Age appropriate social skills	3 Construct viable arguments and critique the reasoning of others.
4 Independent work behaviors	4 Model with mathematics
5 Skills in accessing support systems	5 Use appropriate tools strategically.
	6 Attend to precision.
	7 Look for and make use of structure.
	8 Look for and express regularity in repeated reasoning

National Center and State Collaborative

Universal Design for Learning (UDL) Instructional Unit – Measurement

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ncsc

National Center and State Collaborative

The National Center and State Collaborative (NCSC) is applying the lessons learned from the past decade of research on alternate assessments based on alternate achievement standards (AA-AAS) to develop a multi-state comprehensive assessment system for students with significant cognitive disabilities. The project draws on a strong research base to develop an AA-AAS that is built from the ground up on powerful validity arguments linked to clear learning outcomes and defensible assessment results, to complement the work of the Race to the Top Common State Assessment Program (RTTA) consortia.

Our long-term goal is to ensure that students with significant cognitive disabilities achieve increasingly higher academic outcomes and leave high school ready for post-secondary options. A well-designed summative assessment alone is insufficient to achieve that goal. Thus, NCSC is developing a full system intended to support educators, which includes formative assessment tools and strategies, professional development on appropriate interim uses of data for progress monitoring, and management systems to ease the burdens of administration and documentation. All partners share a commitment to the research-to-practice focus of the project and the development of a comprehensive model of curriculum, instruction, assessment, and supportive professional development. These supports will improve the alignment of the entire system and strengthen the validity of inferences of the system of assessments.

IDEAs that Work
U.S. Office of Special Education Programs

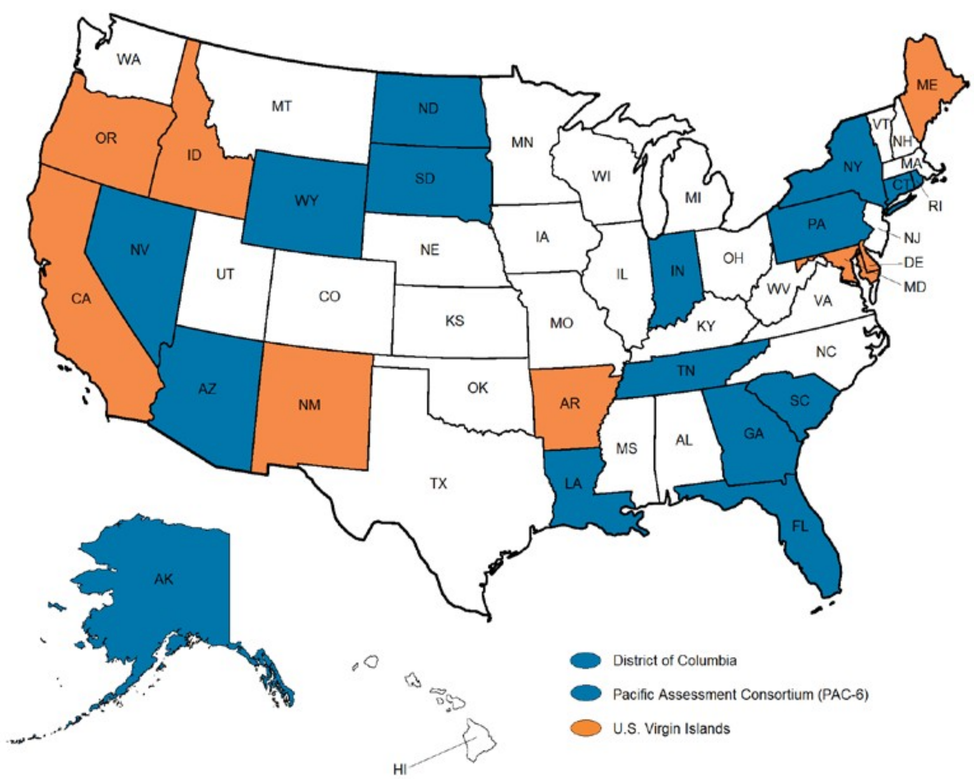
The contents of this document were developed as part of the National Center and State Collaborative for a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made.

National Center and State Collaborative

NCSC is a collaborative of 18 states and five organizations.

The states include (shown in blue on map): Alaska, Arizona, Connecticut, District of Columbia, Florida, Georgia, Indiana, Louisiana, Nevada, New York, North Dakota, Pacific Assessment Consortium (PAC-6)[1], Pennsylvania, Rhode Island, South Carolina, South Dakota, Tennessee, and Wyoming.

Tier II states are partners in curriculum, instruction, and professional development implementation but are not part of the assessment development work. They are (shown in orange on map): Arkansas, California, Delaware, Idaho, Maine, Maryland, New Mexico, Oregon, and U.S. Virgin Islands.



[1] The Pacific Assessment Consortium (including the entities of American Samoa, Commonwealth of the Northern Mariana Islands, Federated States of Micronesia, Guam, Republic of Palau, and Republic of the Marshall Islands) partner with NCSC as one state, led by the University of Guam Center for Excellence in Developmental Disabilities Education, Research, and Service (CEDDERS).

ncsc

National Center and State Collaborative

The five partner organizations include: The National Center on Educational Outcomes (NCEO) at the University of Minnesota, The National Center for the Improvement of Educational Assessment (Center for Assessment), The University of North Carolina at Charlotte, The University of Kentucky, and edCount, LLC.

NATIONAL CENTER ON EDUCATIONAL OUTCOMES

Center for Assessment

UNIVERSITY OF KENTUCKY

UNC CHARLOTTE

edCount LLC because all students count

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www.ncscpartners.org

<u>Description of Instructional Units</u>:
Each of the NCSC instructional units is intended to be used to clarify *what* the academic content is, how it can be made more accessible *for all students*, and what units of study might look like when sequencing skills and concepts along a research-based learning continuum. Towards that goal, the units take a content strand learning progression at grade level and weave within the lessons those skills or waypoints of critical understandings that a student with a significant cognitive disability may not have and may not have ever received instruction on in past grades.

The NCSC units' lesson plan format details how general education lessons can be broken down into steps. Within each step, specific suggestions of how to modify and adapt the lesson and materials are given for Emerging Readers (e.g., students who use oral speech or symbol-based augmentative communication, read sight words) and Emerging Communicators (e.g., students who are learning to use regularized gestures, signs, and symbols to communicate a variety of intents). Additionally, examples of what some of the suggestions might actually look like for individual students are provided so teachers can see possible "student work."

Each unit is composed of multiple lessons. Each lesson consists of the following components:

- Materials and Vocabulary
- Lesson Introduction
  - Build background knowledge
  - Review of Lesson objectives
- Body of the Lesson
- Practice
- Closure
  - Review of Lesson Objectives
  - Exit Assessment (Formative Assessment)

The Unit Lesson Plans represent the concepts and big ideas of the grade-specific Common Core State Standards (CCSS) and provide models of universally designed instruction for all students. The lessons also provide examples of additional supports that may be used for emerging reading and emerging communication. The Unit Lesson Plans also illustrate how to target the CCCs within general education lessons. In RTI thinking, these models provide "Tier One" plans. That is, they offer a model for how to engage all students in well-designed instruction for the CCSS. Many examples are offered for meeting the unique needs of students with significant cognitive disabilities.

With that in mind, read through each lesson as follows:

1. Review the materials for the standards addressed
   a. Common Core State Standards

   b. Instructional Families
      i. Learning Progressions Learning Targets
      ii. Core Content Connectors
2. Review the materials and vocabulary specific for the lesson.
   a. Prepare supports to access the materials
   b. Ensure vocabulary is in the student's mode of communication and available in the student's communication system
3. Read General Education Lesson 1.
   a. Introduction
      i. Consider the supports your student may need to actively engage in the activity.
         1. Means of presenting information to your student
         2. Means your student has for expressing information
         3. Means your student has for engaging in the activity and materials
      ii. If you are unsure of how to provide access, find corresponding suggestions for supports for each step of an activity/lesson in the corresponding "Additional Considerations for Emerging Readers and Emerging Communicators". Some samples of supports or student work may be found at the end of the unit. Any additional materials will be labeled with the corresponding lesson number (1-5) and component name (Intro, Body, Practice, Closure)
   b. Repeat for Lesson 1 Body
   c. Repeat for Lesson 1 Practice
   d. Repeat for Lesson 1 Closure
4. Once the units are fully electronic, inks will be provided to the other NCSC resource materials. In the meantime, you can reference the additional resource materials.
   a. When you need additional information about a concept
      i. Content Modules
      ii. Curriculum Resource Guides
   b. When you need additional ideas and strategies for supporting students
      i. Element Cards
      ii. Instructional resource guides
      iii. MASSIs
5. Always return to the general education lesson/unit after providing supplemental instruction and continue teaching where you left off.
6. Repeat for these steps for the remainder of the unit.

Table of Contents

Key Vocabulary

Measurement Unit
Lesson #1

Objectives

- Students will be able to choose the appropriate tool to measure various lengths.
- Students will be able to correctly measure various objects using inches and feet.
- Students will be able to explain why using standardized units is important and how measuring is practical in real life situations.

Essential Questions

- What are standard units of measure?
- Why is it important to use appropriate standard units of measure?
- How are standard units of measure important to our everyday lives?

Vocabulary

Centimeter – a standard unit of length in the metric system equal to 1/100 of a meter (may be mentioned in Lesson 1 but not defined until Lesson 3)

Distance – the amount of space between two things, places, or people

Foot – a standard unit of length in the US Customary system equal to 12 inches

Height – the vertical (up and down) distance from the top of an object or figure to its base

Inch – a standard unit of length in the US Customary system equal to 1/12 of a foot

Length – a measurement of the distance from one point to another

Measure – to determine a quantity/amount

Measurement – a determined quantity/amount

Meter – a standard unit of length in the metric system equal to 100 centimeters (may be mentioned in Lesson 1 but not defined until Lesson 3)

Nonstandard – not the same (e.g., the width of your thumb versus the width of my thumb) or not marked (e.g., a piece of string used to measure length)

Ruler – a wooden or plastic tool used to measure lengths up to 12 inches

Standardized – the same

Tape Measure – a tool consisting of a flexible ribbon of cloth or metal used to measure lengths up to several feet

Unit – a set amount used to consistently determine quantities

US Customary System of Measurement – a system of measurement that includes units such as inches, feet, yards and miles (for length); also referred to as empirical units or English units; this system is not commonly used outside of the United States

Width – measurement of the distance from one side or edge to the opposite side or edge

Yard Stick – a usually wooden tool used to measure lengths up to 3 feet (1 yard)

Materials

- Masking tape or painter's tape
- Math journals
- Chart paper
- *How Big is a Foot?* Storybook
  Myller, R. (1991). *How big is a foot?* New York, NY: Random House Inc.
- A piece of blank paper (just to demonstrate how to measure using a ruler)
- Rulers (enough for each student)
- Tape measure (ideally, at least one for each pair of students and one for each of the 8 stations in the Practice section)
- Yard sticks (ideally, at least one for each pair of students and one for each of the 8 stations in the Practice section)
- Classroom supplies/features
  - Stapler
  - Bookshelf
  - Doorway
  - Window
  - Filing cabinet
  - Computer screen
  - Hardcover book
  - Picture frame
- One blank sheet of paper (or could use grid paper)

Introduction

a. **Activate Previous Knowledge**

1) Using masking or painter's tape, tape a 5- to 10- foot line on the floor, depending on the amount of floor space you have. *(This could be done in the*

gym or other room if you want a longer length than you have room for in your classroom. This same line will be used again in Lesson 3.)

2) Ask students to take turns "measuring" the taped line using just their feet (heel-toe-heel-toe-heel-toe). As each student takes his/her turn, the other students should help count and record each "measurement" in their math journals, or each student can record his/her "measurements" on chart paper displayed in the classroom.
3) After all students have measured the line with their feet, you, the teacher, should do the activity as well and have the student(s) record your measurement.
4) Have the students look at and compare all of the "measurements".
   –Guiding questions should include:
   • "Are all of the measurements the same?"
   • "Why aren't all of the measurements the same?"
   • "How could the line be measured so that no matter who measures it, the same measurement will be determined?"
5) Ask the students to verbally list the kinds of tools they have used to measure things (e.g., string, rulers, and tape measures). Have them add this list to their math journals or to a new piece of chart paper displayed in the classroom.

b. **Establish Goals/Objectives for the Lesson**

- Share with the students that in this lesson they will learn about why "standardized" units of measure are important. Verbally define "standardized units" for the students. For each "tool" listed on the chart made in step 5 of part a, have the students decide if it uses standardized units or if it is "nonstandardized."
- Share with the students that they will be asked to measure a variety of things throughout this unit of study and that they will be using various tools, such as the ones they listed in Step 5 of part a.

Additional Considerations for Emerging Readers and Emerging Communicators

a. 1) Attach a tactile support alongside the tape, such as a rope or stick.
Place corresponding tape (with or without the additional tactile support) on the wall. (This does not take the place of the tape on the floor, which will be used by most students and allows easier access for students whose gross motor skills do not allow them to access the floor measurement activity.)

2) As the student walks the tape:
i. Have a partner place a dot (removable) or other marker for each step along the tape, which the student can then count at the end.

ii. Have the student or peer place a cube, token, or other manipulative for each step in a container.
iii. Have the student place a tick mark or tally or write the numeral for each step in his/her math journal.
iv. Have the student enter each step into a calculator using the “+1” function.

If the student cannot access the floor tape, trace his/her foot and use any of the above suggestions as he/she “walks” his/her foot outline along the tape on the wall.

On the chart paper or in the journal (these could have the “measurers’” pictures as well as the names) for his/her own measurement, peers’ measurements, and the teacher’s measurement, the student can:
i. writes his/her measurement;
ii. trace the numeral (if the student is working on fine motor skills);
iii. paste on a sticky note with the numeral on it (if the student is working on number recognition, he/she can select the correct number from two or more options); and/or
iv. access and work in a digital journal using assistive technology.

3) As the teacher measures using his/her own foot, the student can use any of the methods in step 2) above to count and record the teacher’s measurement.

4) Allow the student to answer the teacher’s questions by using his/her preferred mode of communication (e.g., verbal, sign, written, speech generating device). Student might select from multiple options by using touch, eye-gaze, assistive technology, etc. Options may be presented as text, symbols, pictures, tactile representations, concrete objects, etc., or any combination of those.

(Pre-teach concepts such as “same” that may not be consistently demonstrated using systematic instructional techniques, such as time delay, shaping, prompt hierarchy, etc.)

5) Allow the student to answer the teacher’s questions by using his/her preferred mode of communication (e.g., verbal, sign, written, speech generating device). Student might select from multiple options by using touch, eye-gaze, assistive technology, etc. Options may be presented as text, symbols, pictures, tactile representations, concrete objects, etc., or any combination of those.

Provide a premade list for students of tools and what each tool measures as headings; have students participate by selecting the appropriate tool and what the tool measures when asked/prompted.

The student can record these measuring tools in his/her journal by:
i. writing, tracing (if the student is working on fine motor skills), or keyboarding;
ii. drawing;
iii. cutting and/or pasting pictures, symbols, icons, text, or any combination of those;
iv. selecting pictures, symbols, icons, text, tactile representations, concrete objects or any combination of those which a partner scribes; and
v. saving symbols, icons, pictures, etc. into a digital journal using assistive technology.

b. 1) In the journal, have the student categorize the measurement tools in a. 5) into standardized vs. non-standardized (e.g., concrete objects may be placed into “standardized” or “non-standardized” containers, such as boxes, baskets, etc.). Pre-teach these concepts as necessary.

2) no accommodations

Body

1) Read aloud to the students the story- *How big is a foot?* Question the students about the issue related to the story. – Guiding questions should include:
    - How did the lack of a standard unit of measure impact the main character?
    - What tools would have been better for measurement? *(Refer back to the list of tools created in the Introduction of this lesson.)*
2) Model for the students how to measure the length of a piece of paper with your thumb (nonstandard) and then with a ruler (standardized, in inches). Then model how to measure the distance from a desk to the doorway with a tape measure and a yardstick (in inches and feet). Be sure to point out the units of measure and the reasons for using a ruler vs. a yardstick. Be sure to point out the unit named on the tool, especially if using a ruler that has metric as well as US Customary units, and for now, specify that they should use the US Customary units. Also, for now, stick to whole number measurements.

3) Lead the students in a discussion about what tools would be necessary to measure the length of various objects (e.g., a pencil vs. the line on the floor used in the Introduction). In fact, demonstrate how to use a ruler to measure the length of a pencil, but do not demonstrate measuring the line on the floor because the students will do that later in the lesson. Remind students that the US Customary System uses inches and feet. Ask students to name all of the units of which they may have heard or used pertaining to length/distance (US Customary and/or metric). Using chart paper, make a list of all of the tools used, referring back to the list devised in the Introduction, adding to it if necessary (e.g., ruler, yardstick, tape measure, odometer, etc.) and then a list of all of the units named (e.g., inches, feet, yards, miles, centimeters, meters, kilometers). *(It is OK if the students do not name them all. This is just to get an idea of what tools and units with which they are already familiar.)*

Additional Considerations for Emerging Readers and Emerging Communicators

1) Provide a personal copy of the book for the student to follow as the teacher reads aloud. Options include:
   i. text supplemented with symbols;
   ii. text with sentences or words affixed with hook-and-loop tape so the student can manipulate the text as the teacher reads;
   iii. text (or supplement) with Braille and/or tactile representations [resources such as Standard Tactile Symbol List available from the Texas School for the Blind www.tsbvi.edu/tactile-symbols may be useful in determining representations; some representations may need to be pre-taught (Rowland, 2012), but use of the same representations/symbols (or the system) during this and other lessons will reduce the need for pre-teaching]; or
   iv. illustrations in the text with -
      a) hook-and-loop tape so the student can manipulate the illustrations
      b) tactile qualities added such as outlining with glue, puffy paint, yarn, etc.

   Allow the student to answer the teacher's questions by using his/her preferred mode of communication (e.g., verbal, sign, written, speech generating device). Student might select from multiple options by using touch, eye-gaze, assistive technology, etc. Options may be presented as text, symbols, pictures, tactile representations, concrete objects, etc., or any combination of those.

2. Provide the same materials that the teacher is using:
   a. adapted materials such as a ruler with tactile qualities;
   b. a piece of paper laminated on card stock; and/or
   c. other rulers that accommodate the student more effectively such as digital rulers, bendable/foldable rulers, tactile rulers, transparent/translucent rulers, simplified rulers with only inches marked)

This may allow the student to mirror what the teacher is demonstrating. Provide adaptations, such as hook-and-loop tape or a “handle” to the ruler if the student’s fine motor skills do not allow him/her to manipulate the tool(s); provide decreasing physical assistance to help the student manipulate the materials and mirror the skill.
Allow student to move closer to the teacher during modeling or have a nearby classmate model with the student by measuring a piece of paper using your thumb and then using a ruler.

Pre-teach concepts and skills the teacher is modeling (include systematic instruction techniques, such as task analysis, prompting, shaping, time delay, etc.).

3) Preplan by finding out what tools and objects to be measured will be used as examples; then provide those tools and objects for the student to interact with during the discussion (make adaptations to the materials such as those described in 2) above so the student can meaningfully interact with them).

Allow the student to participate in the discussion by using his/her preferred mode of communication (e.g., verbal, sign, written, speech generating device). Student might contribute to the discussion from multiple options by using touch, eye-gaze, assistive technology, etc. Options may be presented as text, symbols, pictures, tactile representations, concrete objects, etc., or any combination of those.

Preplan at least one contribution for the student to make to the classroom discussion, at least one measurement tool, and at least one unit of measure.

Practice

1) Have students work in pairs to find the actual measurement of the tape on the floor used in the Introduction. Each student pair should first select a tool to use then measure the length of the tape and record their measurement in their math journals. Instruct the student pairs not to share their measurement with anyone else. As each pair selects a tool and measures the line, the other students should observe but not try to direct or interrupt the pair.

2) Once all pairs have taken their turn measuring the line, record each pair’s measurement on chart paper. Discuss any differences.
   - Guiding questions should include:
     - Are any of the measurements different this time?
     - Why might they be different? (e.g., different tools used, different units, an error in measuring, etc.)
     - How did you decide which tool to use?

• Why would it be helpful to use one tool over another?
• Why would it be helpful to use one kind of unit over another?

3) Divide the class into two groups. The first group will take turns at 8 stations where they will choose from a selection of tools and then measure a given object at each station:
   - Station 1 – the length of a stapler
   - Station 2 – the height and/or width of a bookshelf
   - Station 3 – the height of a doorway
   - Station 4 – the width of a window
   - Station 5 – the height and/or width of a filing cabinet
   - Station 6 – the height and/or width of a computer screen
   - Station 7 – the length of the spine of a book
   - Station 8 – the length and/or width of a picture frame)

   Note: Since we are still working with whole number measurements at this point, have the students round to the nearest whole number measurement if the objects do not measure right at the whole number mark.
   In their math journals, the remaining students should brainstorm and list 5 instances in which they might need to measure things in real life as well as identify the tool they would need to measure each thing named. When everyone in the first group has been to each of the 8 stations, they should switch places and activities with the second group.

4) Once all students have visited each station, as a class, go over each station and the correct measurement of each object. Model how to correctly measure each object. Using chart paper, chart the students' answers concerning real life situations in which measuring objects would be useful.

Additional Considerations for Emerging Readers and Emerging Communicators

1) Allow the student to participate in the tool selection within his/her pair by using his/her preferred mode of communication (e.g., verbal, sign, written, speech generating device). Student might select from multiple options by using touch, eye-gaze, assistive technology, etc.

   When the measurement activity occurs, one student could do the actual measurement by moving the selected measurement tool along the tape while the other student records each measurement through:
      a) moving the tool – adapt the tool with hook-and-loop tape, a "handle", etc. so the student can more effectively and independently maneuvers the tool;
      b) recording the measurement -

i. place a dot (removable) or other marker for each measurement along the tape or on a note card, which the students can then count at the end;
ii. place a tick mark or tally or write the numeral for each measurement on a note card;
iii. enter each measurement into a calculator using the "+1" function;
iv. place a manipulative (cube, token, etc.) in a container so they can be counted for the final measurement;
v. write the final measurement;
vi. trace the numeral;
vii. paste on a sticky note with the numeral on it; and
viii. access and work in a digital journal using assistive technology.

c) Paste card with tick marks and total counted by student into math journal or paste picture of tool used and circle correct measurement in math journal.

2) Allow the student to answer the teacher's questions by using his/her preferred mode of communication (e.g., verbal, sign, written, speech generating device). Student might select from multiple options by using touch, eye-gaze, assistive technology, etc.

Preplan at least one answer to one of the guiding questions.

3) Stations:

Allow student to select, using his/her preferred mode of communication (eye-gaze, verbal, touch, sign, written, speech generating device, assistive technology, etc.), a measuring tool at each station. Depending on the student's skill at selecting an appropriate tool, the number of tools offered could vary:

- two tools for a student who has difficulty with selection;
- several tools for a student who can make a selection; or
- the "appropriateness" could vary from widely discrepant (e.g., Station 3 – six inch ruler vs. yardstick) to more discreet differences (e.g., Station 6 – six inch ruler vs. 12-inch ruler).

Use the station activities as opportunities to instruct and practice performance. Provide effective accommodations to the materials such as those found in Lesson 1 – Body, 2), adapted materials:

- a ruler with tactile qualities or a piece of paper;
- digital rulers;
- bendable/foldable rulers;

- tactile rulers;
- transparent/translucent rulers;
- simplified rulers with only inches marked; and
- rulers adapted with hook-and-loop tape or a “handle”.

Other effective adaptations include laminated card. He/she should be allowed to manipulate the tool(s) with the provision of decreasing physical assistance and to mirror the skill (similar to those found in Lesson 1 – Introduction a. 2 - place a manipulative in a container every time the tool moves, place a tick/tally mark on paper every time the tool moves, etc.)

Journals:

The student can record instances where he/she might need to measure and corresponding measuring tools selected in his/her journal using his/her preferred mode of communication (verbal, sign, written, speech generating device to select (by touch, eye-gaze, assistive technology, etc.) from multiple options (presented by text, symbols, pictures, tactile representations, concrete objects, or any combination of these, etc.)]. The student can record these measuring tools in his/her journal by:

- o writing,
- o tracing (if the student is working on fine motor skills),
- o keyboarding (in a digital journal),
- o drawing, cutting and/or pasting pictures, symbols, icons, text, or any combination of those.

The student selects pictures, symbols, icons, text, tactile representations, concrete objects, or any combination of these which a partner scribes, saving symbols, icons, pictures, etc. into a digital journal using assistive technology.

Be consistent in the supports the student uses for journaling; refer back to the accommodations the student used in Lesson 1 – Introduction a. 5).

4) As the teacher models correctly measuring the objects at each station, quietly remind the student how he/she measured them. (Because the station activity was instructional practice, the student should have received guidance, possibly through errorless learning techniques, at each station resulting in correctly measuring each object.)

Allow the student to participate in the math journal discussion by using his/her preferred mode of communication (e.g., verbal, sign, written, speech generating device). Student might select from multiple options by using touch, eye-gaze, assistive technology, etc. Preplan at least one contribution to the discussion.

Closure

a. Revisit/Review Lesson and Objectives –

1) Ask the students to, as a class, verbally explain what "standardized unit" means and why using standardized units is important/helpful. Remind the students that, that was one of the goals of this lesson—to learn about how and why standardized units of measure are used.
2) Ask students to brainstorm and suggest alternative ways to measure that would still be standardized (i.e., if you were creating your own standardized system of measurement, what would you use?)
3) Share with students a bit of the history behind the development of the United States Customary System, also called empirical units or the English System of Measurement (i.e., barleycorns, digits, fingers, hands, inches, nails, palms, shaftments, links, spans, feet, cubits, yards, etc. – *The purpose of this is just to show the students that this system of measure was devised to help people, particularly farmers, measure/judge lengths and distances and that it was, at first, non-standardized. They do not need to add these words to their working vocabulary*.) Share with students that there are, indeed, other systems of measurement, such as the metric system, which they will be learning about later in the unit.

Additional Considerations for Emerging Readers and Emerging Communicators:

a. 1) Allow the student to participate in the class discussion by using his/her preferred mode of communication (e.g., verbal, sign, written, speech generating device). Student might select from multiple options by using touch, eye-gaze, assistive technology, etc. Preplan at least one contribution to the discussion.

2) Allow the student to participate in the brainstorming using his/her preferred mode of communication e.g., verbal, sign, written, speech generating device). Student might select from multiple options by using touch, eye-gaze, assistive technology, etc. Preplan at least one contribution to the brainstorming.

3) Provide engagement supports as necessary, which may include pictures of key concepts being presented, concrete objects to manipulate, and/or positive behavioral supports.

b. Exit Assessment –

1) Have each student draw one line of any length on a piece of blank paper using a ruler (could use grid paper). Tell the students to use their rulers to measure the line they drew but not to write the measurement down.
2) Group the students in threes. Each trio should take turns measuring each group mate's line. Each trio should compare their measurements to make sure they came up with the same measurement. They should resolve any inconsistencies by re-measuring the lines together.

Additional Considerations for Emerging Readers and Emerging Communicators:

b. 1) The student could draw his/her line digitally using assistive technology, choosing a straw, pipe cleaner, etc. to measure, or placing two dots on the paper and have someone draw the line. Allow the student to use (if necessary) any of the adapted materials he/she used in Lesson 1 – Body 2), such as a ruler with tactile qualities, low vision ruler, audio measuring tape, or a piece of paper laminated on card stock which support student needs. Other rulers may accommodate the student more effectively, such as digital rulers, bendable/foldable rulers, tactile rulers, transparent/translucent rulers, simplified rulers with only inches marked) or a ruler adapted with hook-and-loop tape or a "handle" if the student's fine motor skills do not allow him/her to manipulate the tool(s).

2) Allow the student to use (if necessary) any of the adapted materials he/she used in Lesson 1 – Body 2). These include rulers which may accommodate the student needs more effectively such as:

- rulers with tactile qualities;
- digital rulers;
- bendable/foldable rulers;
- transparent/translucent rulers;
- simplified rulers with only inches marked; and
- a ruler adapted with hook-and-loop tape or a "handle" if the student's fine motor skills do not allow him/her to manipulate the tool(s).

As this is now assessment as opposed to all previous activities in this lesson, the student's first independent attempt at measuring the three lines should be observed and recorded for accuracy.

Measurement Unit
Lesson #2

Scaling and Unit Conversion:
4.ME.2f1 Complete a conversion table for length and mass within a single system

Objectives
- Students will be able to convert inches to feet and feet to inches.

Essential Questions
- How can a measurement expressed in one unit be expressed in another unit without changing the quantity?
- How is expressing equal quantities in different units practical/useful in real world situations?

Vocabulary
Convert – to change

Foot – a standard unit of length in the US Customary system equal to 12 inches

Inch – a standard unit of length in the US Customary system equal to 1/12 of a foot

Length – a measurement of the distance from one point to another

Measure – to determine a quantity/amount

Measurement – a determined quantity/amount

Quantity – an amount of something

Ruler – a wooden or plastic tool used to measure lengths up to 12 inches

Standardized – the same

Tape Measure – a tool consisting of a flexible ribbon of cloth or metal used to measure lengths up to several feet

Unit – a set amount used to consistently determine quantities

US Customary System of Measurement – a system of measurement that includes units such as inches, feet, yards and miles (for length); also referred to as imperial units or English units; This system is not commonly used outside of the United States.

Width – measurement of the distance from one side or edge to the opposite side or edge

Yard Stick – a usually wooden tool used to measure lengths up to 3 feet (1 yard)

Materials

- Charts from lesson 1
- Two 8-packs of juice boxes
- Individual whiteboards (one per student) and dry erase markers
- Lesson 2 Conversions PowerPoint presentation (print out slides for students)
- Lesson 2 Practice Conversions PowerPoint (do not need to print out the slides for the students, but if you do remove the answers from the slides)

Introduction

a. Activate Previous Knowledge

1) Review the chart used in Lesson 1 whereby units of length were listed (inches, feet, yards, and miles) as well as the tools used to measure length (ruler, tape measure, yardstick, odometer, etc.). Remind the students how in Lesson 1 they were asked to decide which tool and which unit was most appropriate given what they were measuring.
2) Show the students an 8-pack of juice boxes. Open the pack and have the students count aloud each box as you remove each from the pack.
3) Ask the students, "How many boxes of juice are there in 1 pack?" Then ask the students, "What if I had 2 packs of juice? How many boxes of juice would I have all together?"
4) Ask the students, "Is there a difference between saying that I have 1 pack of juice or that I have 8 boxes of juice? Is there a difference between saying that I have 2 packs of juice or that I have 16 boxes of juice? Is there the same amount either way I say it?"
5) Finally, ask the students, "Why might I want to say that I have 2 packs of juice rather than saying that I have 16 boxes of juice? Or, why might I want to say that I have 16 boxes of juice rather than saying that I have 2 packs?" Remind them of how in Lesson 1 it was determined, for example, that using feet to measure the line of the floor was better than using inches, although it is certainly possible to measure the line in inches—it would just likely take a little longer and involve larger numbers. Remind them also of how this is the same principle they learned about when they learned why multiplication is handy even though it is actually nothing more than repeated addition.

Additional Considerations for Emerging Readers and Emerging Communicators

1) Provide reminders in the form of accommodations used in Lesson 1- Practice. Refer student to math journal information recorded in Lesson 1.

2). Provide 2-dimensional copies (enhanced with tactile qualities such as glue or puffy paint outlines) of the juice, boxes (or a set of actual juice boxes) to count as the teacher demonstrates. The copies or actual juice boxes may be supplemented with hook-and-loop tape if necessary. Mark the copies or the actual boxes with numerals 1-8. Add tactile qualities as necessary including touch math dots, sets of textured dots corresponding to the numerals, outlined numerals, etc. Provide a grid or number line for the student to use when counting. It may have tactile qualities added.

3) allow the student to answer using his/her preferred mode of communication. Provide answer options from which the student can choose. Provide a second set of copies or actual boxes so the student can come up with the answer. Provide an equation (either addition or multiplication) the student can use to come up with the answer. The student may use a calculator if necessary.

4) Have the student compare the individual copies or the actual boxes to unopened packs to answer the questions. Allow the student to answer using his/her preferred mode of communication.

5) Allow student to answer using his/her preferred mode of communication. Provide options of answers from which to choose. Depending on the student's skills at this point, you may or may not include distractors or "wrong" answers. However, since this is still learning and not being assessed, "errorless learning" is still an option, so giving all appropriate choices is acceptable. Preplan a question and answer so the student can contribute to the discussion.

b. Establish Goals/Objectives for the Lesson

1) Tell the students that in this lesson, they will be using inches and feet and that, like the packs of juice and the juice boxes, they will learn how to use those two units to express the same amount/quantity.

Additional Considerations for Emerging Readers and Emerging Communicators

b. 1) As the teacher explains the goals to the class, provide access to the same measuring tools (with any supports necessary) that he/she used in previous activities in this unit (once a student demonstrates that a support of any kind is successful in helping him/her learn, it is important to use that same support whenever appropriate; consistency is important). Provide a digital or graphic representation of inches compared to feet.

<u>Body</u>

1) Provide students with handouts of the Lesson 2 Conversions PowerPoint.
2) Guide students through the PowerPoint. *(The PowerPoint is very short and is scripted. You will need to print out the speaker's notes or at least read them ahead of time so that you know when to click for each narration.)* The PowerPoint shows students that:
   a. 12 inches = 1 foot and 1 foot = 12 inches
   b. To convert feet to inches, multiply by 12
   c. To convert inches to feet, divide by 12
   d. Reference charts (12's times table and basic division by 12) are included in the presentation.

Additional Considerations for Emerging Readers and Emerging Communicators

1) Provide a digital or graphic representation of inches compared to feet in addition to the PowerPoint.

2) Allow student to use a calculator (talking calculator, large button calculator, digital calculator, etc.) to practice multiplying and dividing by 12.

   Pre-teach any concepts (multiplication, division, calculator use, etc.) that the student may not have learned completely.

<u>Practice</u>

1) Present the students with the Lesson 2 Practice Conversions PowerPoint presentation. *(You do not necessarily need to give handouts of this one. If you do, remember to remove the answers from the slides before printing.)*
   a. Each slide presents the students with a conversion problem.

    b. Students are asked to decide whether they would need to multiply or divide in order to find the equivalent measurement. (Remind students they may refer to their PowerPoint notes.)
2) Students are asked to write and complete each problem on their whiteboards.
3) The final slide tasks the students to complete a chart whereby they must convert inches to feet and feet to inches. Again, students should write the problems and their answers on their whiteboards.

Additional Considerations for Emerging Readers and Emerging Communicators

1) As the practice conversion PowerPoint is being presented
    a. Provide models for the problems that the student can use to copy on the whiteboard.
    b. Provide options of math function for the student to choose. Allow him/her to choose using the preferred mode of communication.

2) Provide prewritten cards with the numbers and functions that the student can use to "write" the problems (touching, moving, selecting by eye-gaze, etc.).
    a. Prewrite the problems on the whiteboard.
    b. Provide the problems digitally and allow the student to solve the problems digitally. Allow the student access to the digital problems using assistive technology.
    c. Provide options of math function for the student to choose. Allow him/her to choose using the preferred mode of communication.
    d. Allow the student to use a calculator (traditional or digital, talking, large button, etc.) to solve the problems.
    e. Provide number cards that the student can use as answers for the problems, process cards, and math function cards instead of writing them.
    f. Reduce the number of problems that the student must solve.
    g. Reduce the difficulty of the problems. For example, only give the student problems that have feet in single digits (1-9).

3) Use the same supports as in step 2

<u>Closure</u>

a. Revisit/Review Lesson and Objectives –

1) Remind students that the goal for this lesson was to learn how to convert inches to feet and feet to inches.
2) Ask students to verbally explain how to convert feet to inches.
3) Ask students to verbally explain how to convert inches to feet.

Additional Considerations for Emerging Readers and Emerging Communicators

1. During the review, provide access to the same measuring tools (with any supports necessary) that he/she used in previous activities in this unit (once a student demonstrates that a support of any kind is successful in helping him/her learn, it is important to use that same support whenever appropriate; consistency is important).
2. Allow the student to answer the questions using his/her preferred mode of communication (including AAC and AT).
   - Allow the student to create the formulas for conversion (feet to inches and inches to feet) by writing, manipulating prewritten labels, numbers, and functions, etc. and use those formulas to answer the questions.
   - Provide formulas and allow the student to select them as answers to the teacher's questions.
3. Use the same supports as in step 2

b. Exit Assessment –

1) Group the students in pairs.
2) Have each student come up with a conversion problem and write it on his/her whiteboard.
3) Have students exchange whiteboards with their partners and solve their partners' problem.
4) Have each pair present both of their problems and of answers to the class and explain how they solved them.

Additional Considerations for Emerging Readers and Emerging Communicators

Allow the student to write a conversion problem using any of the supports or accommodations that he/she used in previous activities in this unit (once a student demonstrates that a support of any kind is successful in helping him/her learn, it is important to use that same support whenever appropriate; consistency is important). Give the student a chance to do this independently (this is assessment, so it is important to determine the accuracy of the student's independent performance). If the student cannot do this step independently, assist as necessary so the other student has a correct problem to solve. Allow the student to solve the partner's conversion problem using any of the supports or accommodations that he/she used in previous activities in this unit (once a student demonstrates that a support of any kind is successful in helping him/her learn, it is important to use that same support whenever appropriate; consistency is important). Give the student a chance to do this independently (this is assessment, so it is important to determine the accuracy of the student's independent performance). If the student cannot do this step independently, make a note of this and assist as necessary so the problem is solved correctly. . Have the student point to each step of the equation sequentially. Have the student present his/her solution digitally using AT.

Measurement Unit
Lesson #3

Scaling and Unit Conversion:
4. E.2f1 Complete a conversion table for length and mass within a single system

Objectives
- Students will be able to measure objects using centimeters and meters.
- Students will be able to convert measurements within the metric system.

Essential Questions
- How is the metric system like/unlike the US Customary System?
- What are the most commonly used units of measure within the metric system?
- How do the metric units compare to one another?

Vocabulary
Centimeter – a standard unit of length in the metric system equal to 1/100 of a meter

Convert – to change

Foot – a standard unit of length in the US Customary system equal to 12 inches

Inch – a standard unit of length in the US Customary system equal to 1/12 of a foot

Length – a measurement of the distance from one point to another

Measure – to determine a quantity/amount

Measurement – a determined quantity/amount

Meter – a standard unit of length in the metric system equal to 100 centimeters

Meter Stick – a usually wooden tool used to measure lengths of up to 1 meter

Quantity – an amount of something

Ruler – a wooden or plastic tool used to measure lengths up to 12 inches

Standardized – the same

Tape Measure – a tool consisting of a flexible ribbon of cloth or metal used to measure lengths up to several feet

Unit – a set amount used to consistently determine quantities

US Customary System of Measurement – a system of measurement that includes units such as inches, feet, yards and miles (for length); also referred to as imperial units or English units; this system is not commonly used outside of the United States.

Width – measurement of the distance from one side or edge to the opposite side or edge

Yard Stick – a usually wooden tool used to measure lengths up to 3 feet (1 yard)

Materials

- Chart paper
- Rulers (one for each student)
- Meter sticks (ideally, at least one for each of the 8 stations in the Practice section)
- Charts from lesson 1
- Taped line on the floor from Lesson 1
- Classroom supplies/features from Lesson 1
    - Stapler
    - Bookshelf
    - Doorway
    - Window
    - Filing cabinet
    - Computer screen
    - Hardcover book
    - Picture frame
- Lesson 3 Metric Conversions PowerPoint presentation (print out slides for students)
- Lesson 3 Practice Metric Conversions PowerPoint (do not need to print out the slides for the students, but if you do remove the answers from the slides)
- Individual whiteboards (one per student) and dry erase markers

Introduction

a. Activate Previous Knowledge

1) Have students name all of the ways they know of to say, "Hello" (e.g., Hi! Hey! Bonjour! Hola!, Konnichiwa!, Ciao!, Aloha!, etc.). *If no one knows "Hello!" in any other language than English, you can just write the words on the chart paper and tell him or her language each is.*
2) Ask students to think about how each of these words communicates the same thought (a greeting) just in a different way.
3) Show students a ruler and a meter stick.
4) Have students recall how in Lesson 1 they practiced measuring various objects using inches and feet.
5) Explain that in this lesson, they will learn about the metric system, which is like another language people can use to measure objects. Explain that the metric

system does not use inches and feet but does use different standardized units, just as if different languages use different words to communicate the same idea, such as "hello".
6) Review the chart made in Lesson 1 whereby different units of measure were listed. Point out which ones, if any, is part of the metric system. *(Recall how in Lesson 1 students were asked to list all of the different units of which they had heard. If no one listed any metric units, that's OK. Just point out that the ones on the chart are from the US Customary System and that in this lesson, they'll be learning about metric units.)*

Additional Considerations for Emerging Readers and Emerging Communicators

1) Pre-teach a small number of words for the student to contribute. Allow the student to contribute using the preferred mode of communication. Provide pre-recorded (either digital or with assistive technology) contributions for the student to activate.

2) No accommodations.

3) Provide a ruler and meter stick.

4) Provide the (accommodated) tools to measure inches and feet that the student used in Lesson 1.

5) Provide the ruler and meter stick. Provide a graphic representation (symbolic and/or concrete) of basic metric measurements (meters, centimeters, decimeters, etc.) which the student may be familiar.

6) Provide the information previously developed in Lesson 1-Body 3).

b. Establish Goals/Objectives for the Lesson
1) Continuing with numbers 5 and 6 in section a., let the students know that in this lesson they will measure objects in centimeters and meters, which are basic units of the metric system, and they will learn how to convert units within the metric system, just like they did for inches and feet in the US Customary System. *(Note: Students will NOT be asked to convert between systems.)*

Additional Considerations for Emerging Readers and Emerging Communicators

1) Provide a graphic or 3-dimensional representation of basic metric measurements (meters, centimeters, decimeters, etc.). Enhance the representations with texture, color, etc.

Body

1) Demonstrate how to measure objects using centimeters and meters with a ruler and meter stick. (Measure the length of a pencil in centimeters, and measure the taped line on the floor from Lesson 1 in meters.) Relate to the students that just as sometimes measuring in feet is more practical/useful than measuring in inches; the same is true for centimeters and meters. *Note: Again, for now, stick to whole number measurements.*
2) Using the same 8 objects that were used in the Practice section for Lesson 1, pair the students and have each pair practice measuring the 8 objects in centimeters or meters, depending on the object. Once all students have been to each of the 8 stations, go over the actual measurements of each object. *Note: For now, have students round to the nearest whole number if/when they are measuring objects that do not measure right at the whole number mark.*
    - Station 1 – the length of a stapler
    - Station 2 – the height and/or width of a bookshelf
    - Station 3 – the height of a doorway
    - Station 4 – the width of a window
    - Station 5 – the height and/or width of a filing cabinet
    - Station 6 – the height and/or width of a computer screen
    - Station 7 – the length of the spine of a book
    - Station 8 – the length and/or width of a picture frame
3) Provide students with handouts of the Lesson 3 Metric Conversions PowerPoint presentation.
4) Guide students through the presentation. *(The PowerPoint is very short and is scripted. You will need to print out the speaker's notes or at least read them ahead of time so that you know when to click for each narration.)* The PowerPoint shows students that:
    a. 100 centimeters = 1 meter and 1 meter = 100 inches
    b. To convert meters to centimeters, multiply by 100
    c. To convert centimeters to meters, divide by 100
    d. Reference charts (100's times table and basic division by 100) are included in the presentation.
    e. The last slide is a summary of how to convert meters to centimeters (and vice versa) and how to convert feet to inches (and vice versa).

Additional Considerations for Emerging Readers and Emerging Communicators

- Provide a centimeter ruler and a pencil exactly the same length as the teacher's so the student can practice using the teacher as a model. Other rulers that accommodate the student more effectively can be used, such as digital rulers, bendable/foldable rulers, tactile rulers, transparent/translucent rulers, simplified rulers with only centimeters marked) so the student can mirror what the teacher is demonstrating. . Provide adaptations such as hook-and-loop tape or a "handle" to the ruler if the student's fine motor skills do not allow him/her to manipulate the tool(s); provide decreasing physical assistance to help the student manipulate the materials and mirror the skill. As the teacher measures the tape:
  i. place a dot (removable) or other marker for each meter along a number line, which the student can then use for counting at the end.
  ii. have the student place a tick mark, tally, or write the numeral for each meter in his /her math journal.
  iii. have the student enter each meter into a calculator (or talking calculator, large button calculator, etc.) using the "+1" function. Pre-teach concepts and skills the teacher is modeling (include systematic instruction techniques, such as task analysis, prompting, shaping, time delay, etc.).

- Use the station activities as opportunities to instruct and practice performance. Provide accommodations to the materials such as those found in Lesson 1 – Body, 2) adapted materials which accommodate the student effectively, such as:
  a. a ruler with tactile qualities
  b. or a piece of paper laminated on card stock may be substituted;
  c. digital rulers,
  d. bendable/foldable rulers,
  e. tactile rulers,
  f. transparent/translucent rulers,
  g. simplified rulers with only centimeters marked,
  h. rulers adapted with hook-and-loop tape or a "handle" to allow him/her to manipulate the tool(s),

the provision of decreasing physical assistance is necessary to help the student manipulate the materials and demonstrate the skill, similar to those found in Lesson 1 – Introduction a. 2) (place a manipulative in a container every time the tool moves, place a tick/tally mark on paper every time the tool moves, etc.). As the teacher models correctly measuring the objects at each station, quietly remind the student how he/she measured them. Because the station activity was instructional practice, the student should have received guidance, possibly through errorless learning techniques, at each station resulting in correctly measuring each object.

- Provide a copy of the PowerPoint.

- Guide the student through his/her personal PowerPoint copy, focusing on foundational reading skills as appropriate (however, the main focus here is mathematics, so do not emphasize reading if it is "getting in the way").

Practice

1) Present the students with the Lesson 3 Practice Metric Conversions PowerPoint presentation. *(You do not necessarily need to give handouts of this one. If you do, remember to remove the answers from the slides before printing.)*
   a. Each slide presents the students with a conversion problem.
   b. Students are asked to decide whether they would need to multiply or divide in order to find the equivalent measurement. (Remind students they may refer to their PowerPoint notes.)
   c. Students are asked to write and complete each problem on their whiteboards.
   d. The final slide tasks the students to complete a chart whereby they must convert meters to centimeters and centimeters to meters. Again, students should write the problems and their answers on their whiteboards.

Additional Considerations for Emerging Readers and Emerging Communicators

Provide models for the problems that the student can use to copy on the whiteboard. Provide prewritten cards with the numbers and functions that the student can use to "write" the problems (touching, moving, selecting by eye-gaze, etc.). Prewrite the problems on the whiteboard. Provide the problems digitally and allow the student to solve the problems digitally. Provide math function options for the student to choose from. Allow him/her to choose using the preferred mode of communication. Allow the student to use a calculator (traditional or digital, talking, large buttons, etc.)) to solve the problems. Provide number cards that the student can use as answers for the problems, process cards, and math function cards instead of writing them. Reduce the difficulty of the problems. For example, only give the student problems that have meters in single digits (1-9).

Closure

a. Revisit/Review Lesson and Objectives –

1) Remind students that the goal for this lesson was to learn how to convert meters to centimeters and centimeters to meters.
2) Ask students to verbally explain how to convert meters to centimeters.
3) Ask students to verbally explain how to centimeters to meters.

4) Ask students to verbally compare the units of the US Customary System and the metric system:
   - Which metric unit is similar to feet?
   - Which metric unit is similar to centimeter?
   - Are the US Customary units and the metric units the same?
   - What did you notice about converting metric units versus converting US Customary units?

Additional Considerations for Emerging Readers and Emerging Communicators

During the review, provide access to the same measuring tools (with any supports necessary) that he/she used in previous activities in this unit (once a student demonstrates that a support of any kind is successful in helping him/her learn, it is important to use that same support whenever appropriate; consistency is important). Allow the student to answer the questions using his/her preferred mode of communication (including AAC and AT). Allow the student to create the formulas for conversion (meters to centimeters and centimeters to meters) by writing, manipulating prewritten labels, numbers, and functions, etc., and use those formulas to answer the questions. Provide formulas and allow the student to select them as answers to the teacher's questions. Provide the tools the student used to measure feet, inches, meters, and centimeters. Have him/her categorize each tool as either "big" or "little" ("large"/"small"). Have him/her use that categorization to determine that feet are similar to meters and inches are similar to centimeters. Allow the student to answer the questions using his/her preferred mode of communication. Have the student physically or digitally compare the sizes of the tools in each set to determine that the units are not the same. Allow the student to answer the questions using his/her preferred mode of communication. Allow the student to look at both the metric and customary conversion charts and formulas and point out that when converting "big" to "little", you must multiply and when converting "little" to "big", you have to divide. Allow the student to answer the questions using his/her preferred mode of communication. Pre-teach any concepts that the student may not yet fully understand (big/little, same/different, etc.). Preplan a piece of information for the student to contribute.

b. Exit Assessment –

1) Group the students in pairs.
2) Have each student come up with a (metric) conversion problem and write it on his/her whiteboard.
3) Have students exchange whiteboards with their partners and solve their partners' problem.
4) Have each pair present both of their problems and answers to the class and explain how they solved them.

Additional Considerations for Emerging Readers and Emerging Communicators

Allow the student to write a conversion problem using any of the supports or accommodations that he/she used in previous activities in this unit (once a student demonstrates that a support of any kind is successful in helping him/her learn, it is important to use that same support whenever appropriate; consistency is important). Give the student a chance to do this independently (this is assessment, so it is important to determine the accuracy of the student's independent performance). If the student cannot do this step independently, assist as necessary so the other student has a correct problem to solve. Allow the student to solve the partner's conversion problem using any of the supports or accommodations that he/she used in previous activities in this unit (once a student demonstrates that a support of any kind is successful in helping him/her learn, it is important to use that same support whenever appropriate; consistency is important). Give the student a chance to do this independently (this is assessment, so it is important to determine the accuracy of the student's independent performance). If the student cannot do this step independently, make a note of this and assist as necessary so the problem is solved correctly. Have the student point to or verbally describe each step of the equation sequentially. Have the student present his/her solution digitally using AT.

Measurement Unit
Lesson #4

Objectives
- Students will be able to distinguish between perimeter and area.
- Students will be able to calculate perimeter and area.
- Students will be able to relate the concepts of area and perimeter to real life situations.

Essential Questions
- What is perimeter, and how is it calculated?
- What is area, and how is it calculated?
- How could perimeter and area be used in real life situations?

Vocabulary
Area – the amount of space an object occupies

Centimeter – a standard unit of length in the metric system equal to 1/100 of a meter

Foot – a standard unit of length in the US Customary system equal to 12 inches

Inch – a standard unit of length in the US Customary system equal to 1/12 of a foot

Length – a measurement of the distance from one point to another

Measure – to determine a quantity/amount

Measurement – a determined quantity/amount

Meter – a standard unit of length in the metric system equal to 100 centimeters

Perimeter – the distance around a figure along its edges

Quantity – an amount of something

Rectangle – a 4-sided figure in which opposite sides are equal and angles measure 90°

Ruler – a wooden or plastic tool used to measure lengths up to 12 inches

Square – a figure that has four equal sides and angles that measure 90°

Standardized – the same

Tape Measure – a tool consisting of a flexible ribbon of cloth or metal used to measure lengths up to several feet

Unit – a set amount used to consistently determine quantities

US Customary System of Measurement – a system of measurement that includes units such as inches, feet, yards and miles (for length); also referred to as imperial units or English units; This system is not commonly used outside of the United States.

Width – measurement of the distance from one side or edge to the opposite side or edge

Yard Stick – a usually wooden tool used to measure lengths up to 3 feet (1 yard)

Materials

- Masking or painter's tape
- Math journals
- Index cards (Each student will get one card. Some cards should have a "P" on them, while others should have an "A" on them. It is recommended that you have more "A" cards than "P" cards.)
- Chart paper
- Lesson 4 Perimeter Area PowerPoint presentation (print out slides for the students)
- Lesson 4 Practice Perimeter Area PowerPoint presentation (do not need to print out the slides for the students, but if you do remove the answers from the slides)
- Whiteboards and dry erase markers
- Lesson 4 Practice Worksheet document printed for each student
- Grid paper
- Lesson 4 Word Problem document printed for each student

Introduction

a. Activate Previous Knowledge

1) Using masking or painter's tape, tape off a large rectangle on the classroom floor. *(If you still have the tape on the floor from Lessons 1 and 2, you could just use that initial line as one of the sides of the rectangle. If you need more space than you have available in your classroom, you could do this activity in the cafeteria or gym.)*
2) Divide the class into two groups. Give each student in the first group an index card with the letter "P" written on it. Give each student in the second group an index card with the letter "A" written on it. *(It is a good idea to have more students in the "A" group than in the "P" group.)*
3) Have the students in the "P" group get up and walk along/on the taped edges of the rectangle while the students in the "A" group watch.

4) Have the students in the “A” group get up and fill the inside space of the rectangle while the students in the “P” group watch.
5) Ask the students, “What is the difference between the “P” group and the “A” group?
6) Explain to the students that the “P” group demonstrated the word perimeter whereas the “A” group demonstrated the word area.
7) Have the students verbally come up with definitions for perimeter and area, and write those definitions on chart paper to be displayed in the classroom.

Additional Considerations for Emerging Readers and Emerging Communicators

1) Provide a graphic representation, enhanced with tactile (e.g. outlined with glue, rough and smooth textures) and color qualities (e.g. perimeter in pink, area in blue) as appropriate, of the area and perimeter of the taped off section of the floor. A picture frame with a think frame could be used as a model of perimeter and area.

2) No accommodations.

3) No accommodations.

4) No accommodations.

5) Provide vocabulary options (word bank) for the student to choose from (“perimeter”“, area”“, outside”, “inside”, “more”, “less”, etc.). Put them in a format that allows the student to use his/her preferred mode of communication, including AAC and/or AT.

6) As the teacher explains, show the student the concepts on his/her graphic representation.

7) Have the student put the class definitions of perimeter and area in his/her math journal. The student can enter the definitions by writing, tracing (if the student is working on fine motor skills), keyboarding, drawing, cutting and/or pasting pictures, symbols, icons, text, selecting pictures, symbols, icons, text, tactile representations, concrete objects or any combination of those which a partner scribes, or saving symbols, icons, pictures, etc. into a digital journal using assistive technology.

Body

1) Present the students with the Lesson 4 Perimeter Area PowerPoint presentation. (The presentation is scripted, so be sure to consult the speaker's notes before you begin for information concerning how the animations are sequenced as you explain each step.) Provide the students with handouts of the PowerPoint. The presentation guides students in:
    a. finding perimeter of a square given the lengths of all four sides
    b. finding perimeter of a square given only one length
    c. finding area of a square using tiles and the formula for area (length x width)
    d. finding perimeter of a rectangle given the lengths of all four sides
    e. finding perimeter of a rectangle given one length and one width
    f. finding area of a rectangle using tiles and the formula for area (length x width)
2) As you go through the part of the presentation concerning rectangles, you will need to point out the difference/similarity of "length" and "width". Up until now, students have primarily been dealing with length only, so they will need to add "width" to their vocabulary.
3) Be sure to note the importance of including the appropriate units when providing answers to perimeter and area problems (i.e., perimeter = in vs. area = in^2).

Additional Considerations for Emerging Readers and Emerging Communicators

1) Provide a copy of the PowerPoint. Enhance with tactile qualities as necessary. Provide the PowerPoint digitally with animations and sound. Guide the student through his/her personal PowerPoint copy, focusing on foundational reading skills as appropriate (however, the main focus here is mathematics, so do not emphasize reading if it is "getting in the way"). Provide a 3-dimensional representation that the student can manipulate during the presentation, such as a geoboard or pegboard that can be altered to reflect the shapes in the PowerPoint where pegs can be used to count perimeter and the grid cells to count area. Cubes or tiles might also be used to fill in the area. Provide a personal digital device that can be programmed with the shapes from the PowerPoint, and have the student manipulate the perimeter and area measurements, with or without AT. Allow student to use a calculator (talking, large button, etc.) to mirror the calculations during the PowerPoint. Have the student place tiles or cubes used in the perimeter/area demonstrations on a number line to count to the correct answer.

2) Have the student put the definitions of length and width in his/her math journal. The student can enter the definitions by writing, tracing (if the student is working on fine motor skills), keyboarding, drawing, cutting and/or pasting pictures,

symbols, icons, text, selecting pictures, symbols, tactile representations, concrete objects, or any combination of these which a partner scribes, or saving symbols, icons, pictures, etc. into a digital journal using assistive technology.

3) Have the student add to the definition of area that a “2” representing “square” should always be used in area calculations. Have the student use the same strategy as in 2) above to add this information to the math journal.

Practice

1) Present the students with the Lesson 4 Practice Perimeter Area PowerPoint presentation. *(You do not necessarily need to give handouts of this one. If you do, remember to remove the answers from the slides before printing.)*
    a. Each slide presents the students with a perimeter and/or area problem.
    b. Students are asked to use the information provided to find the perimeter and/or area as shown. (Remind students they may refer to their PowerPoint notes. You should point out that the figures shown on screen are to scale drawings.)
    c. Students are asked to write and complete each problem on their whiteboards.
2) Have students complete the Lesson 4 Practice Worksheet individually.
    a. There are only 4 problems on the worksheet.
    b. Remind the students that they may refer to their PowerPoint notes if needed.
    c. Students may show their work in the blank space on the worksheet or on a separate sheet of paper.
    d. Remind students to include the appropriate units in each of their answers.

Additional Considerations for Emerging Readers and Emerging Communicators

1) Provide models for the problems that the student can use to copy on the whiteboard. Provide prewritten cards with the numbers and functions, which the student can use to “write the problems (touching, moving, selecting by eye-gaze, etc.) Prewrite the problems on the whiteboard. Provide the problems digitally and allow the student to solve the problems digitally. Allow the student access to the digital problems using assistive technology. Provide math function options for the student to choose from. Allow the student to use a calculator (traditional or digital, talking, large button, etc.) to solve the problems. Provide number cards that the student can use as answers for the problems instead of writing them.
2) Divide the shapes into cells or grids so that the student may count versus calculate. Provide cutout cells or other manipulatives that the student can place on the worksheet and then count. Adapt the worksheet by adding cells to each

formula/equation for the student to fill in. Provide number cards that the student can use as answers for the problems instead of writing them. Provide visual cues to help the student distinguish between length and width. Allow the student to use a calculator (traditional or digital) to solve the problems. Reduce the number of problems to solve, remembering that the student may need extra practice at a later time. As this is still instructional, not assessment, provide guidance so that the student completes the worksheet correctly. Techniques such as errorless learning and systematic instruction could be used.

Closure

a. Revisit/Review Lesson and Objectives –

1) Remind students that the goal for this lesson was to learn the difference between perimeter and area as well as how to "find" perimeter and area of squares and rectangles.
2) Review the definitions of perimeter and area that were charted at the beginning of the lesson. Ask the students to verbally explain any additional information they learned about perimeter and area throughout the lesson (e.g., perimeter is calculated in units, area in squared units; when calculating perimeter and area of a square, you really only need to know the length of one side since all four sides are the same; when calculating the perimeter and area of a rectangle, you really only need to know the length of one side and the width of one side since opposite sides are the same, etc.)

Additional Considerations for Emerging Readers and Emerging Communicators

1) & 2) Provide access to math journal and PowerPoint presentations. Assist the student in finding the review information as the teacher goes over it and the class discusses. Provide answer options to the questions. Have the student select an answer(s) and use his/her preferred mode of communication to contribute to the review. Pre-plan a question and answer for the student to contribute.

b. Exit Assessment –

1) Have each student draw a square or a rectangle on a sheet of grid paper and illustrate the difference between perimeter and area (e.g., outline the figure in one color to demonstrate perimeter, and shade the inside of the figure in another color to demonstrate area).
2) Provide each student with a copy of the Lesson 4 Word Problem document. Read the problem to the students, and have the students solve the problem in their math journals. (You'll go over the problem and its answer at the beginning of the next and final lesson in this unit.)

Jane and Luis are in charge of the 4th grade booth at their school's Spring Festival, which will be held on the school's parking lot. Space on the parking lot is very limited, and so the parking lot has been divided so that each grade will have a section that measures only 2 feet by 3 feet. Jane and Luis want to make sure they will have room for their popcorn popper and snow cone machine as well as the serving table.

A) What is the perimeter of class' section of the parking lot?
B) Convert the perimeter of the class' section into inches.
C) What is the area of the class' section of the parking lot?
D) Which is likely more helpful to Jane and Luis in determining whether or not they have enough space for all of their equipment—knowing the perimeter of their section or knowing the area of their section? Why?
E) Based on your calculations, do you think Jane and Luis will have enough room for the popcorn popper, the snow cone machine, and the table? How do you know?

Additional Considerations for Emerging Readers and Emerging Communicators

1) Provide grid paper (enlarged or raised line if either of those has been used successfully by the student during instruction). Allow the student to draw or trace the square or rectangle (shape and size should be the student's choice). Allow the student to create or select shape, digitally using AAC and/or AT. Have the student indicate where the perimeter and area would be found or measured. This could be done verbally/vocally, by touch, selection from an indicated choice (e.g., "Is this the perimeter or is this the perimeter?"), outlining, coloring, digitally, etc. Allow the student to use the method of performance that will give him/her the most success. For instance, if the student has difficulty with fine motor skills, he/she might answer more successfully by answering questions (including use of AAC/AT) rather than coloring in answers. Of course, after a successful response, the student could color in if that is his/her choice, but the accuracy of the response would be on the answer given, not the coloring. Since this is now assessment and not instruction, it is important to assess the student's independent performance. If the answer is incorrect, he/she can be prompted to give a correct performance, but the data should reflect the student's independent (incorrect) response.

2) Provide an adapted story problem worksheet:

a. read the problem to the student
b. simply/summarize text of story problem
c. highlight the important information
d. provide a graphic or 3-dimensional representation of the problem (with manipulatives if necessary)
e. provide formulas for perimeter and area
f. provide visual cues for determining perimeter and area
g. provide answer selections for the student to choose from (these should include plausible distractors or incorrect answers, the number of which could vary from student to student)
h. provide a number line
i. provide a conversion chart

Provide and allow the student to use any of the accommodations or supports he/she used in this lesson (Lesson 4), Practice 2) to solve the problem. Reduce number of questions required for assessment. Since this is now assessment and not instruction, it is important to assess the student's independent performance. If the answer is incorrect, he/she can be prompted to give a correct performance, but the data should reflect the student's independent (incorrect) response.

Measurement Unit
Culminating Activity

Materials

- Math journals
- Lesson 4 Word Problem document (just for reference)
- Unit Review PowerPoint presentation
- Whiteboards and dry erase markers
- 10 Activity Stations (may use sidewalk chalk, tape, poster board, bulletin board paper, etc. to present the squares and rectangles to the students)
- Rulers with inches on one side and centimeters on the other (one for each student)
- Tape measure, yard stick, meter stick (ideally, one for each of the 10 stations)
- Culminating Assessment Worksheet
- 1 ft^2 pieces of poster board (one for each student) *This is only if you have the students do the final mural activity*.

Introduction

b. **Activate Previous Knowledge**
   1) Have students open their math journals to their answers for the Lesson 4 Word Problem they were to complete as an exit assessment for Lesson 4. Reread the word problem to the students, and then go through each question, polling the class to see what their answers were and why. Have students correct any mistakes.
   2) Have students play the Unit Review PowerPoint game. (They can write their answers, which will all be either True or False, on their whiteboards.)
   3) As the students play the game, be sure to take time to review each concept pertaining to each slide. Refer students to their PowerPoint notes from the previous lessons.

Additional Considerations for Emerging Readers and Emerging Communicators

1) Provide math journal, Lesson 4 Word Problem worksheet (adapted version used in Lesson 4, Exit Assessment 2), and any supports or accommodations the student used in completing the worksheet. Go through each section of the problem and solution as the teacher reviews.

2) Provide a copy of the PowerPoint with or without text-based symbols. Provide interactive graphics on the PowerPoint as appropriate. For instance, slide 2 could have scaled representations of an inch and a foot that wobble when clicked. Provide true and false text cards (with or without symbols for the student to

choose from). Rephrase the questions as appropriate (and provide different answers to choose from if the rephrased questions require those). For example, Slide 2 could be rephrased as, "Which is bigger - an inch or a foot?" requiring the answer choices to be "inch" and "foot"; or it could be rephrased as, "Is an inch bigger than a foot?" requiring the answer choices to be "yes" and "no". Allow the student to answer using his/her preferred mode of communication.

3) Provide PowerPoint notes from previous lessons. Provide any supports previously used in those lessons so the student can use them again as the teacher reviews the concepts.

b. Establish Goals/Objectives for the Activity

1) Explain to the students that they are going to complete some problems involving all of the skills they have learned throughout this unit:

- measuring in US Customary Units (inches/feet)
- converting US Customary Units (inches/feet)
- measuring in metric units (centimeters/meters)
- converting metric units (centimeters/meters)
- calculating perimeter of squares and rectangles
- calculating area of squares and rectangles

Additional Considerations for Emerging Readers and Emerging Communicators

1) Provide any supports including definitions, examples, and materials or tools regarding the concepts of inches, feet, centimeters, meters, perimeter, area, squares, and rectangles to remind the student of these during the review.

<u>Body</u>

1) Set up 10 stations (considering the amount of space you have available, this could be done outside using sidewalk chalk, inside using tape on the floor, or you could cut out squares and rectangles using poster board and post them on the wall/bulletin board, etc. You could also cut out the squares and rectangles using bulletin board paper then set up the stations along the wall in a hallway or in a room that has enough space for the figures and the students).
    - Station #1 – a 5 cm square
    - Station #2 – rectangle that is 5 ft. by 7 ft.
    - Station #3 – rectangle that is 3 in by 1 in
    - Station #4 – a 4 in square
    - Station #5 – a 2 m square
    - Station #6 – a rectangle that is 2 m by 4 m
    - Station #7 – an 8 in square
    - Station #8 – a rectangle that is 11 cm by 10 cm

- Station #9 – a 7 cm square
- Station #10 – a rectangle that is 11 in by 9 in

2) At each station, each student will be tasked to
   a. choose an appropriate measurement tool (ruler marked in inches on one side and centimeters on the other, yard stick/meter stick, tape measure);
   b. measure the given figure (length and width);
   c. calculate the perimeter of the figure;
   d. calculate the area of the figure

   Students should record their work and answers in their math journals. They may refer back to their PowerPoint notes from previous lessons.
3) Students should circulate until they have each been to all 10 of the stations and have recorded their answers in their math journals.
4) As a class, go to each station, have a student demonstrate how he/she measured the figure and explain how he/she calculated the perimeter and area of the figure. (Select a different student for each station.)

Additional Considerations for Emerging Readers and Emerging Communicators

1) Provide tactile qualities and color to shapes as appropriate.

2) Follow the same instructional procedures as used in Lesson 1- Practice 3).

3) Follow the same instructional procedures as used in Lesson 1- Practice 3).

4) Follow the same instructional procedures as used in Lesson 1- Practice 3).

Final Assessment

1) Students will individually complete an assessment worksheet whereby they will utilize the skills learned throughout this unit. *See Culminating Assessment Worksheet document.*
2) The worksheet contains three sections: a section whereby students must convert units, a section whereby the students must measure a rectangle and find its perimeter and area (then convert the perimeter from inches to feet), and a final section that presents the students with a story problem.
   - After solving the story problem, you could have the students do the activity described in the problem. Provide each student with a 1-square foot piece of paper and instruct each student to draw a picture of himself/herself. Collect the finished drawings as they turn in their finished worksheets, and piece the drawings together to make either a large square or rectangle, depending on the number of students in the class. If you have an odd number of students in the class, complete a block for yourself so

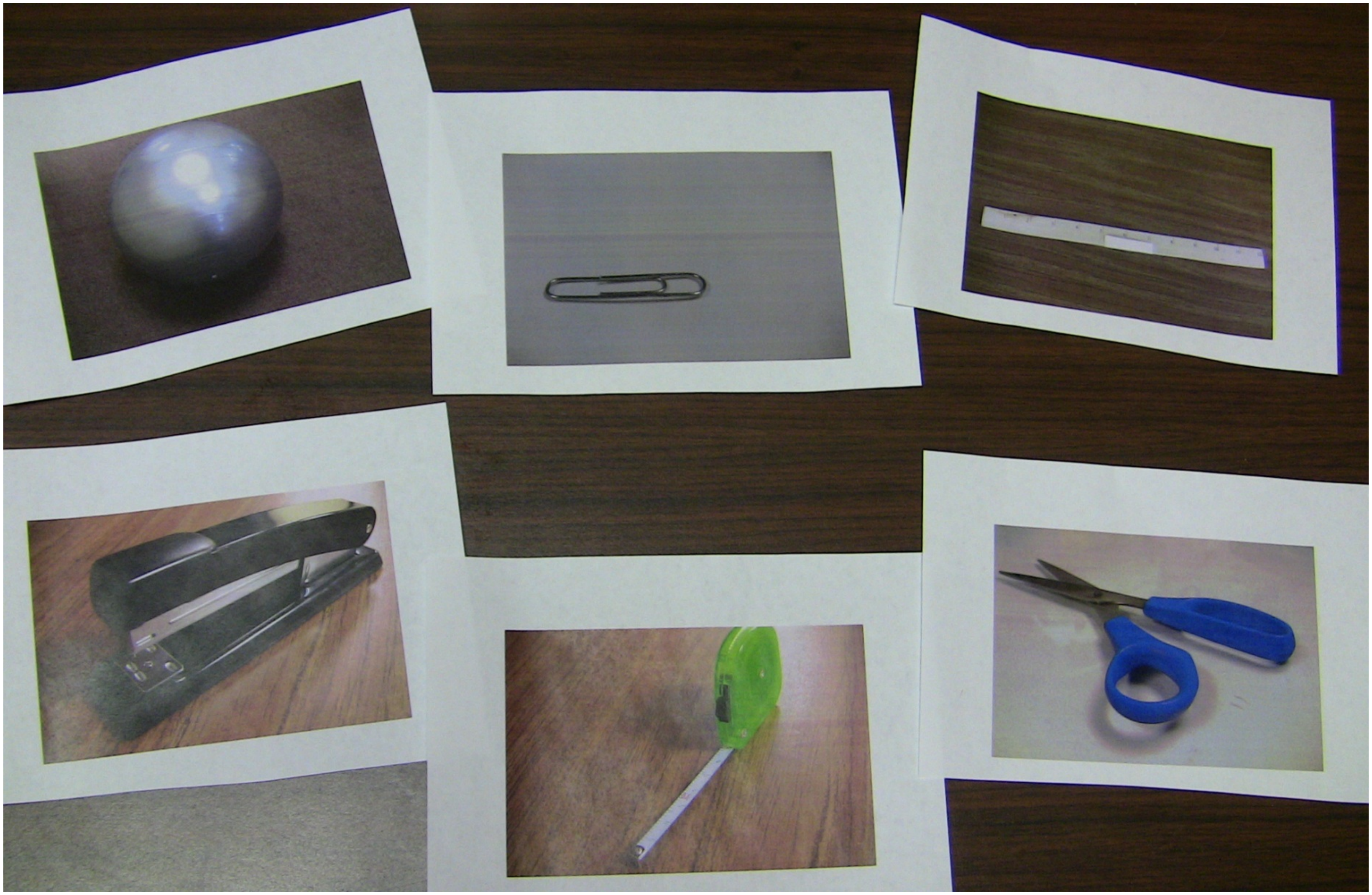
that the completed mural will be a square or a rectangle. Once the mural is complete, have the students calculate the perimeter and area of the mural by counting the individual blocks.

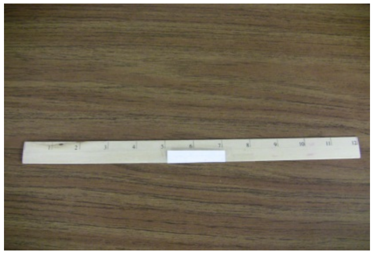
Additional Considerations for Emerging Readers and Emerging Communicators

1) Provide adapted worksheet:
   a. with or without symbol-based text
   b. fewer problems
   c. less difficult problems selected

2) Follow the same procedures as found in
   a. (for worksheet section on conversions) Lesson 3- Practice 1)
   b. (for worksheet section on perimeter/area) Lesson 4- Practice 2)
   c. (for worksheet section on solving word problems) Lesson 4- b. Exit Assessment 2)

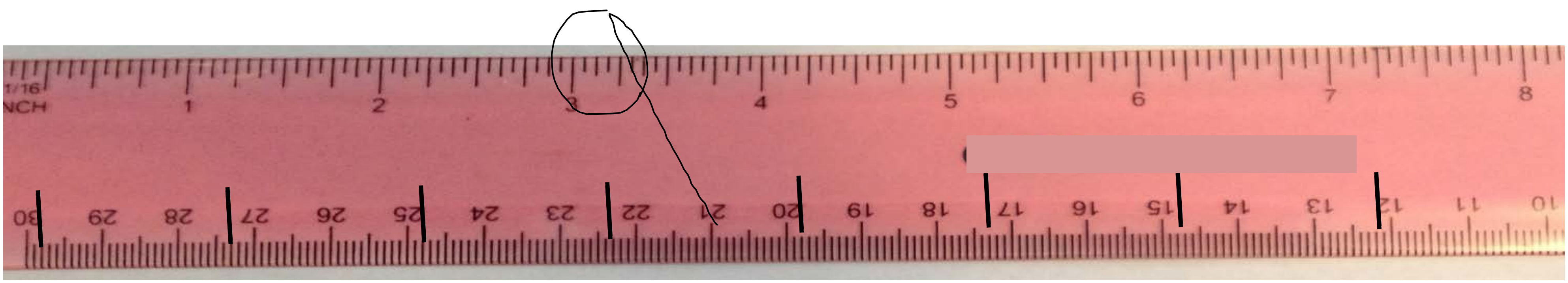
Choices used by student to add to brainstorm discussion.





Eye gaze board

6



9-30-12

1. Choose a tool.
2. Measure the tape on the floor.

|||||

5 ruler

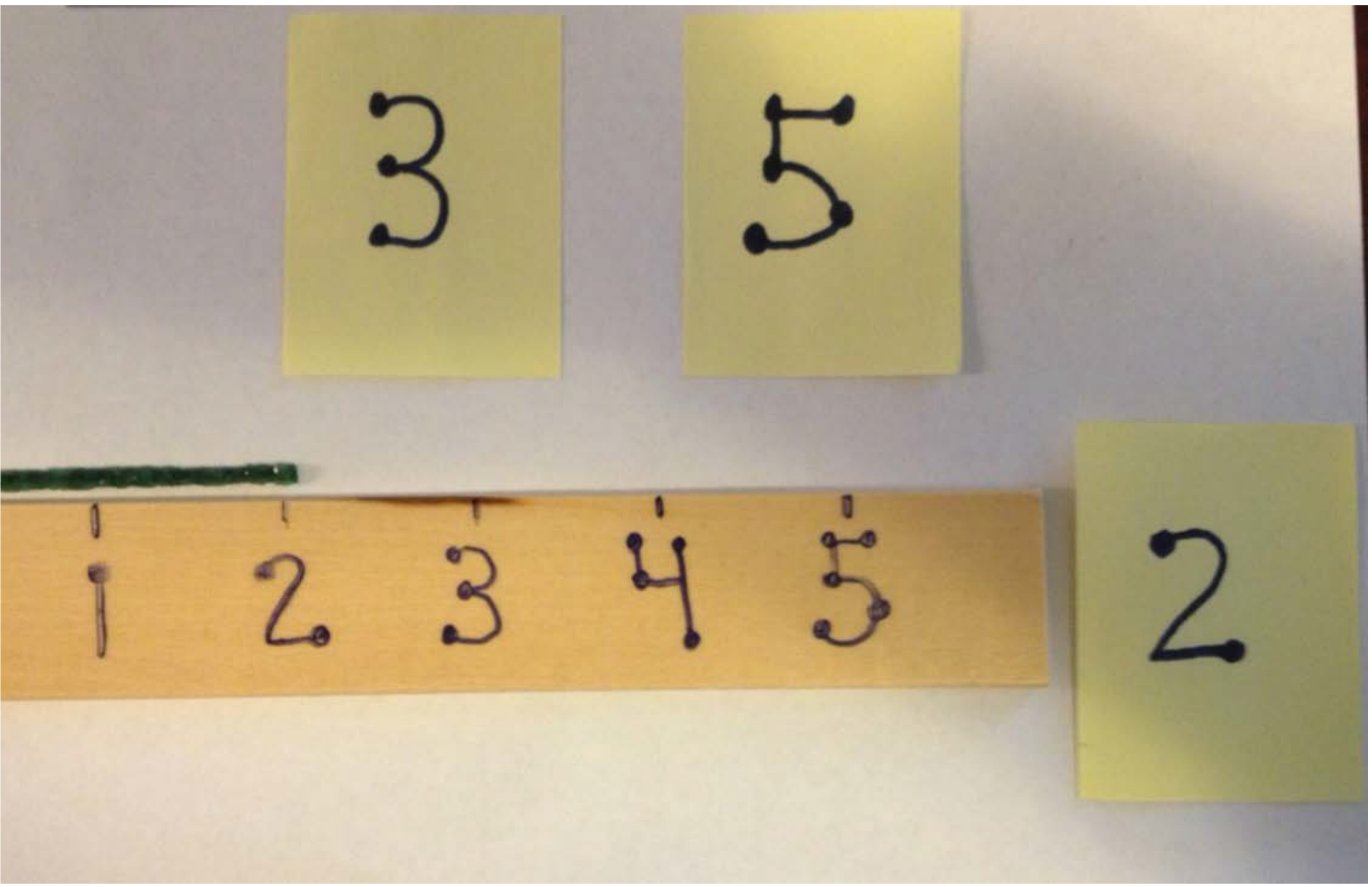
Unit 1 – Lesson 1 – Practice EC

Math Journal

October 12, 2012

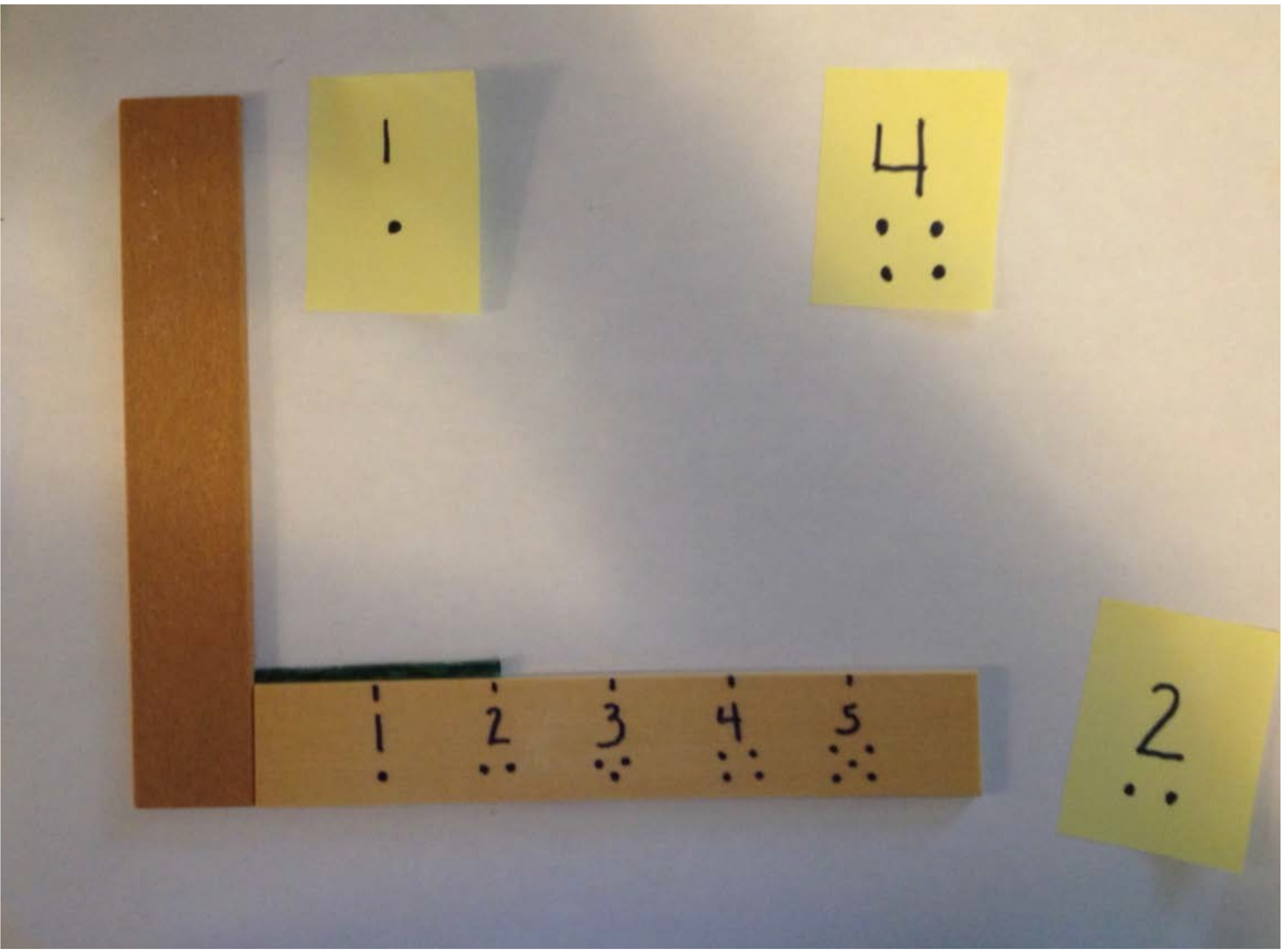
2 feet   5 feet   8 feet

Objects to compare standardization of measuring tools



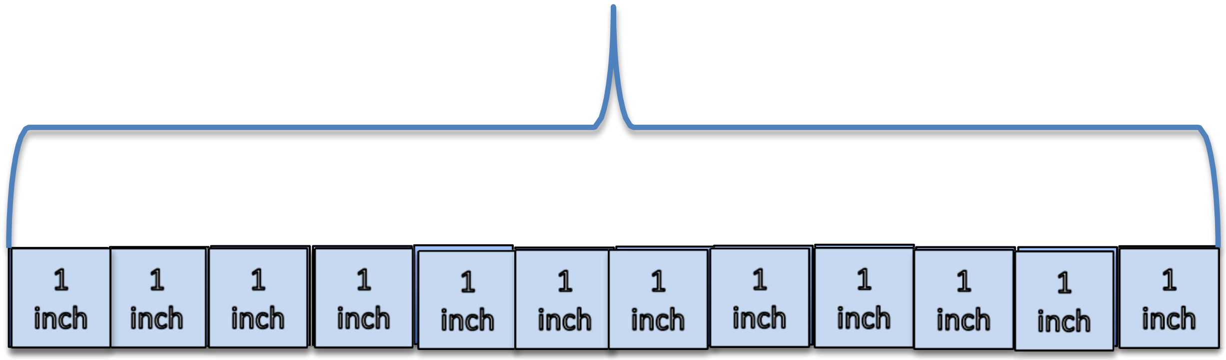
3
5
1 2 3 4 5
2

Ruler with guide and dot cards to choose answer.



Converting Units Within the US Customary System (inches and feet)

1 foot (ft)



1 foot (ft) = 12 inches (in)

1 x 12 = 12

larger unit smaller unit

feet to inches – MULTIPLY BY 12

Remember: inches are smaller units than feet, so it takes more of them to equal a foot, which is why you multiply to convert feet to inches.

Feet to Inches – Reference

MULTIPLY

2 feet x 12 = 24 inches

3 feet x 12 = 36 inches

4 feet x 12 = 48 inches

5 feet x 12 = 60 inches

6 feet x 12 = 72 inches

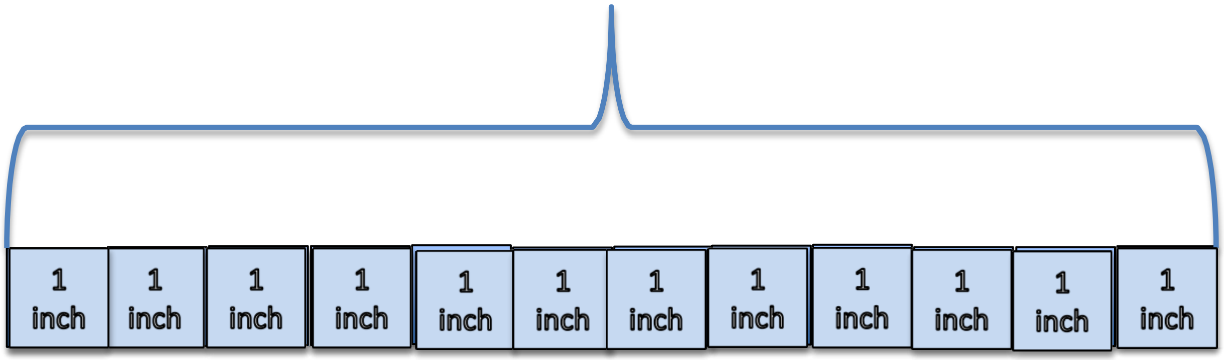
7 feet x 12 = 84 inches

8 feet x 12 = 96 inches

9 feet x 12 = 108 inches

10 feet x 12 = 120 inches

1 foot (ft)



12 inches (in) = 1 foot (ft)

$12 \div 12 = 1$

smaller unit　　larger unit

inches to feet – DIVIDE BY 12

Remember: inches are smaller units than feet, so it takes less feet to equal the same amount in inches, which is why you divide to convert inches to feet.

Inches to Feet – Reference

DIVIDE

24 inches ÷ 12 = 2 feet

36 inches ÷ 12 = 3feet

48 inches ÷ 12 = 4 feet

60 inches ÷ 12 = 5 feet

72 inches ÷ 12 = 6 feet

84 inches ÷ 12 = 7 feet

96 inches ÷ 12 = 8 feet

108 inches ÷ 12 = 9 feet

120 inches ÷ 12 = 10 feet

Quick Review

- To convert *ft* to *in*, MULTIPLY BY 12
- To convert *in* to *ft*, DIVIDE BY 12

Practice Conversions of Length Within the US Customary System

Multiply or Divide

48 inches = 4 feet

By what? 12

Answer: $48 \div 12 = 4$

Multiply or Divide

108 inches = 9 feet

By what? 12

Answer: $108 \div 12 = 9$

Multiply or Divide

5 feet = ___60___ inches

By what? 12

Answer: 5 x 12 = 60

Multiply or Divide

10 feet = ____120____ inches

By what? 12

Answer: 10 x 12 = 120

Multiply or Divide

3 ft = $\underline{\quad 36 \quad}$ in

By what? 12

Answer: 3 x 12 = 36

Multiply or Divide

24 in = 2 ft

By what? 12

Answer: $24 \div 12 = 2$

Multiply or Divide

7 ft = 84 in

By what? 12

Answer: 7 x 12 = 84

Multiply or Divide

6 ft = ____72____ in

By what? 12

Answer: 6 x 12 = 72

Multiply or Divide

84 in = 7 ft

By what? 12

Answer: $84 \div 12 = 7$

Multiply or Divide

8 ft = ____96____ in

By what? 12

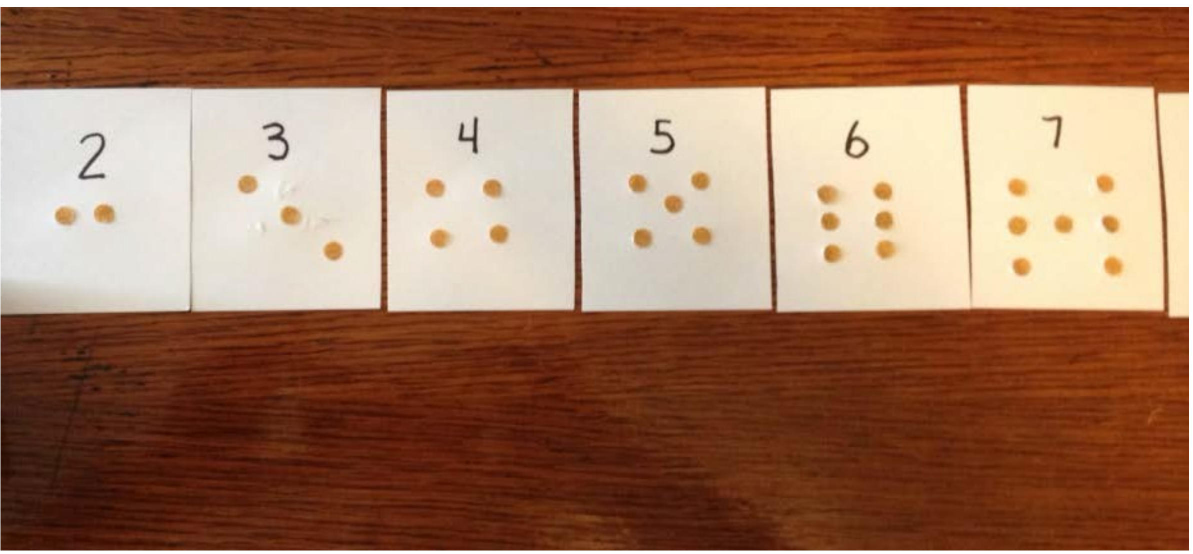
Answer: 8 x 12 = 96

Complete the Chart

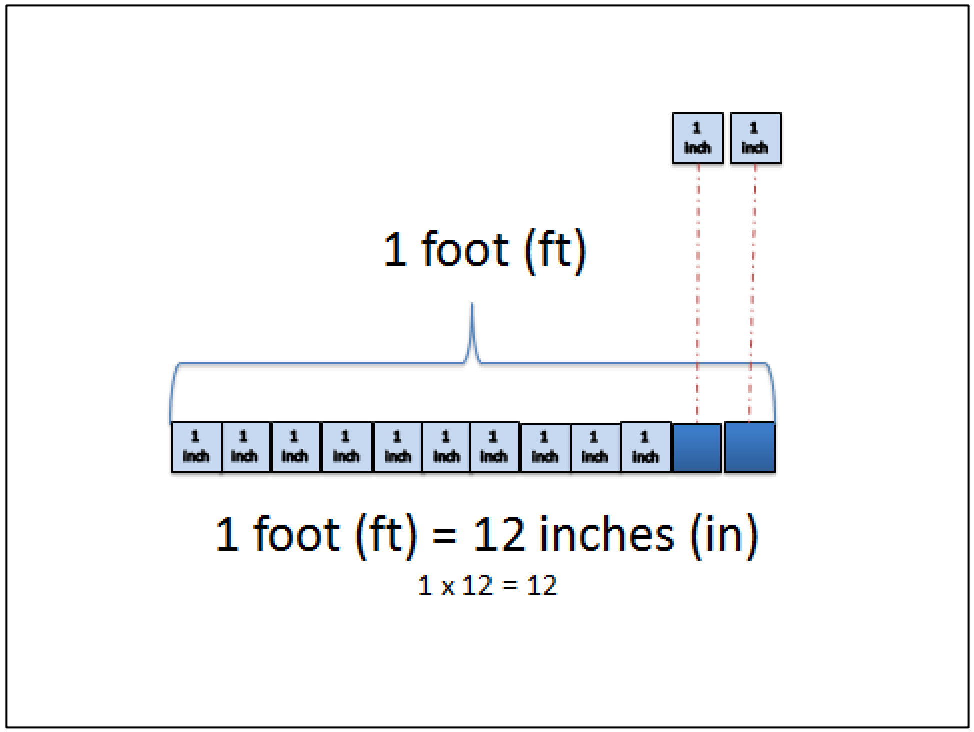
FEET		INCHES
1	=	12
5	=	60
3	=	36
9	=	108
7	=	84

Move juice box to cards and count as teacher unpacks the juice boxes.

A number line using sand paper dots provide both tactile and visual stimuli.



A one inch unit moves down on each click of the mouse to form one foot.



48 inches = 4 feet	multiply or divide
108 inches = 9 feet	multiply or divide
5 feet = 60 inches	multiply or divide
10 feet = 120 inches	multiply or divide
3 feet = 36 inches	multiply or divide
24 inches = 2 feet	multiply or divide
7 feet = 84 inches	multiply or divide
6 feet = 72 inches	multiply or divide
84 inches = 7 feet	multiply or divide
8 feet = 96 inches	multiply or divide
	multiply

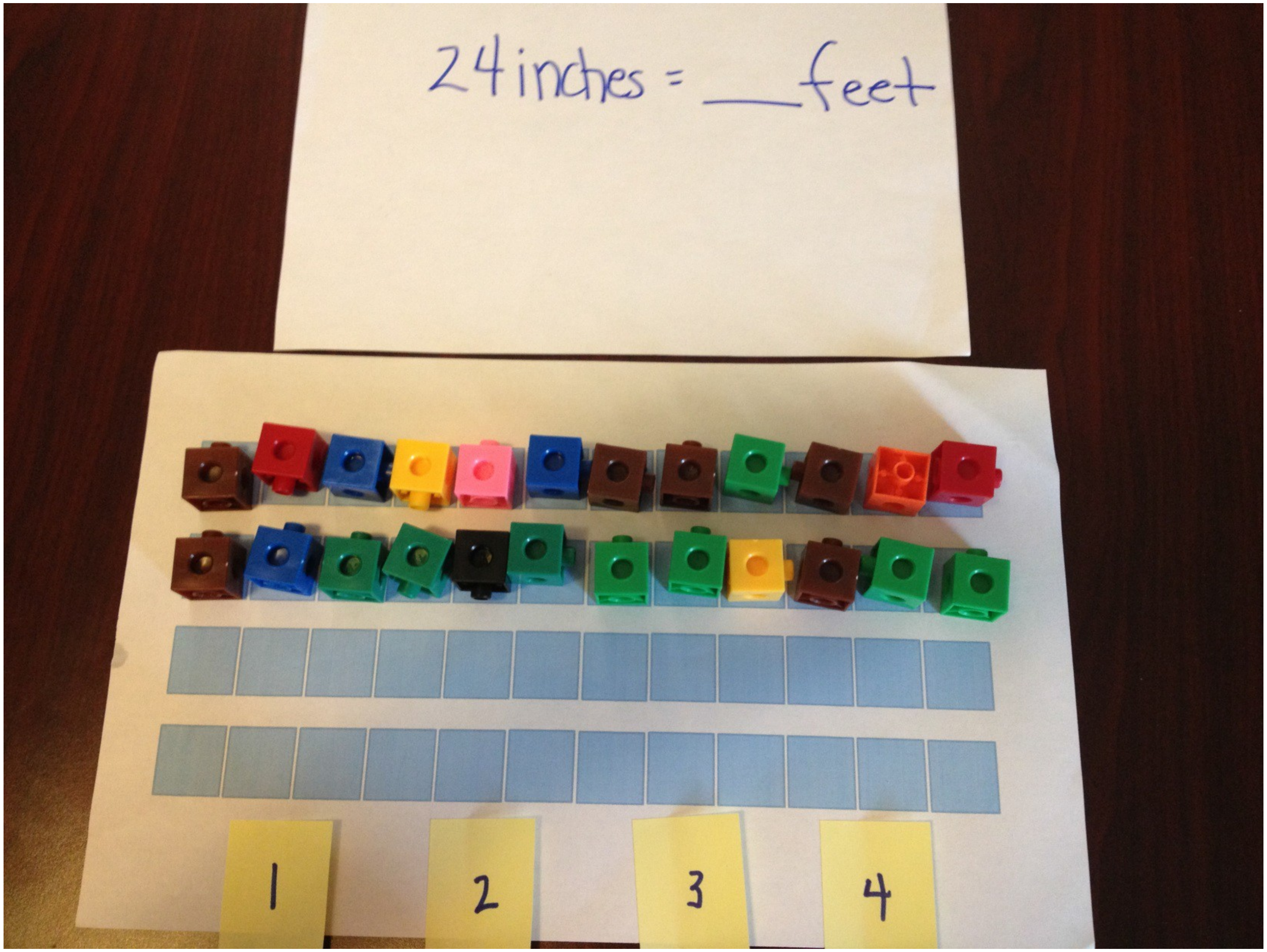
Student:

1. Chooses if the conversion number should be higher or lower than the number of the original measurement.
2. Chooses if the number should be multiplied or divided to make the conversion.
3. Chooses what number to multiply or divide by.

1.	Higher ↑	Lower ↓
2.	Multiply ×	Divide ÷
3.	12	10

Use conversion guide and calculator to solve classmate's conversion problem.

feet → inches	multiply by 12
inches → feet	divide by 12


24 inches = ___ feet
1
2
3
4

Converting Units Within the Metric System (length)

The Metric System

- The metric system does not use feet and inches; it uses **meters** and **centimeters**.

"Cent" means "100," which is why there are 100 <u>cent</u>imeters in a meter.

1 meter (m)

10 cm	10 cm	10 cm	10 cm	10 cm	10 cm	10 cm	10 cm	10 cm	10 cm

1 meter (m) = 100 centimeters (cm)

1 x 100 = 100

larger unit smaller unit

meters to centimeters – MULTIPLY BY 100

Remember: centimeters are smaller units than meters, so it takes more of them to equal a meter, which is why you multiply to convert meters to centimeters.

Meters to Centimeters – Reference

MULTIPLY

2 meters x 100 = 200 centimeters

3 meters x 100 = 300 centimeters

4 meters x 100 = 400 centimeters

5 meters x 100 = 500 centimeters

6 meters x 100 = 600 centimeters

7 meters x 100 = 700 centimeters

8 meters x 100 = 800 centimeters

9 meters x 100 = 900 centimeters

10 meters x 100 = 1000 centimeters

1 meter (m)

10 cm	10 cm	10 cm	10 cm	10 cm	10 cm	10 cm	10 cm	10 cm	10 cm

100 centimeters (cm) = 1 meter (m)

$100 \div 100 = 1$

smaller unit — centimeters **to** larger unit — meters – DIVIDE BY 100

Remember: centimeters are smaller units than meters, so it takes less meters to equal the same amount in centimeters, which is why you divide to convert centimeters to meters.

Centimeters to Meters – Reference

DIVIDE

200 centimeters ÷ 100 = 2 meters

300 centimeters ÷ 100 = 3 meters

400 centimeters ÷ 100 = 4 meters

500 centimeters ÷ 100 = 5 meters

600 centimeters ÷ 100 = 6 meters

700 centimeters ÷ 100 = 7 meters

800 centimeters ÷ 100 = 8 meters

900 centimeters ÷ 100 = 9 meters

1000 centimeters ÷ 100 = 10 meters

Quick Review

- To convert *m* to *cm*, MULTIPLY BY 100
- To convert *cm* to *m*, DIVIDE BY 100

- To convert *ft* to *in*, MULTIPLY BY 12.
- To convert *in* to *ft*, DIVIDE BY 12.

Practice Conversions of Length Within the Metric System

Multiply or Divide

400 centimeters = ___4___ meters

By what? 100

Answer: 400 ÷ 100 = 4

Multiply or Divide

900 cm = ___9___ m

By what? 100

Answer: $900 \div 100 = 9$

Multiply or Divide

5 meters = 500 centimeters

By what? 100

Answer: 5 x 100 = 500

Multiply or Divide

10 m = ____1000____ cm

By what? 100

Answer: 10 x 100 = 1000

Multiply or Divide

3 meters = 300 centimeters

By what? 100

Answer: $3 \times 100 = 300$

Multiply or Divide

200 centimeters = ___2___ meters

By what? 100

Answer: 200 ÷ 100 = 2

Multiply or Divide

7 m = ______700______ cm

By what? 100

Answer: 7 x 100 = 700

Multiply or Divide

6 m = ___600___ cm

By what? 100

Answer: 6 x 100 = 600

Multiply or Divide

800 cm = ____8____ m

By what? 100

Answer: 800 ÷ 100 = 8

Multiply or Divide

5 m = _____500_____ cm

By what? 100

Answer: 5 x 100 = 500

Complete the Chart

METERS		CENTIMETERS
1	=	100
7	=	700
4	=	400
9	=	900
2	=	200

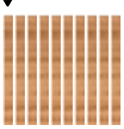
Metric

1 meter

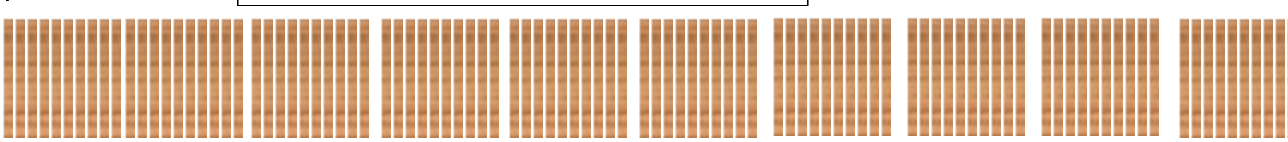
10 decimeters = 1 meter

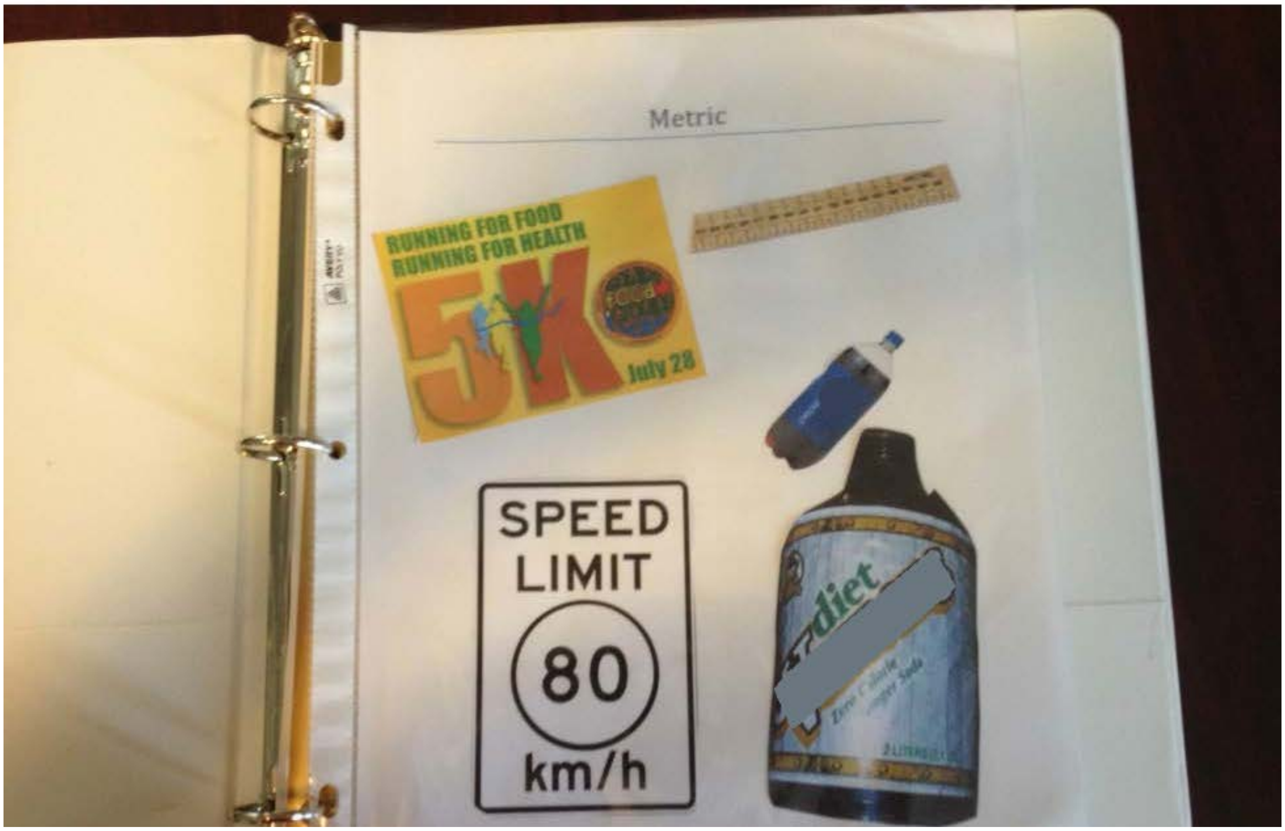
1 decimeter

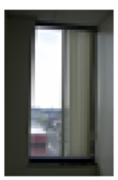
1 centimeter

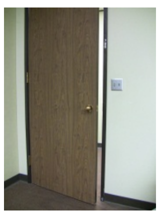


1 centimeter




Metric
RUNNING FOR FOOD
RUNNING FOR HEALTH
5K
July 28
SPEED
LIMIT
80
km/h
diet

 Stapler	☐ Meter stick ☒ Ruler 17	 Computer	☐ Meter stick ☒ Ruler 20
 Bookshelf	☒ Meter stick ☐ Ruler 90	 Filing Cabinet	☒ Meter stick ☐ Ruler 121
 Door	☒ Meter stick ☐ Ruler 213	 Books	☐ Meter stick ☒ Ruler 14
 Window	☒ Meter stick ☐ Ruler 105	 Picture Frame	☐ Meter stick ☒ Ruler 12

 Stapler	☐ Meter stick ☐ Ruler	 Computer	☐ Meter stick ☐ Ruler
 Bookshelf	☐ Meter stick ☐ Ruler	 Filing Cabinet	☐ Meter stick ☐ Ruler
 Door	☐ Meter stick ☐ Ruler	 Books	☐ Meter stick ☐ Ruler
 Window	☐ Meter stick ☐ Ruler	 Picture Frame	☐ Meter stick ☐ Ruler

Meters		Centimeters
4	=	400
9	=	900
5	=	500
10	=	1000
3	=	300

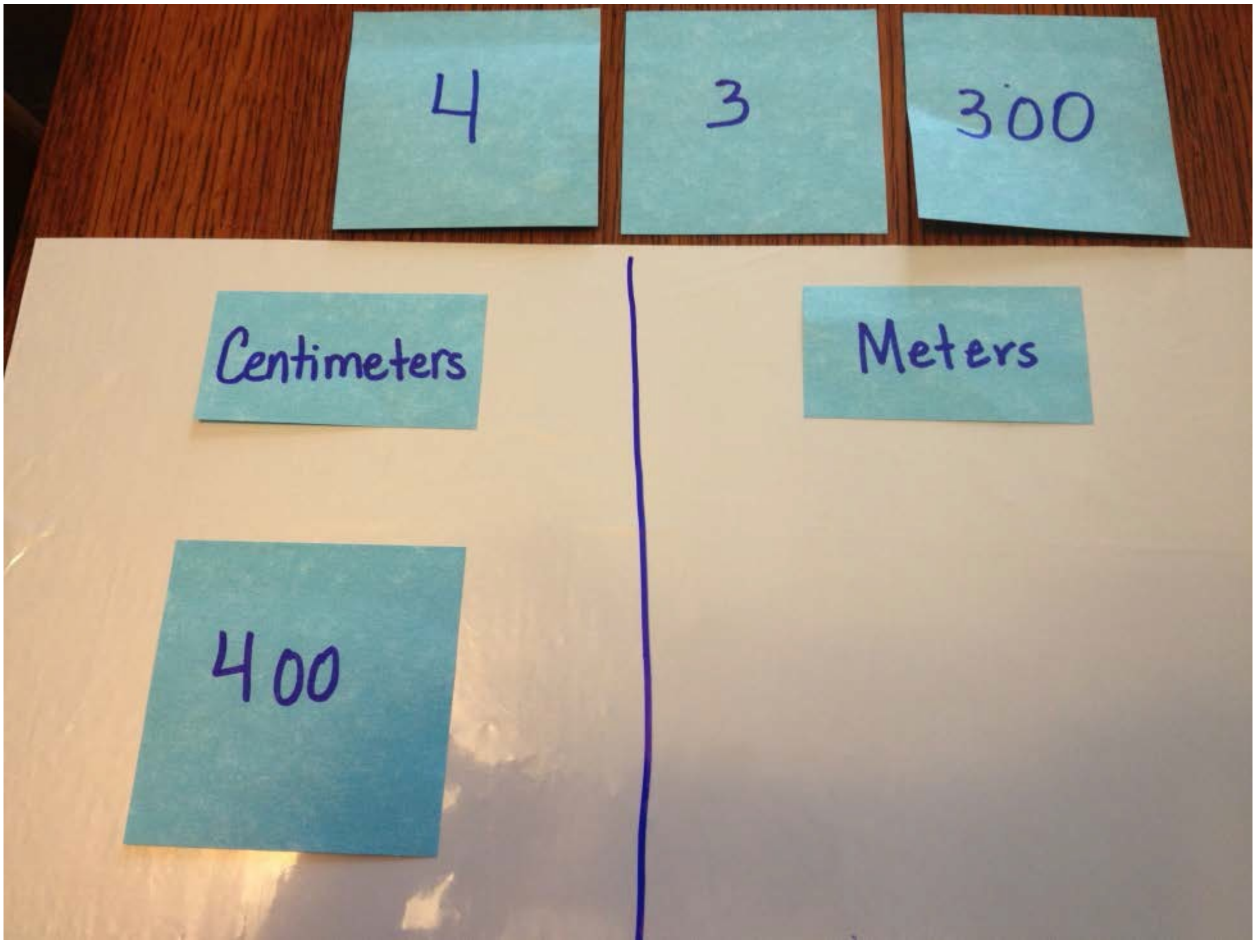
Multiply →

← Divide

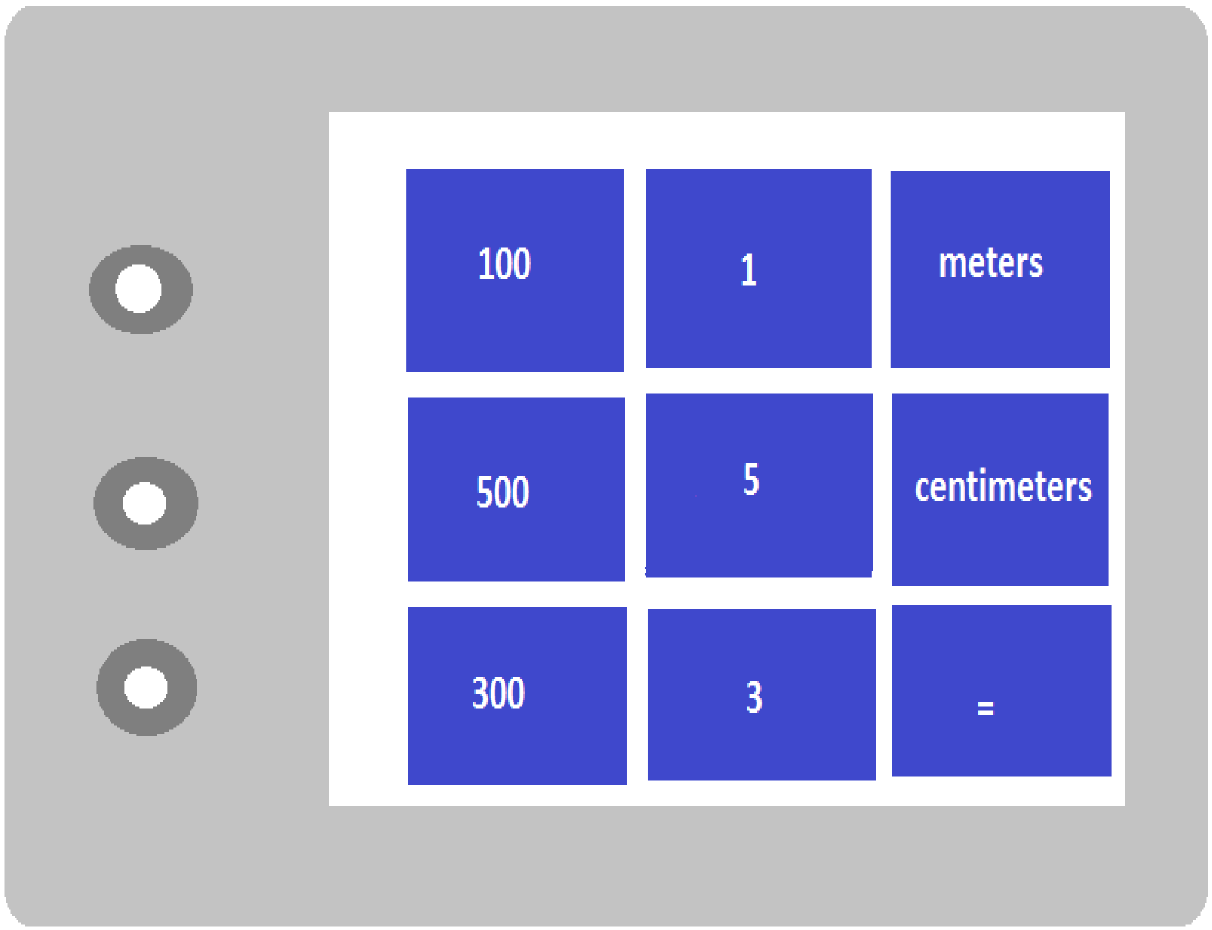
Meter		Centimeter
3 4 5	=	400
5	=	300 400 500
3	=	300 400 500

Multiply

Divide


4
3
300
Centimeters
Meters
400

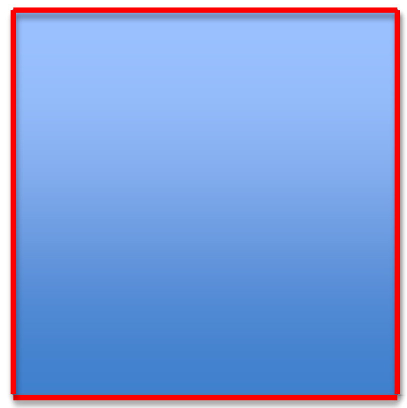
Adapted keyboard to create conversion problem.



Perimeter and Area

Perimeter – "Distance Around"

square = polygon with four equal sides and four 90º angles

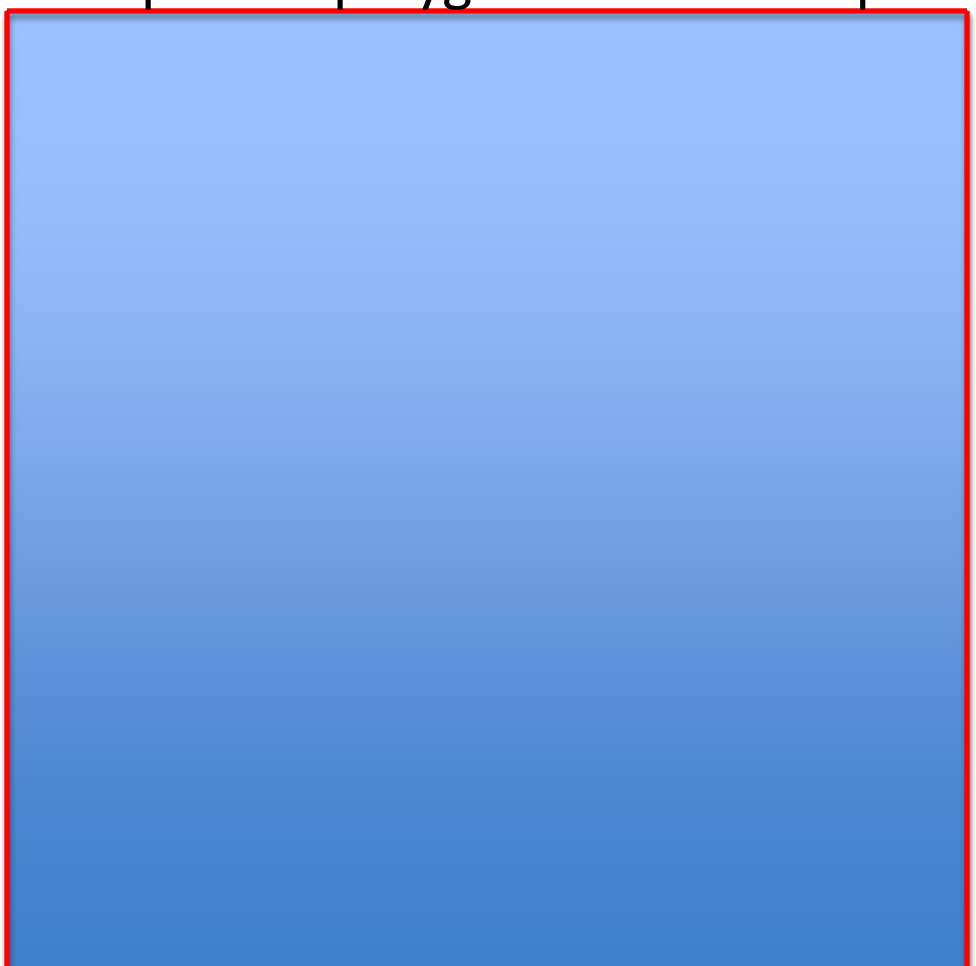


Perimeter: 2 + 2 + 2 + 2 = 8 inches

2 x 4 sides = 8 inches

Perimeter – "Distance Around"

square = polygon with four equal sides and four 90º angles



5 in

Perimeter:

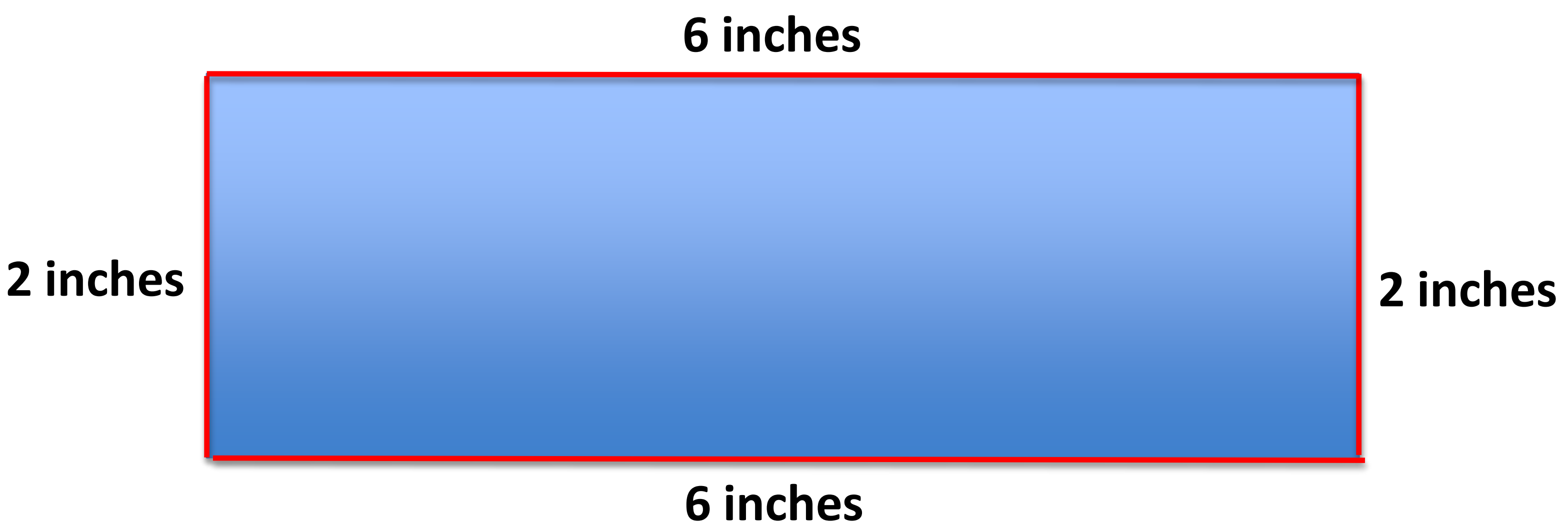
5 + 5 + 5 + 5 = 20 in

OR

5 x 4 sides = 20 in

Perimeter – "Distance Around"

rectangle = polygon with four 90º angles and opposite sides of equal lengths



Perimeter: $2 + 6 + 2 + 6 = 16$ inches

$2 \times 2 = 4$ $2 \times 6 = 12$ $4 + 12 = 16$ inches

Perimeter – "Distance Around"

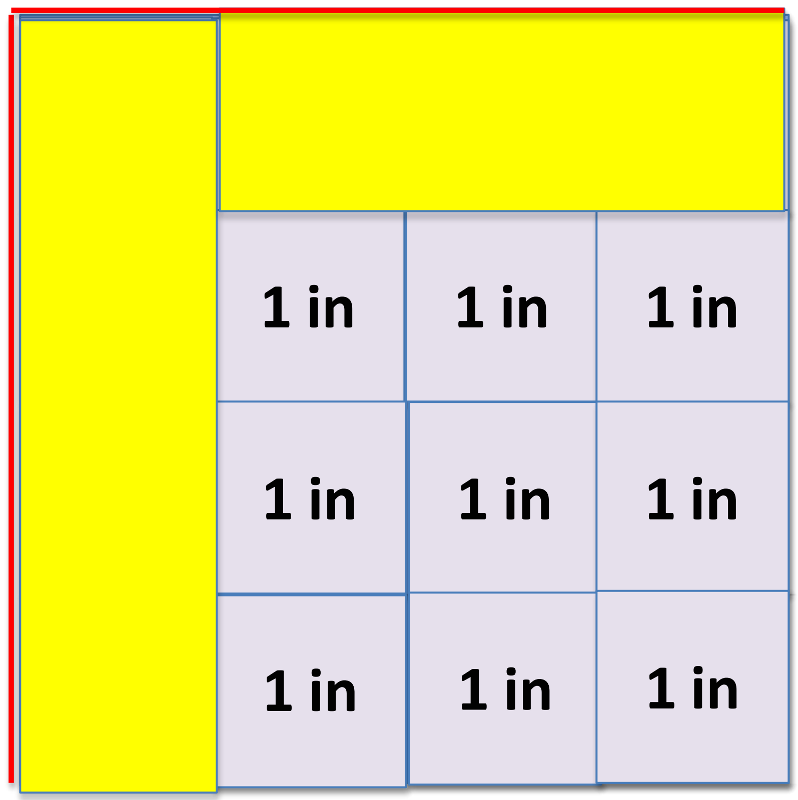
rectangle = polygon with four 90º angles and opposite sides of equal lengths

Perimeter:

1 x 2 = 2 7 x 2 = 14 2 + 14 = 16 inches

Area – "Space Inside"

4 inches



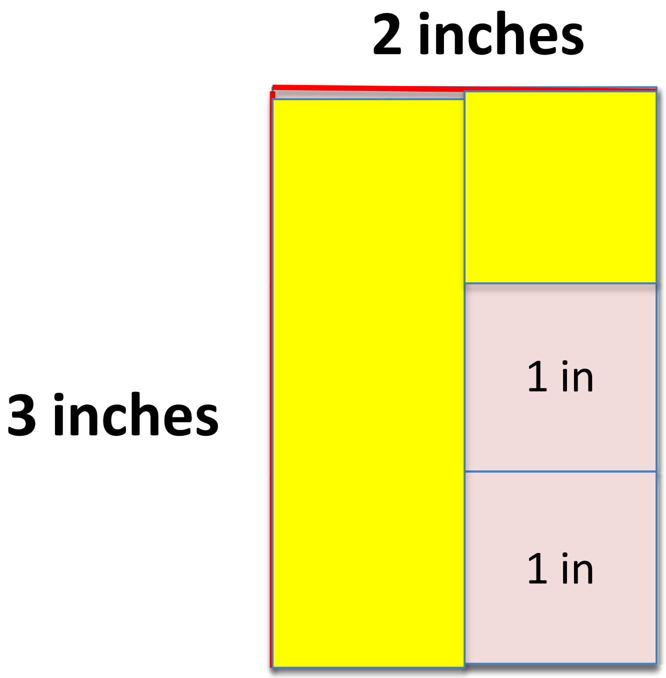
4 inches

16 square inches

Area = Length x Width

$4 \times 4 = 16\ in^2$

Area – “Space Inside”



6 square inches

Area = Length x Width $3 \times 2 = 6\ in^2$

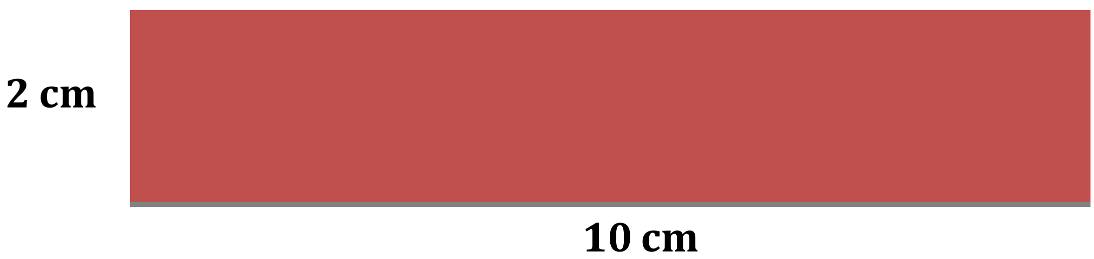
Summary

- Perimeter – Add up all of the sides
  - Don’t forget to include the units with your answer.
- Area – Multiply the length times the width
  - Answer needs to be in square units (such as in^2)

Name ____ Date ____________

Find the perimeter and area of each figure using the information provided.

1.



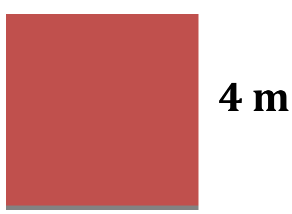
P = side + side + side + side

P = _______________________ _______

A = length x width

A = ____________________ ________

2.



P = side + side + side + side

P = _______________________ _______

A = length x width

A = ____________________ ________

3.

P = side + side + side + side

P = ________________________ _______

A = length x width

A = ____________________ ________

4.

P = side + side + side + side

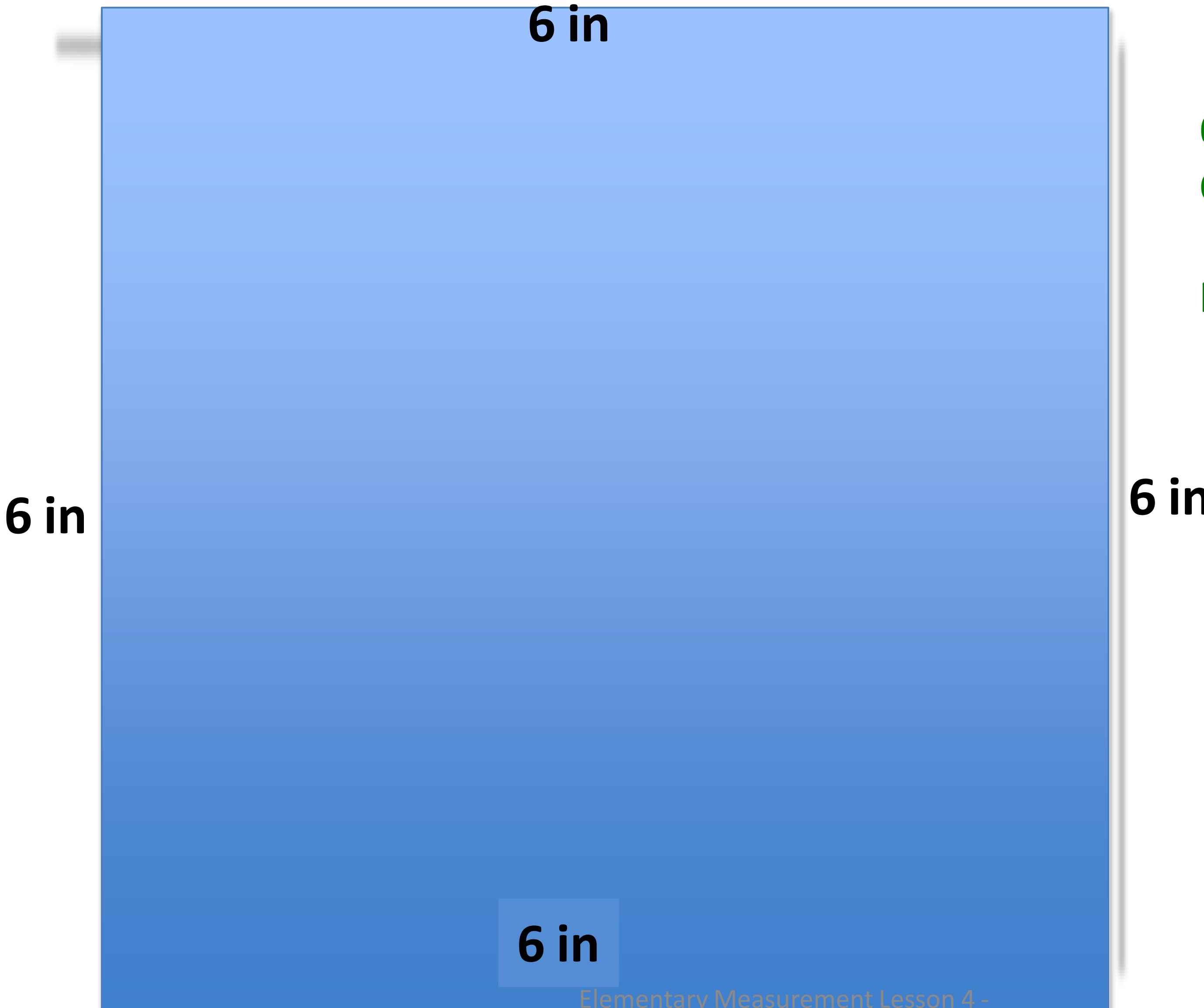
P = ________________________ _______

A = length x width

A = ____________________ ________

Practice Finding Perimeter and Area

What’s the Perimeter of This Square?



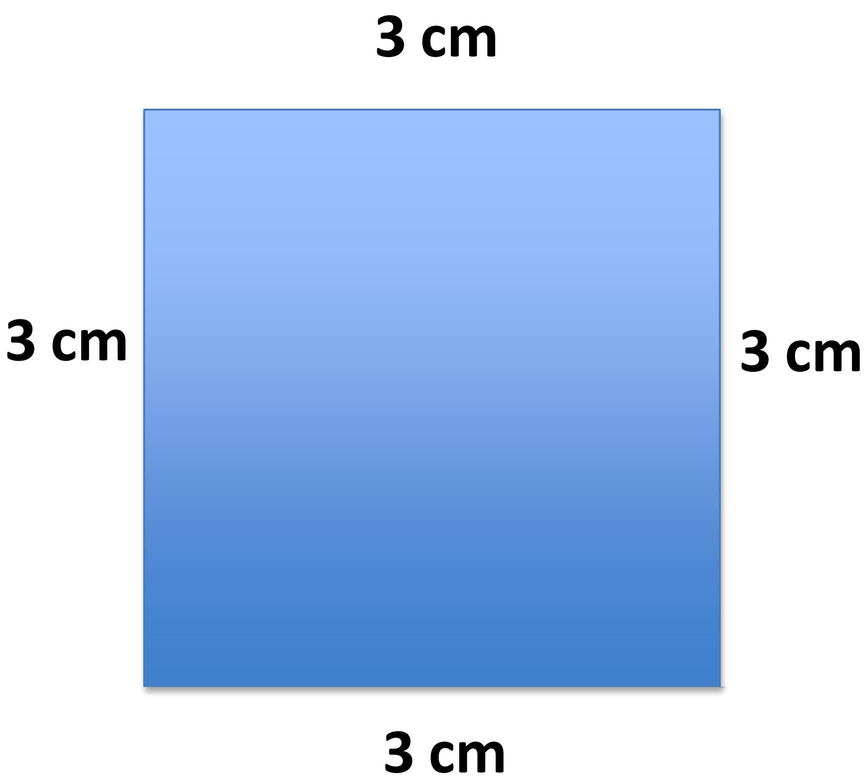
Answer:

6 + 6 + 6 + 6 = **24**

Or 6 x 4 = **24**

P = **24** in

What's the Perimeter of This Square?



Answer:

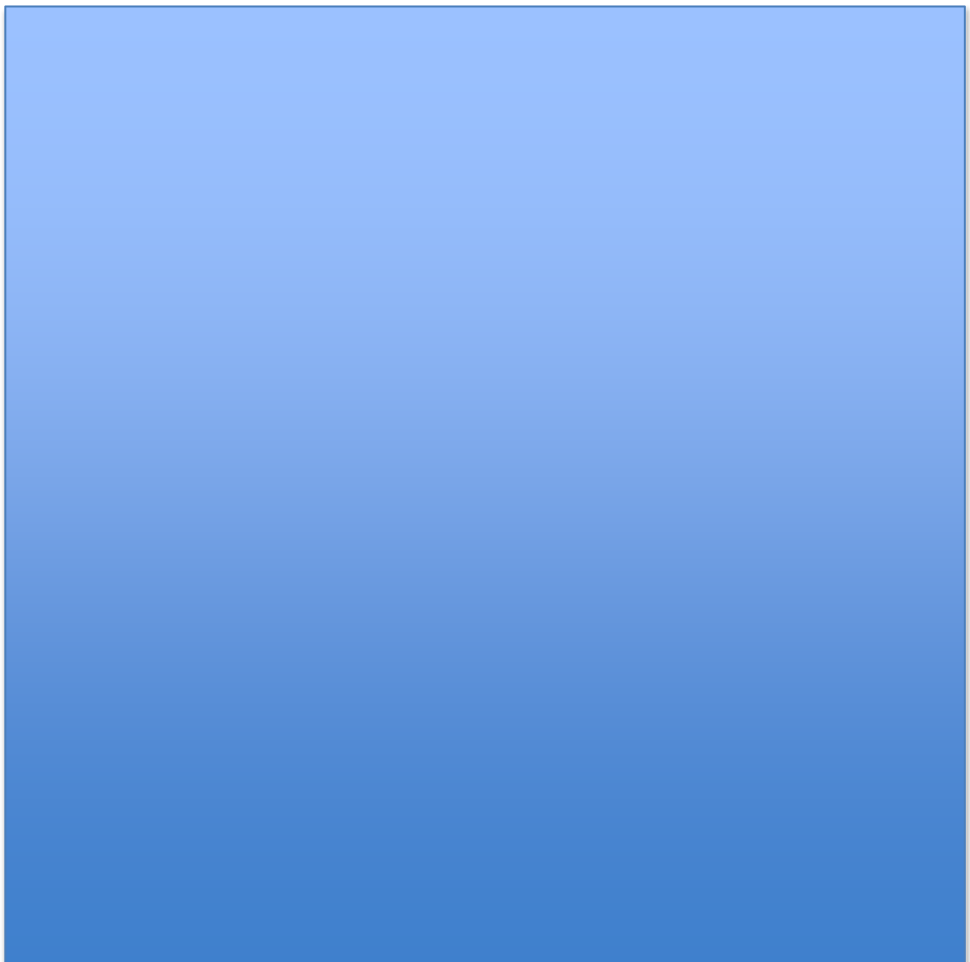
3 + 3 + 3 + 3 = **12**

Or 3 x 4 = **12**

P = **12** cm

What’s the Perimeter of This Square?

7 cm



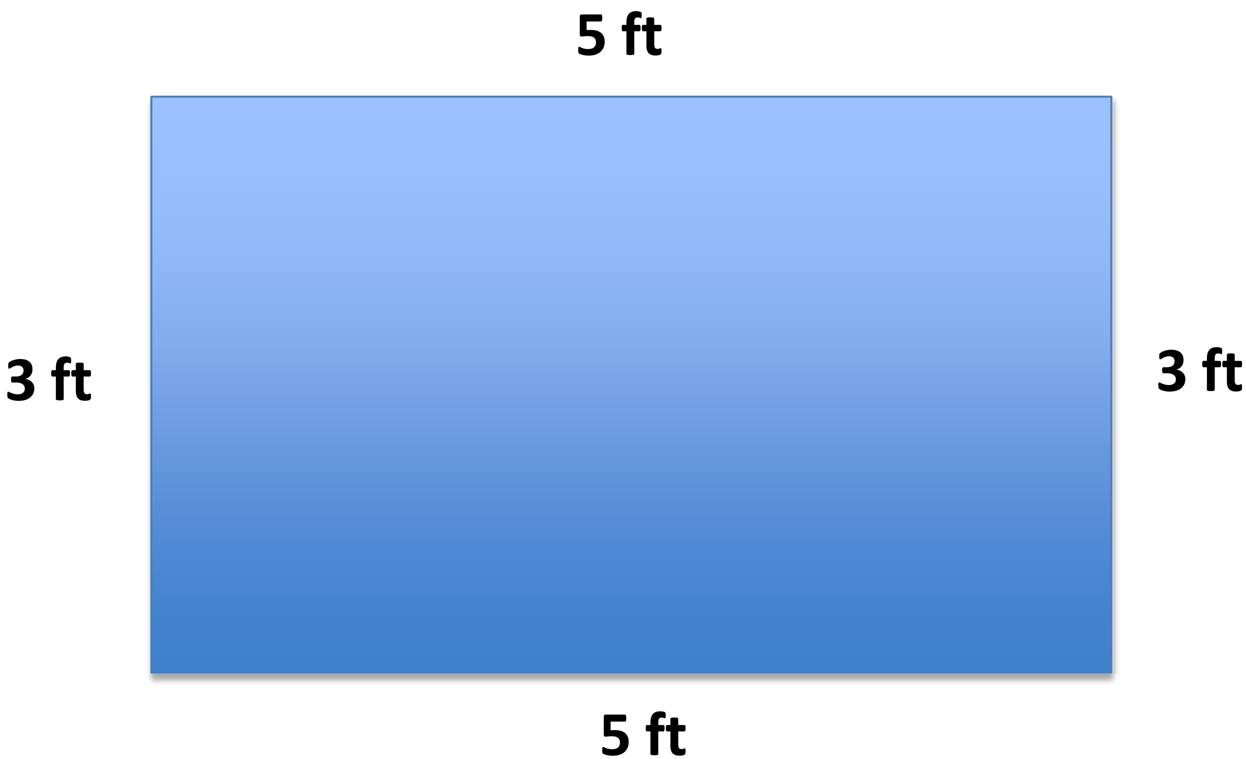
Answer:

7 + 7 + 7 + 7 = **28**

Or 7 x 4 = **28**

P = **28** cm

What's the Perimeter of This Rectangle?



Answer:

3 + 5 + 3 + 5 = **16**

Or

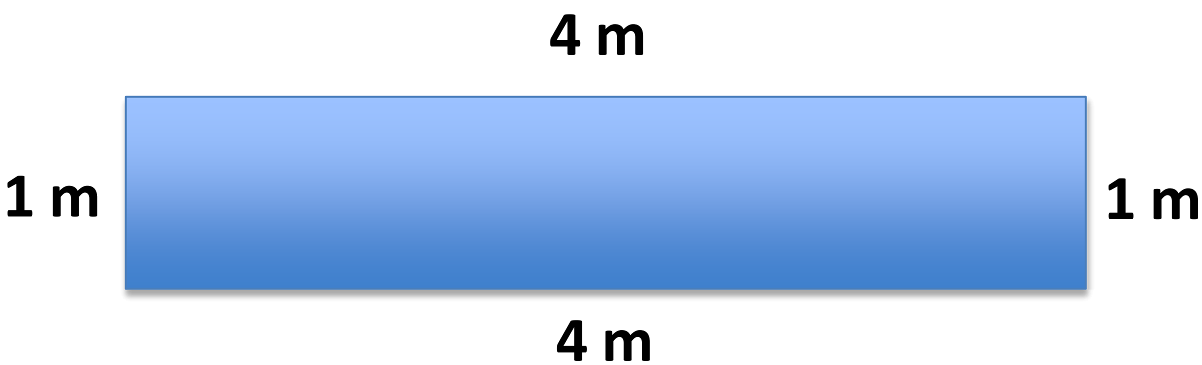
2 x 3 = 6

2 x 5 = 10

6 + 10 = **16**

P = **16** ft

What's the Perimeter of This Rectangle?



Answer:

1 + 4 + 1 + 4 = **10**

Or

2 x 1 = 2

2 x 4 = 8

2 + 8 = **10**

P = **10** m

What’s the Perimeter of This Rectangle?

Answer:

3 + 8 + 3 + 8 = **22**

Or 2 x 3 = 6   2 x 8 = 16   6 + 16 = **22**

P = **22** ft

What's the Area of This Square?

6 in

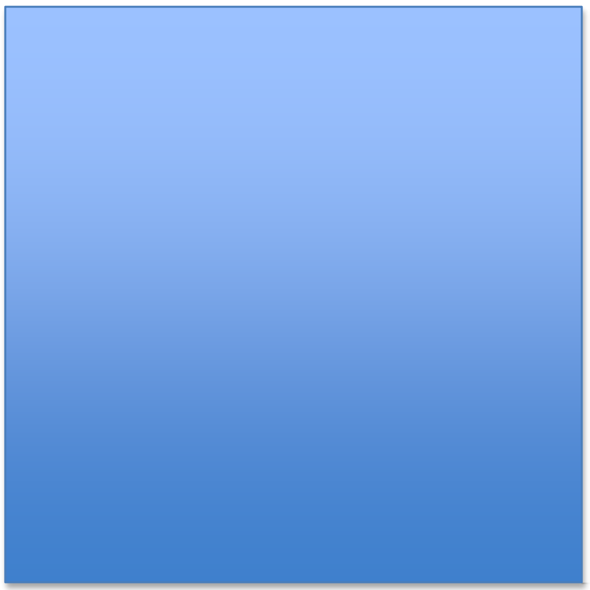
Answer:

6 x 6 = **36**

A = **36** in^2

What's the Area of This Square?

3 cm

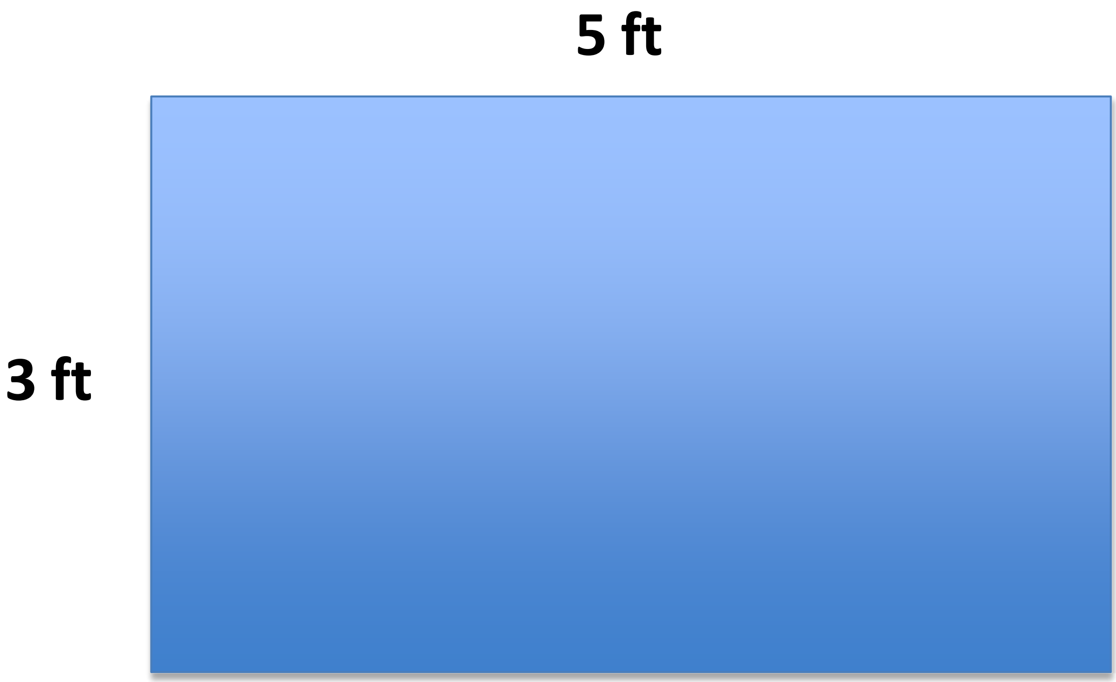


Answer:

3 x 3 = **9**

A = **9** cm^2

What's the Area of This Rectangle?

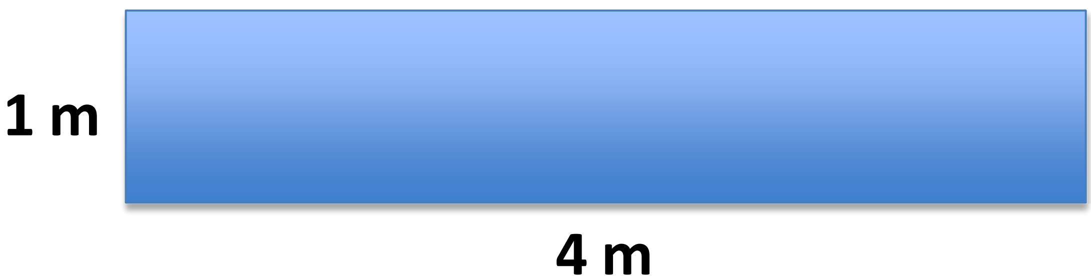


Answer:

$3 \times 5 = \mathbf{15}$

$A = \mathbf{15}\ ft^2$

What’s the Area of This Rectangle?



Answer:

1 x 4 = 4

P = **4** m^2

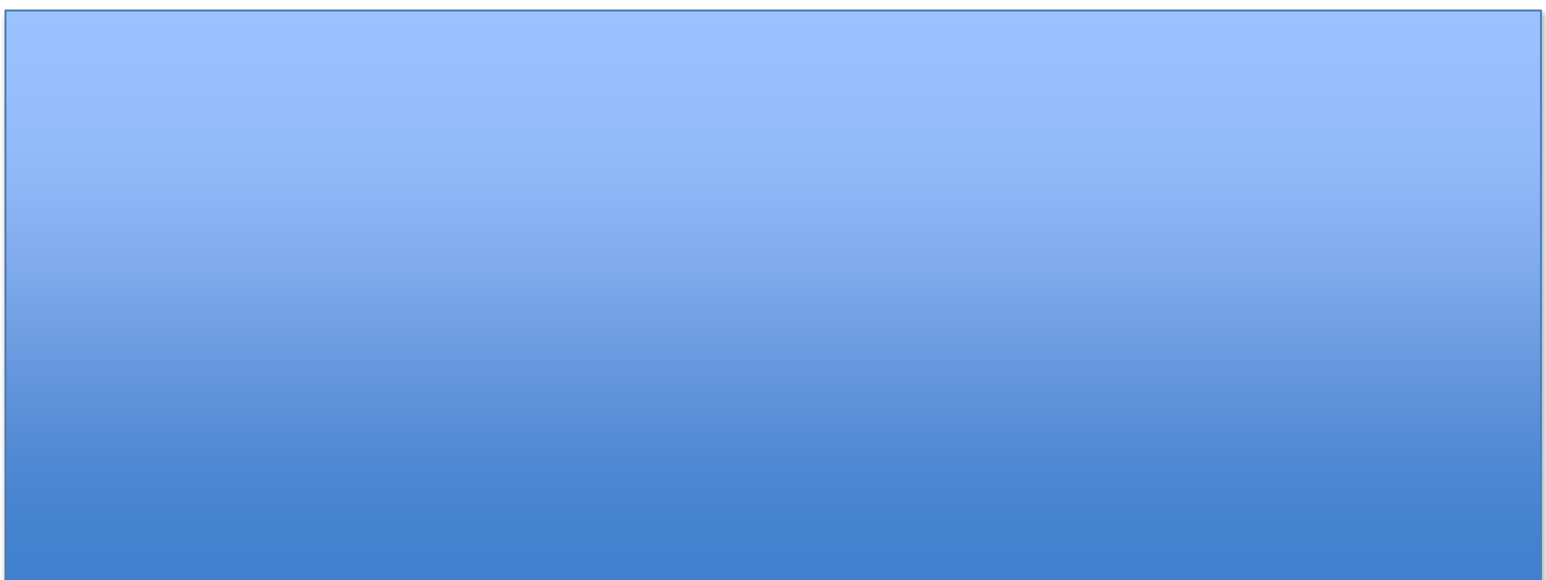
What’s the Perimeter of This Rectangle?

Answer:

8 x 3 = 24

A = **24** ft^2

8 ft



3 ft

What's the Area **and** the Perimeter of This Rectangle?

Answers:

$7 + 2 + 7 + 2 = 18$

OR $2 \times 7 = 14$ $2 \times 2 = 4$ $14 + 4 = 18$

P = **18** cm

$7 \times 2 = 14$

A = **14** cm^2

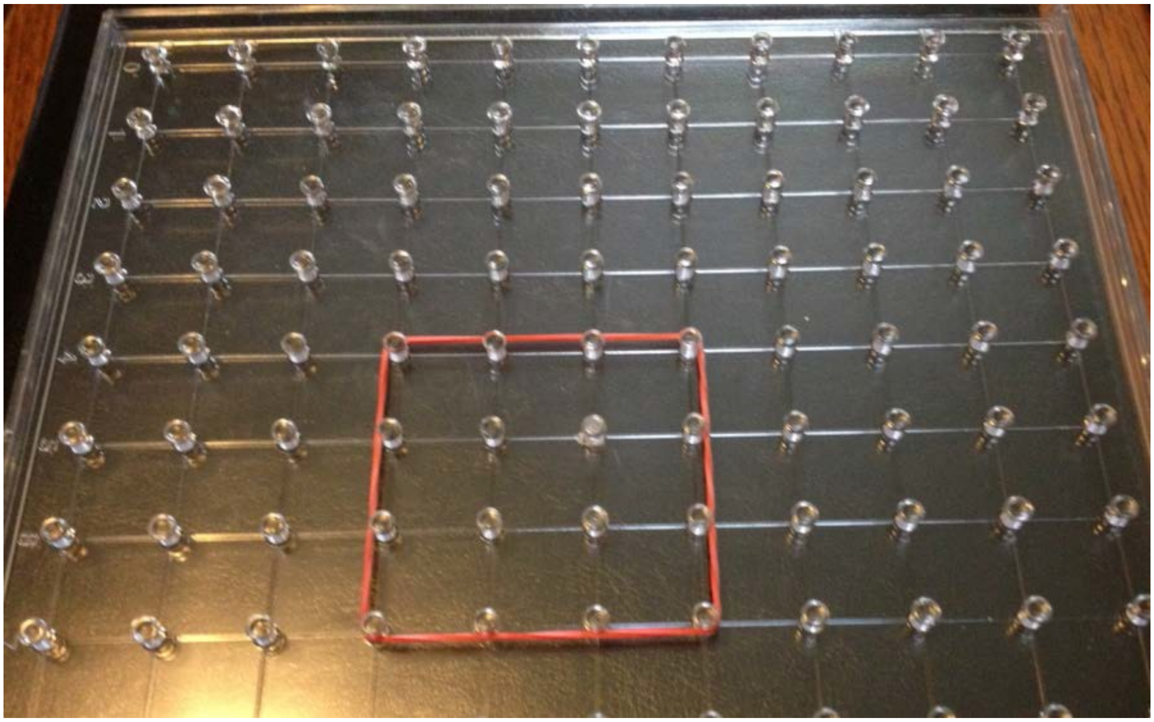
Read the story problem below and answer the questions about the problem in your Math Journal.

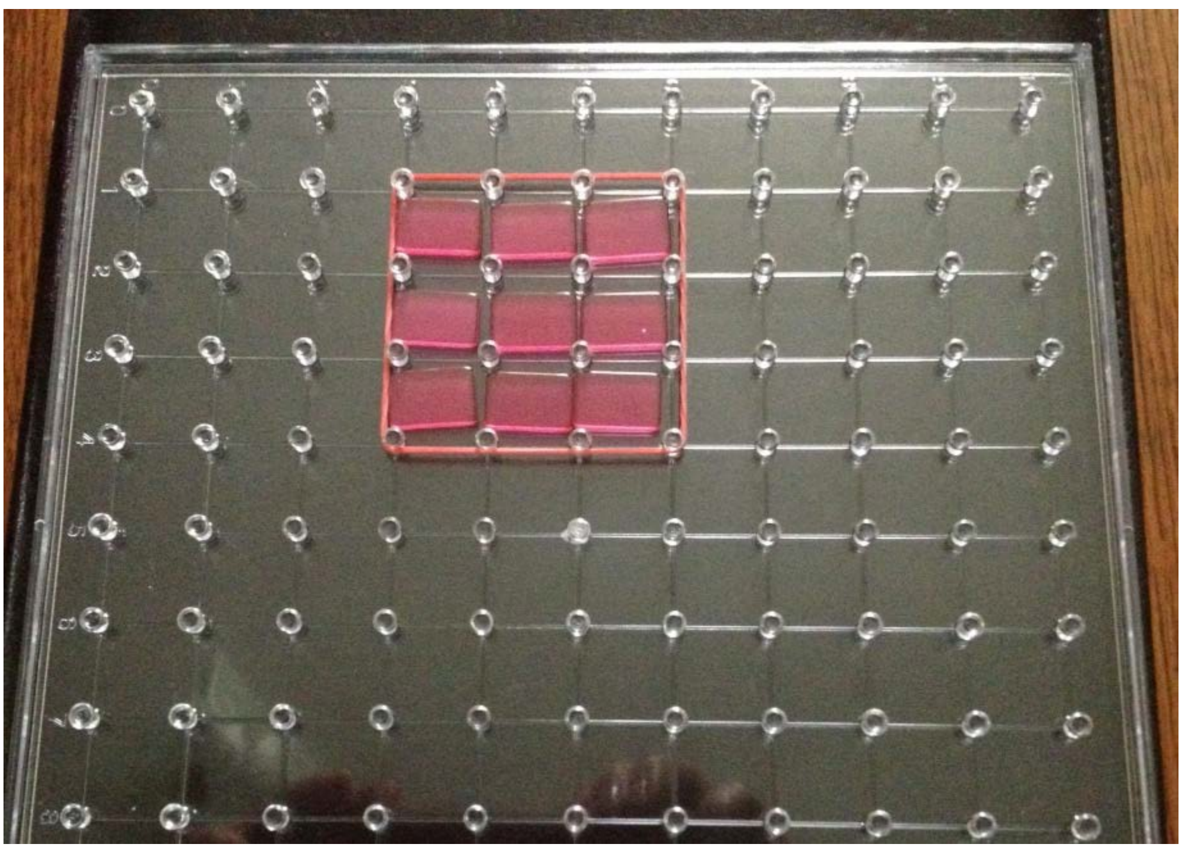
Jane and Luis are in charge of the 4th grade booth at their school's Spring Festival, which will be held on the school's parking lot. Space on the parking lot is very limited, and so the parking lot has been divided so that each grade will have a section that measures only 2 feet by 3 feet. Jane and Luis want to make sure they will have room for their popcorn popper and snow cone machine as well as the serving table.

A) What is the perimeter of class' section of the parking lot?
B) Convert the perimeter of the class' section into inches.
C) What is the area of the class' section of the parking lot?
D) Which is likely more helpful to Jane and Luis in determining whether or not they have enough space for all of their equipment—knowing the perimeter of their section or knowing the area of their section? Why?
E) Based on your calculations, do you think Jane and Luis will have enough room for the popcorn popper, the snow cone machine and the table? How do you know?

Unit 1 – Lesson 4 – ER or EC

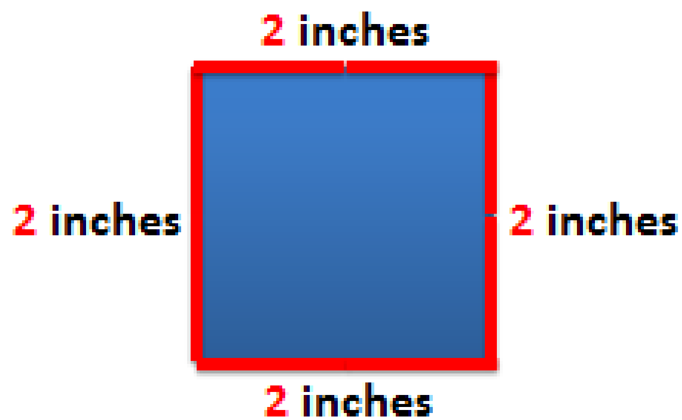
	P	P	P	P	P	P	P	
P	A	A	A	A	A	A	A	P
P	A	A	A	A	A	A	A	P
P	A	A	A	A	A	A	A	P
P	A	A	A	A	A	A	A	P
P	A	A	A	A	A	A	A	P
P	A	A	A	A	A	A	A	P
P	A	A	A	A	A	A	A	P
	P	P	P	P	P	P	P	





Animated slide: Arrows next to "Distance Around" move in a clockwise motion, each one inch section of perimeter appears and the measurement "2 inches" appears after each side has been counted. Each part of the equation " 2 + 2 + 2 + 2 = 8 inches" appears one at a time. Adapted switch may be used.

Perimeter – "Distance Around"



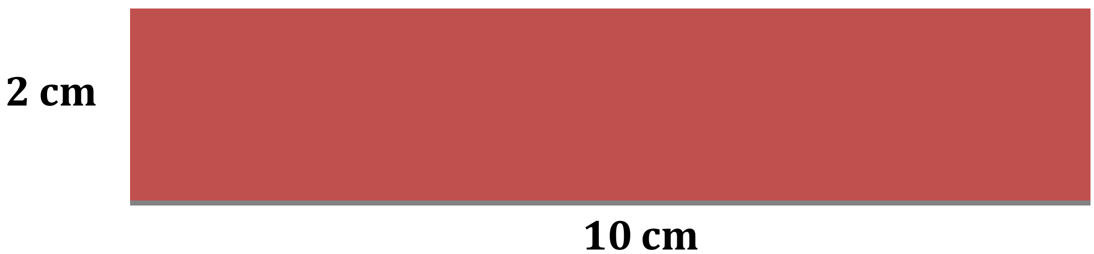
Perimeter: 2 + 2 + 2 + 2 = 8 inches

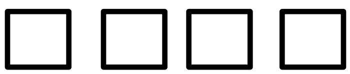
Perimeter and Area Practice Worksheet

Name ____ Date ___________

Find the perimeter and area of each figure using the information provided.

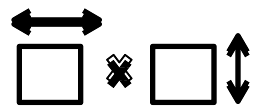
1.





P = side + side + side + side

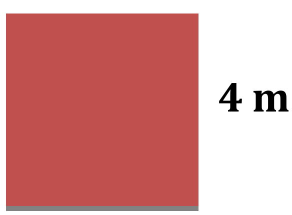
P = ____________________ _____

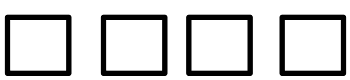


A = length x width

A = _________________ _______

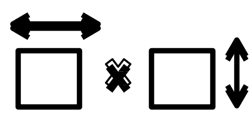
2.





P = side + side + side + side

P = ____________________ _____



A = length x width

A = _________________ _______

3.

P = side + side + side + side

P = ________________________ _______

A = length x width

A = ____________________ ________

4.

P = side + side + side + side

P = ________________________ _______

A = length x width

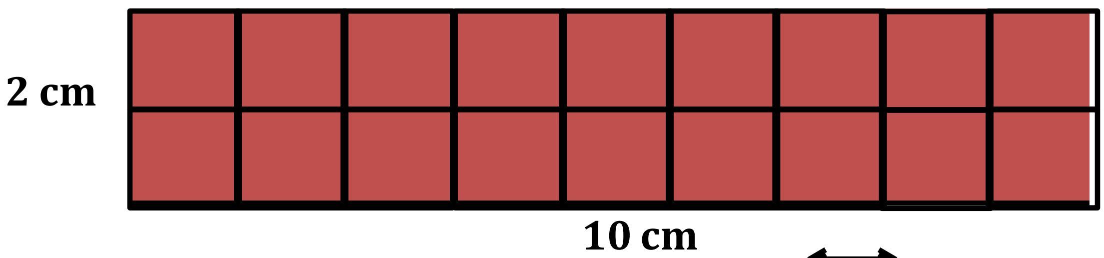
A = ____________________ ________

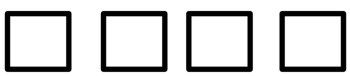
Perimeter and Area Practice Worksheet

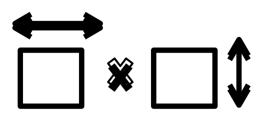
Name ______________________________________ Date __________

Find the perimeter and area of each figure using the information provided.

1.







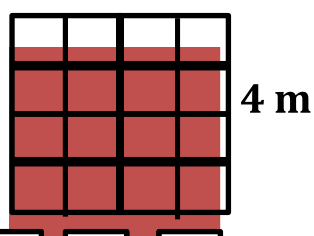
P = side + side + side + side

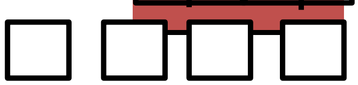
A = length x width

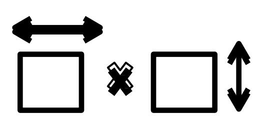
P = ______________________ _______

A = ___________________ ________

2.







P = side + side + side + side

A = length x width

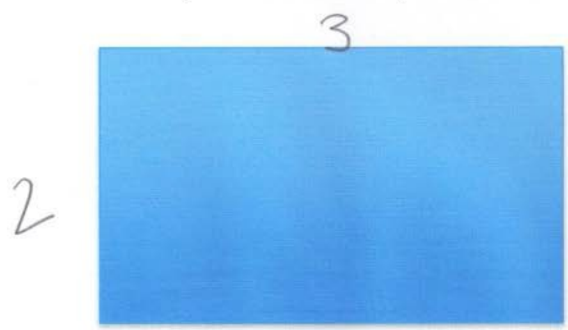
P = ______________________ _______

A = ___________________ ________

Read the story problem below and answer the questions about the problem in your Math Journal.

Jane and Luis are in charge of the 4th grade booth at their school's Spring Festival, which will be held on the school's parking lot. Space on the parking lot is very limited, and so the parking lot has been divided so that each grade will have a section that measures only 2 feet by 3 feet. Jane and Luis want to make sure they will have room for their popcorn popper and snow cone machine as well as the serving table.

A) What is the perimeter of class' section of the parking lot?
B) Convert the perimeter of the class' section into inches.
C) What is the area of the class' section of the parking lot?



$\boxed{2}$ $\boxed{3}$ $\boxed{2}$ $\boxed{3}$

P = side + side + side + side = 10 feet

10 feet = 120 inches

$\boxed{2} \times \boxed{3}$

A = length x width = 6^2 feet

D) Which is likely more helpful to Jane and Luis in determining whether or not they have enough space for all of their equipment—knowing the perimeter of their section or knowing the area of their section? Why?

Perimeter *Area*

E) Based on your calculations, do you think Jane and Luis will have enough room for the popcorn popper, the snow cone machine and the table? How do you know?

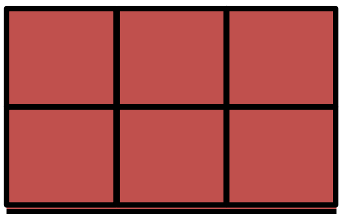
No too big

Jane and Luis want to know if a 2 feet by 3 feet section of the parking lot will be big enough to hold their popcorn popper and snow cone machine as well as a serving table.

1. What is the perimeter?

3 ft.

2 ft.

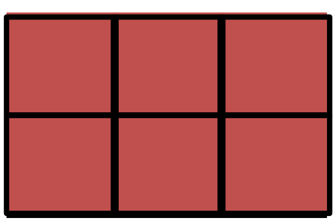


P = $\boxed{10}$

2. Use chart to convert feet to inches.___________feet = __________ inches

3 ft.

2 ft.



A = $\boxed{}$

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**

Name ____ Date _________________

Conversions

1. 36 in =______________ft

2. 3 m =________________cm

3. 7 ft =________________in

4. 800 cm =_______________m

5. 108 in =______________ft

Perimeter and Area

Using a ruler, find the perimeter and area of the figure to the right

using in.

6. P = ______________________

7. A = ______________________

8. Convert your answer for perimeter (number 6) into ft.

_______________________in =________________________ft

Story Problem

Mrs. Hill's 4th grade class is going to make a mural to hang in the hallway. Each student has been asked to draw a picture of himself/herself on a 1-square foot piece of poster board. There are 16 students in the class. Mrs. Hill plans to piece the blocks together in 4 rows with 4 blocks in each row to make the mural one large square.

9. What will the perimeter of the wall mural be?______________________ft.

10. What will the area of the wall mural be?_____________________ft^2

Measurement Unit

Review Game

True or False?

An inch is bigger than a foot.

True or False?

An inch is bigger than a foot.

F A L S E

It takes 12 inches to equal 1 foot.

True or False?

An meter is bigger than a centimeter.

True or False?

T R U E

It takes 100 centimeters to equal 1 meter.

True or False?

To measure the length of a hallway, you would most likely use a tape measure or yard stick rather than a ruler.

True or False?

T R U E

Rulers are marked in inches or centimeters, which are smaller units and less appropriate for measuring a longer length like a hallway.

True or False?

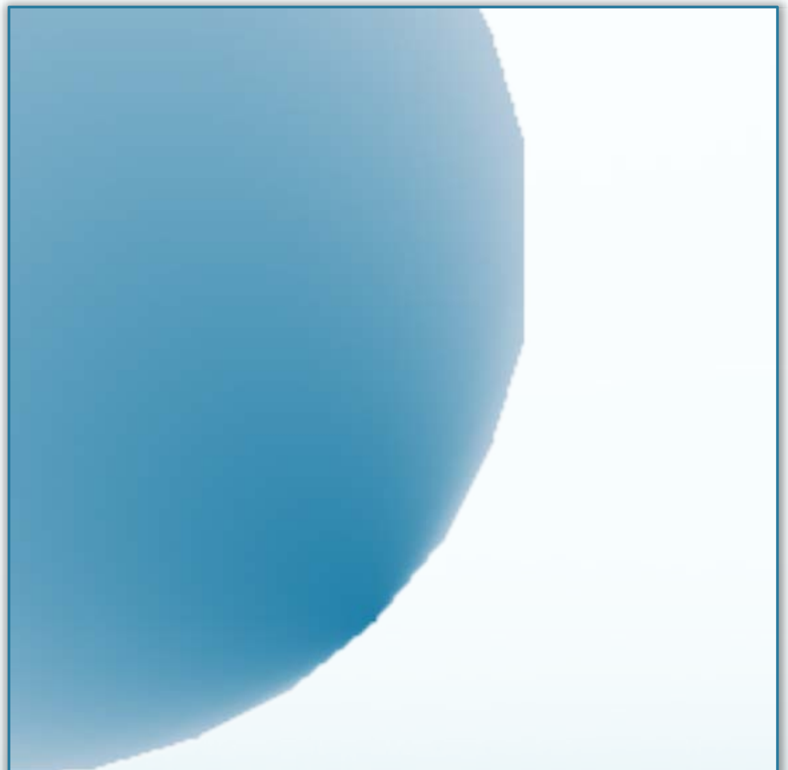
Perimeter means the amount of space inside a figure.

True or False?

F A L S E

The amount of space inside a figure is called area. Perimeter is the distance around a figure.

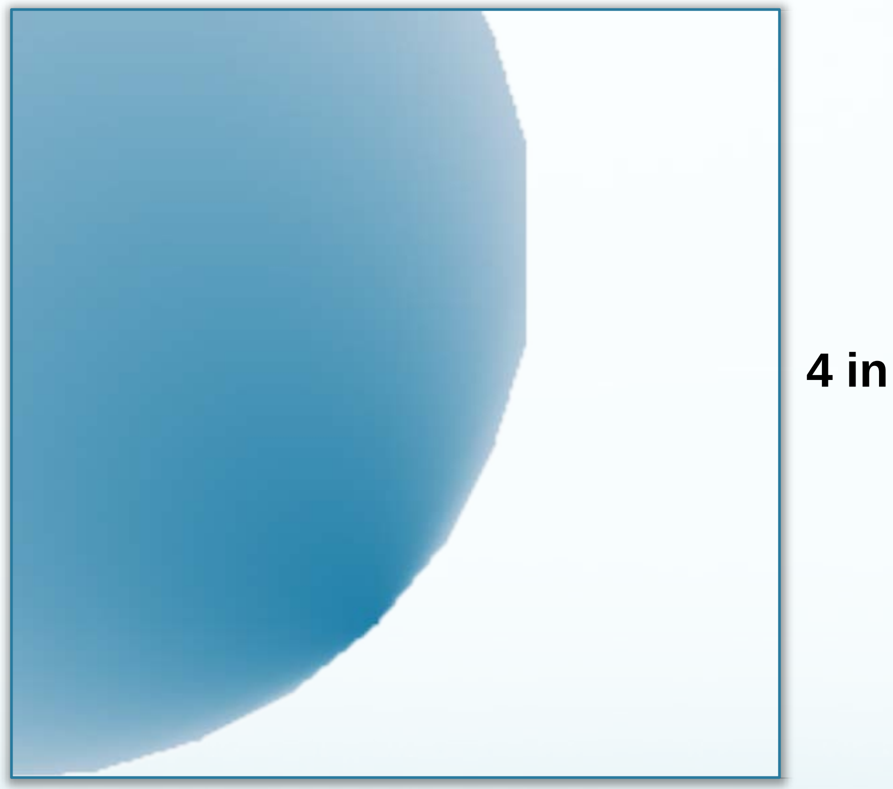
True or False?



4 in

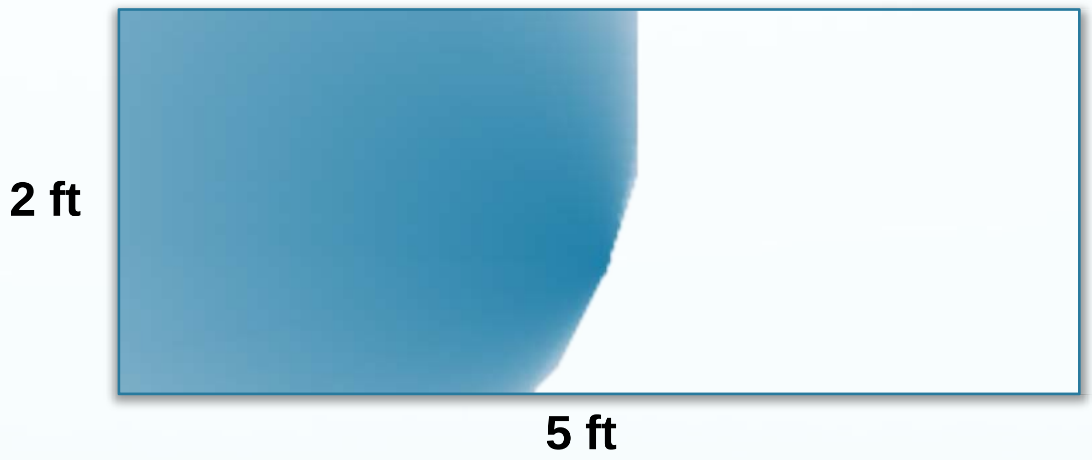
The perimeter of the square above is 16 cm.

True or False?



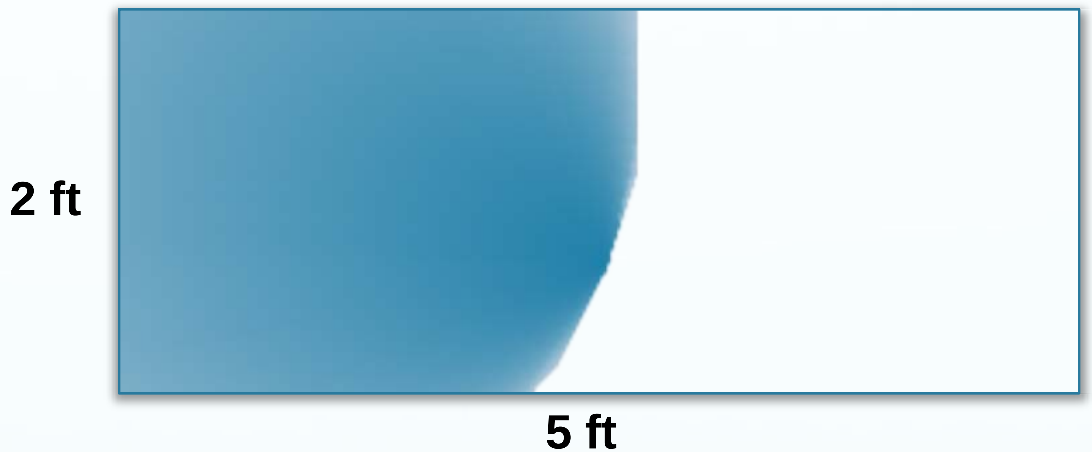
F A L S E – The perimeter of the square above is 16 in, not cm.

True or False?

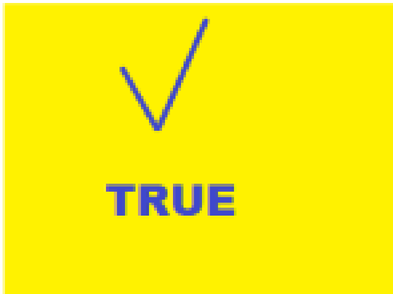


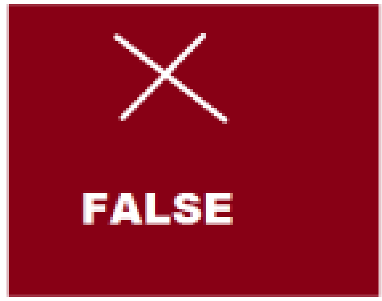
The area of the rectangle above is 14 ft.

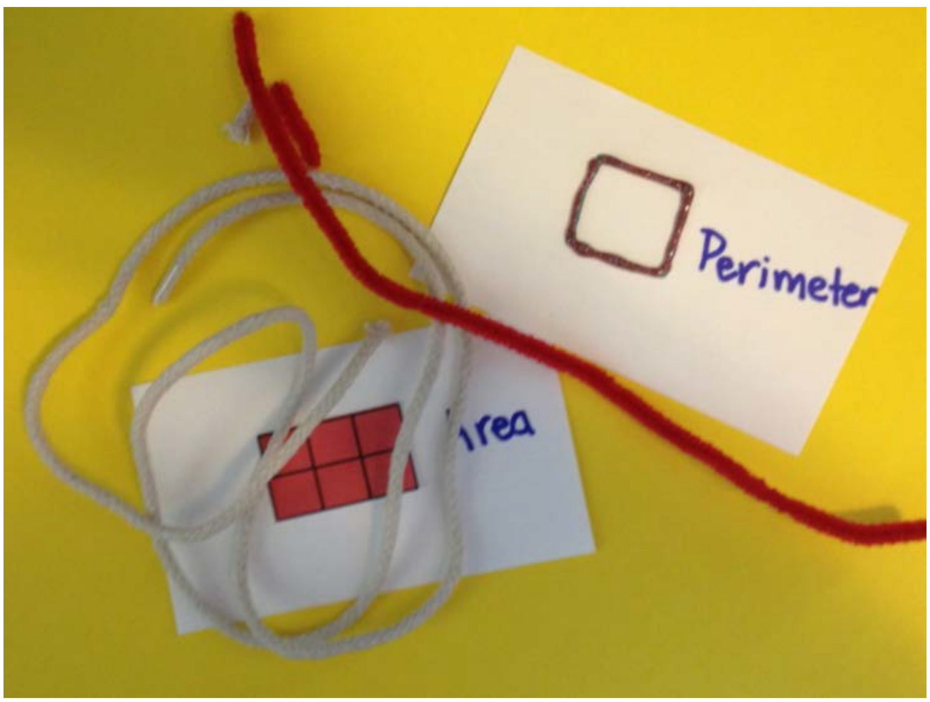
True or False?



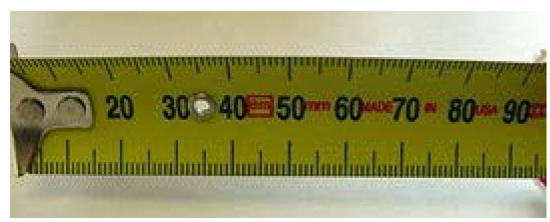
F A L S E The **area** of the rectangle above is 10 ft^2. The *perimeter* is 14 ft.


TRUE

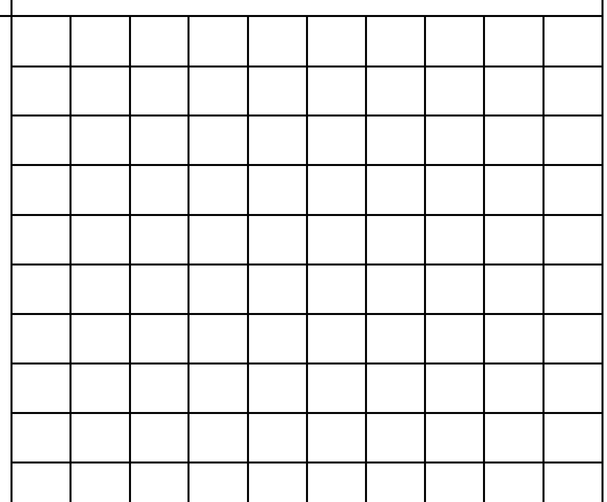

FALSE


Perimeter

Inches ruler



Area grid



Perimeter grid

Culminating Activity Assessment

Name ______________________ Date __________

Conversions

Use conversion guide and calculator to solve classmate's conversion problem.

1. 7 ft = 84 in
2. 108 in = 9 ft

feet → inches	multiply by 12
inches → feet	divide by 12

3. 3 m = 300 cm
4. 800 cm = 8 m

m → cm	multiply by 100
cm → m	divide by 100

Perimeter and Area

Using a ruler, find the perimeter and area of the figure to the right using in.

6. P = $\boxed{4}$ + $\boxed{2}$ + $\boxed{4}$ + $\boxed{2}$ = 12

7. A = $\overleftrightarrow{\boxed{2}}$ x $\boxed{4}\updownarrow$ 8

8. Convert your answer for perimeter (number 6) into ft.

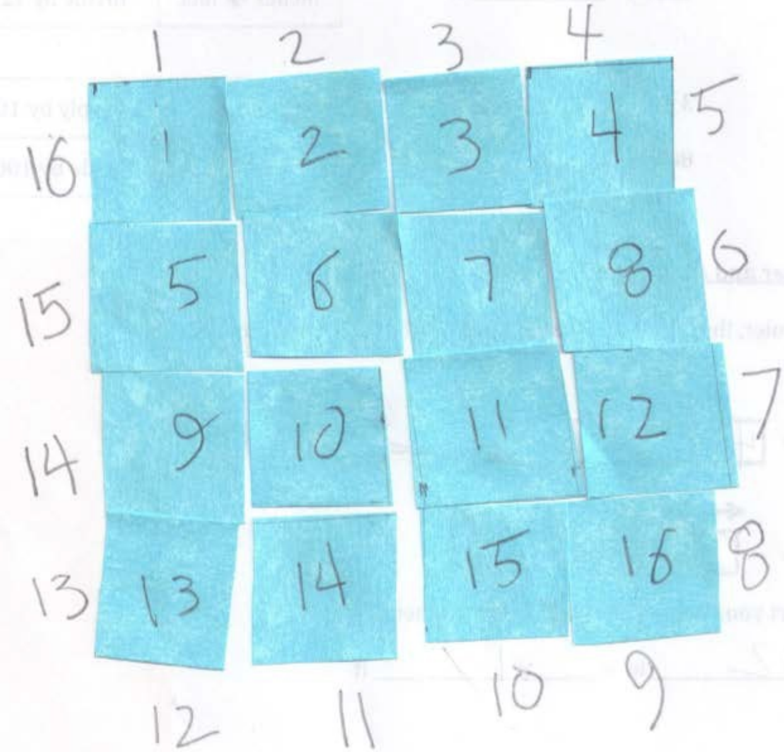
12 in = 1 ft

Story Problem

Mrs. Hill's 4th grade class is going to make a mural to hang in the hallway. Each student has been asked to draw a picture of himself/herself on a 1-square foot piece (1 foot long and one foot wide) of poster board. There are 16 students in the class. Mrs. Hill plans to piece the blocks together in 4 rows with 4 blocks in each row to make the mural one large square.

9. What will the perimeter of the wall mural be? ___16___ ft.

10. What will the area of the wall mural be? ___16___ ft^2



References

Myller, R. (1991). *How big is a foot?* New York, NY: Random House Inc.

Mathematics:

Measurement

9-10

NCSC Sample Instructional Unit

Investigating Measurement in the Real World

National Center and State Collaborative

The contents of this product were developed under a grant from the Department of Education (PR/Award #:H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made.

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Table of Contents

Unit Key Vocabulary

Area. The space that covers a two dimensional figure which is measured in square units.

Centimeter. A metric unit of length equal to one hundredth of a meter.

Conversion. A change from one unit or unit system to another. For example, inches to feet, inches to centimeters.

Dimensions. The length, height, and width of a figure.

Foot. A standard unit of length in the US Customary system of measurement equal to 12 inches.

Inch. A standard unit of length in the US Customary system of measurement equal to 1/12 of a foot.

Length. A measurement of the distance from one point to another.

Meter. The basic Standard International unit of length.

Perimeter. The distance around a closed figure which is measured in units.

Proportion. Equality between two ratios. For example, 8:12 and 4:6.

Ratio. A compared relationship between two numbers. For example, 2:1.

Rectangle. A four-sided figure with four right angles.

Similar Rectangles. Rectangles of different sizes with all corresponding angles congruent and all corresponding sides are proportional.

Unit of Measure. A defined and adopted magnitude of a physical quantity such weight, length, temperature.

Unit Rate. A ratio with the second term being a unit of one. For example, miles per hour; dollars per hour, etc.

Width. Measurement of the distance from one side or edge to the opposite side or edge.

Yards. Measurement of length in which one yard is equal to 3 feet or 36 inches.

Unit Standards Overview

Common Core State Standard:

HSN-Q Number and Quantity

- **HSN-Q.A** Reason quantitatively and use units to solve problems.
  - **HSN-Q.A.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
  - **HSN-Q.A.2** Define appropriate quantities for the purpose of descriptive modeling.

HSG-MG Geometry

- **HSG-MG.A** - Apply geometric concepts in modeling situations.
  - **HSG-MG.A.1** Use geometric shapes, their measures, and their properties to describe objects.
  - **HSG-MG.A.2** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
  - **HSG-MG.A.3** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Learning Progressions Frameworks Progress Indicator

H.ME.1a Making decisions about units and scales that are appropriate for problem-solving situations involving mathematics within or across mathematics disciplines or real-world context.

H.ME.1b Investigating the results when linear dimensions of objects change by some factor (e.g., area and volume change disproportionately: area in proportion to the square of the factor and volume in proportion to its cube)

Instructional Family: Problem solving using measurement process.

Core Content Connectors addressed:

- **H.ME.1a1** Determine the necessary unit(s) to use to solve real world problems.
- **H.ME.1a2** Solve real world problems involving units of measurement.
- 3.ME.1g1 Identify a figure as getting larger or smaller when the dimensions of the figure changes.

Instructional Family: Perimeter, Area, Volume

Core Content Connectors addressed:

- **H.ME.1b1** Describe the relationship between the attributes of a figure and the changes in the area or volume when 1 attribute is changed.
- 8.ME.1e2 Compare area and volume of similar figures.
- 6.ME.1a1 Identify the appropriate formula (i.e., perimeter, area, volume) to use when measuring for different purposes in a real life context.
- 4.ME.2h1 Apply the formulas for area and perimeter to solve real world problems.
- 3.ME.2h Use addition to find the perimeter of a rectangle.

Instructional Family: Scaling and Unit conversion

Core Content Connectors addressed:

- **H.ME.2b5** Apply the formula of geometric figures to solve design problems (e.g., designing an object or structure to satisfy physical restraints or minimize cost)
- **H.ME.2b1** Determine the dimensions of a figure after dilation
- 8.ME.1e1 Describe the changes in surface area, area, and volume when the figure is changed in some way (e.g., scale drawings)
- 7.ME.1d1 Solve problems that use proportional reasoning with ratios of length and area
- 7.PRF.1e1 Determine unit rates associated with ratios of lengths, areas, and other quantities measured in like units
- 5.ME.2a1 Solve problems involving conversions of standard measurement units when finding area, volume, time lapse, or mass.
- 5.ME.1b2 Convert standard measurement of length.

Grade Span: 9-10	**Content Area:** Mathematics – Measurement Investigating Measurement in the Real World
Lesson 1 of the Unit	**Approximate Time Needed:** 50 minutes

Lesson

1

Objective: Student will make decisions about units and scales that are appropriate for problem solving situations involving mathematics within mathematics or across disciplines or contexts.
Essential Question: What are the relationships among the measurements of dimensions, area, and perimeter in problem solving situations?

Lesson 1: Materials

Materials Needed:

Using grid paper or a Geoboard, provide students with rectangles having different dimensions, some of which are similar. Some of the rectangles should have the same perimeter but different areas.

For example:

- 9 x 12, P = 42; A = 108
- 8 x 13 P = 42; A = 104
- 6 x 15 P = 42; A = 90

Some should have the same area but different perimeters.

For example:

- 4 x 5; A = 20; P = 18
- 10 x 2; A = 20; P = 24

Make chart as used in practice note below to reinforce skills.

- Modify chart to include picture and/or representations of headings.
- Provide chart in electronic format for access with switch or alternate keyboard.

Materials needed:

- Grid paper
  - o Grid paper with raised lines
  - o Grid paper created on overhead transparencies for use with light boards
  - o Virtual grid paper
- Geoboards
  - o Virtual Geoboards

- Calculator
- Paper and pencils
- Ruler
- Yardstick
- Foldable ruler
- Conversion charts(for inches to feet, feet to yards, centimeters to meters)

See Resources: See Lesson 1 Resources for example exercises/images.

Lesson Vocabulary
Area
Centimeter
Foot
Inch
Length
Meter
Perimeter
Rectangles
Similar Rectangles
Width
Yards

Lesson 1: Introduction – 10 minutes

A. Activate Previous Knowledge

1. Lead a short discussion about how to find perimeter and area of rectangles.
   - Review the concepts of perimeter and area.
   - Discuss how these concepts are used in real life examples.
     - Example 1: A runner is practicing by running along the fence line of a parking lot.
       - Is he running the perimeter of the parking lot or is he running the area?
     - Example 2: The school is getting new carpet in the classroom.
       - Will the workers need to figure out the area of the classroom or the perimeter?
2. Break class into small groups to answer exercises.
3. Using figures (rectangles and squares) drawn on grid paper or formed on Geoboards, find the perimeters and areas.
4. Remind students that answers should/must include the appropriate units of measure.

Multiple means of representation: Use models and/or drawings during large group instruction. Allow students to have a copy of a drawing or a model at their desks.

Multiple means of expression: Provide a list of formulas to determine area and perimeter or provide options for using manipulatives and/or computer models.

Multiple means of engagement: Allow students to use paper/pencil, manipulatives, computer, etc. to complete exercises.

Additional Considerations for Emerging Readers and Emerging Communicators

1. Provide picture and/or tactile and/or object representations of relevant vocabulary paired with the written word as it is mentioned during presentation or discussion for rectangle, area, perimeter as well as the meanings of each word.
2. Create math journals to record vocabulary, formulas, and notes.
3. Provide the formulas for area and perimeter as the concepts of each are discussed.
4. During discussion, provide picture representation of real world uses for area and perimeter.
5. As students work in small groups or pairs, ensure they have a means for gaining their group members' or partner's attention and a means for contributing to the discussion.

6. Students may use their math journals or a graphic organizer to collect/store information gathered during group.

7. To find area and perimeter, use grid paper, count/mark/tally each unit along the length of the figure to determine length and count/mark/tally each unit along the width of the figure to determine the width.

8. Use the formulas to determine area and perimeter.
    - A list of formulas may be used by the student as a reference.

9. Student may be presented with manipulatives of a unit and the rectangle drawn on grid paper.
    - Students determine area and perimeter by placing the manipulative units on each unit around the rectangle on the grid paper to demonstrate perimeter as well as within the rectangle to demonstrate area.

    - Using manipulatives may be demonstrated electronically by using a computer program or PowerPoint to count units virtually to determine area and perimeter.
        - Each time the student hits the switch, the computer program counts each unit around the rectangle to determine perimeter.
        - To determine area, each time the student hits the switch, the program counts the units within the rectangle or for larger numbers, highlights a row or column of units and skip counts by 5s to determine the total number of units.

See Resources: See PowerPoint, Slides 1 and 2.

10. As answers are reviewed, be sure to reference the appropriate units of measure. For example, if students determine the perimeter of a 3inch by 4inch figure is 14, reply, "That is correct. It is 14 inches." If they determine the area is 12, reply, "That is correct. It is 12 inches square."
    - Remind students to record the appropriate unit.
    - Model how to write the appropriate units.
    - Present students with an alternative representation of unit to record in their math journals or graphic organizers.

Important Note for Communicators Considered Pre-Symbolic: Be sure students have a way to attain peer attention as well as to share and receive information. Limit measurements to one type: standard or metric unit.

B. Establish Goals/Objectives for the Lesson

Inform students that in this lesson they will make decisions about units and scales that are appropriate for problem solving situations within mathematics or across disciplines or contexts, and:

1. Convert units using standard/known conversion units.
2. Use appropriate known formulas for the area.
3. Solve multistep problems involving one unit of measure.

Multiple means of representation: Along with posting lesson objectives in the classroom, provide individual copies for students.

Multiple means of expression: Allow students to record lesson objectives in different formats: mathematics journals, computer, premade or original graphic organizers, etc.

Multiple means of engagement*:* Brainstorm ideas of how and when these skills might be relevant to "me."

Additional Considerations for Emerging Readers and Emerging Communicators
Inform students of expected outcomes. • Provide keys words paired with symbols/images/tactile representations. • Provide the key words in the lesson objectives paired with images or tactile representations, record into mathematics journals, or students may use an electronic picture writer paired with text-to-speech to record the lesson's objectives. • Provide photographs, models or tactile representations of examples of situations in which these concepts are used.

Lesson 1: Body – 15 minutes

Direct Instruction and/or Facilitation of the Lesson

1. Ask students:
   a. "For two rectangles of the same (similar) shape, how do their sizes compare?"
   b. "How do their perimeters compare?"
   c. "How do their areas compare?"

See Resources: Lesson 1, Pages 15 and 16.

2. Identify rectangles that have the same areas.
   a. Ask students: "Do they have the same perimeter?"

See Resources: Lesson 1, Page 17.

3. Lead a discussion on how to find perimeter and area of rectangles when units are different. (e.g., a rectangle measures 8 inches wide, 2 feet long or 80 cm wide, 1 meter long).

4. Review converting units:
   a. From inches to feet
   b. From feet to inches
   c. From feet to yards
   d. From yards to feet
   e. From centimeters to meters
   f. From meters to centimeters

See Resources: Lesson 1, Page 18.

Multiple means of representation: Use models and/or drawings during large group instruction; allow students to have a copy of a drawing or a model at their desks.

Multiple means of expression: Provide a list of formulas to determine area and perimeter or provide options for using manipulatives and/or computer models.

Multiple means of engagement*:* Allow students to use paper/pencil, manipulatives, computer, etc. to complete exercises. Present information within the context of students' interests such as pets, gardening, new bedroom floor plan, etc.

Additional Considerations for Emerging Readers

1. Provide picture representations of relevant vocabulary: similar rectangle, area, perimeter as well as the meanings of each word.

2. Provide students with grid paper with two similar rectangular figures printed on it or grid paper with manipulatives of the figures.

- Ensure students have a means for sharing how the rectangles are the same and/or different. Students use their math journals or a graphic organizer to record information about the attributes and measurements of each rectangle. Students use the same strategies as were used to determine area and perimeter in the introduction.
- Students share how the perimeters are the same or different using the information recorded in the journals or the graphic organizer.
- Students share how the areas are the same or different using the information recorded in the journals or the graphic organizer.

3. Demonstrate how ratios are used to compare the area of each figure and the perimeter of each figure.

4. Demonstrate how ratios are used to compare the area of each figure and the perimeter of each figure.

See Example: Lesson 1, Body 1, Similar Figures.

5. Provide students with two rectangles that have different measurements but the same area drawn on grid paper or as manipulatives of the figures.
   - Students verify that the area is the same and determine the perimeter of each figure.

See Example: Lesson 1, Body 2, Same Area.

6. Provide students with picture representations of relevant vocabulary for discussion: inches, feet, yard, centimeter, and meter as well as the meanings of each word.

7. When discussing different measurements within the same system (inches, feet, and yards, or centimeters and meters), present students with picture/object/tactile representations of the different measures.

8. To build understanding of the relationship between the different measurements, have students measure:
   - a book using inches marked on a ruler;
   - a table using feet while attending to how many inches that would be;
   - the length of the chalkboard/ whiteboard using a yardstick while attending to how many feet that would be.

9. Repeat measurements using centimeters and meters.

10. Emphasize the difference in size of object in relation to the size of unit used.

11. Students measure a given piece of paper that is 8 inches wide and 2 feet long.
    - Allow students to explore other units that can be used.
    - Discuss how the paper can be measured using the same unit of measure (i.e., Inches, 8 inches wide and 24 inches long).

12. Provide students with a calculator, the formulas, and task-analyzed steps for converting from one unit to another. Or provide students with a conversion chart with which to match the measured unit to a converted unit.
    - As a whole class or in small groups, work together to convert the units of measure for a rectangle that measures 8 inches wide and 2 feet long.
    - Students use the conversion formulas or create the figures on grid paper.
    - Students count the units on the grid paper to determine the conversion.

See Example: Lesson 1, Body 4, Converting Units.

Additional Considerations for Emerging Communicators

1. Provide picture and tactile representations of relevant vocabulary: similar rectangle, area, perimeter as well as the meanings of each word.
2. Using manipulatives or a computer program or PowerPoint that contains 2 similar figures (Figure A and Figure B) in a grid, students determine the area of Figures A and B by counting or using one-to-one correspondence to determine the number of units within the figure (area) and number of units around the figure (perimeter).
    - Students compare the areas by moving the smaller figure (Figure B) into the larger figure (Figure A) to see how many are needed to completely cover the larger figure.
    - This activity demonstrates how many times bigger Figure A is than Figure B.

See Example: PowerPoint Lesson 1, Slides 3 and 4.

3. Using a computer program or PowerPoint that contains 2 figures (Figure A and Figure B) with the same area in a grid, students use a switch to determine the area for each figure and verify that they are the same.
    a. Students determine if the perimeters are also the same.

See Example: PowerPoint Lesson 1, Slides 5 and 6.

4. Provide picture and tactile/object representations of relevant vocabulary: inches, feet, yard, centimeter, meter as well as the meanings of each word.

5. When discussing different measurements within the same system (inches, feet, and yards, or centimeters and meters), present students with tactile representations or objects of the different measures.
    - Students work with peer partners to measure a book using inches marked on a foldable ruler so students feel the inches.
    - Students measure a table using feet while attending to how many inches that would be. Again use the foldable ruler.
    - To build understanding of the relationship between the different measurements, students measure the length of the chalkboard/whiteboard using a yardstick while attending to how many feet that would be.

- Emphasize the difference in the size of an object in relation to the size of the unit used.
- Students measure a given piece of paper that is 8 inches wide and 2 feet long.
- To emphasize the inches within the foot, use a foldable ruler to show students that the piece of paper can also be measured as 8 inches wide and 24 inches long.

6. As a whole class or in small groups, work together to convert the units of measure for a rectangle that measures 8 inches wide and 2 feet long.

   a. Provide students with the figure represented on grid paper.
   - Tactilely represent the figure with the original units of inches across and feet down as well as with a rectangle with all measurements converted to inches.
   - Represent the different units of measure (1foot and 12 inches) tactilely with different thicknesses, so when students are tactilely scanning the inches, they feel the difference when a foot has been reached.
   - Students use a computer program with an alternate keyboard or multi-switch access with each tactile representation on the keyboard or one per switch.
   - Students locate the original figure and press it so it is represented on the computer screen.
   - The computer program states the original measurements (8 inches by 2 feet), and students locate and press the tactile representation that has been converted to inches.
   - The figure is displayed on the screen, and the computer states the converted measurements (8 inches by 24 inches).

See Example: PowerPoint Lesson 1, Slide 7.

Important Note for Communicators Considered Pre-Symbolic: Work with only one system: standard or metric units.

Lesson 1: Practice – 20 minutes

1. In small groups, students work on a variety of problems using different given dimensions such as:
    - Alex has 140 feet of fencing to place around a rectangular garden he is making. He wants the area of the garden to be as large as possible. What should the length and width of the garden be?
    - Give each student the chart below.
    - Students may use models to explore the various possibilities and complete the chart.

Rectangle	Length	Width	Perimeter	Area
A	40 ft	30 ft	140 ft	1200 ft^2

See Resources: Pages 20 and 21.

2. Bring the whole group back together.
3. Ask one student from each group to discuss the results.

4. After reviewing students' results, help students realize that when a perimeter is 140 units, length plus width always equals 70 units, but length times width varies.
5. Make a chart with students of the possible lengths and widths of a rectangle in increments of 5 when the perimeter is 140 units. Use information from their charts and organize it in order from smallest length to largest length.

	L + W	L x W
70 units =	5 units + 65 units	5 units x 65 units = 325 $units^2$
70 units =	10 units + 60 units	10 units x 60 units = 600 $units^2$
70 units =	20 units + 50 units	20 units x 50 units = 1000 $units^2$
70 units =	35 units + 35 units	35 units x 35 units = 1225 $units^2$
70 units =	50 units + 20 units	50 units x 20 units = 1000 $units^2$

6. Demonstrate to students that as the dimensions change, the area gets larger, reaching the highest value, and then gets smaller. Students should conclude that when asked to find the largest area when given a perimeter, a square would always have the greatest area.

Multiple means of representation: Provide students with a copy of the word problem and the table. Have drawings and manipulatives available for students to use.

Multiple means of expression: Allow students to solve the problem using formulas and/or models.

Multiple means of engagement*:* Ensure each student is actively involved in the small groups. Present different problems related to students' interests. Use questioning to encourage students to explain their strategies in their groups.

Additional Considerations for Emerging Readers

1. Provide picture representations of the word problem to students as it is being read.
   - Present word problem electronically using text-to-speech.
   - Label each column of the table using picture symbols and include measurements for length and width needed for a perimeter of 140 ft.
   - Remind students how to determine perimeter and area.
     - Provide formulas and task-analyzed steps for using the formulas:
       - Perimeter = L+L+W+W or 2L + 2W
       - Area = L x W
       - Perimeter = 140ft: 2L+2W = 140ft or L+W = 70ft
     - Provide students with models of the various fenced-in areas (rectangles).
       - Students use the given formula and calculators to determine area.
     - Students use the grid paper to identify the rectangle that provides the most area.

2. For steps 3-6, students discuss results by presenting the length and width of the rectangle with the largest area that they discovered in their group, or students point to the actual picture representation of the rectangle with the largest area.

Additional Considerations for Emerging Communicators

1. Modify the word problem to include smaller whole numbers.
   - For example, Alex has 16 yards of fencing to place around a rectangular garden he is making. He wants the area of the garden to be as large as possible. What should the length and width of the garden be?
   - Provide students with picture and/or tactile representations of the important aspects of the problem as it is being read.
   - As information is provided, students organize the information in a graphic organizer, baskets, etc.

2. Provide students with a computer program or premade PowerPoint as used in previous portions of the lesson to explore rectangles with the same perimeter but different areas.

3. Using the computer program or PowerPoint with switch access, students verify that the perimeter is 16 yd. in the modified word problem and count the area units to determine the area.

4. After students have verified the perimeter or determined the area for a particular rectangle,

the program should insert the number of units into the correct portion of the chart. Remember, the alternate keyboard or switches used should have picture or tactile representations of area and perimeter so students determine the correct measurement.

See Example: PowerPoint Lesson 1, Slide 8.

5. Students indicate the rectangle with the largest area by choosing rectangle A or B.

Lesson 1: Closure – 5 minutes

A. Review Lesson and Objectives

Remind students that they were to make decisions about units and scales that are appropriate for problem solving situations within mathematics or across disciplines or contexts, and:

1. Convert units using standard/known conversion units.
2. Use appropriate known formulas for the area.
3. Solve multistep problems involving one unit of measure.

Multiple means of representation: Along with posting lesson objectives in the classroom, students may refer to their individual copies.

Multiple means of expression: Students share what they have learned in different formats: writing, drawing, creative expression, etc.

Multiple means of engagement*:* Brainstorm ideas of how and when these skills might be relevant to "me."

Additional Considerations for Emerging Readers and Emerging Communicators

1. When reviewing the expected outcomes, have students refer to the lesson objectives they recorded in their mathematics journals or their electronic picture versions.

2. Students use the information recorded in their journals to refer back to the lesson objectives key words paired with images. From that information, they share what they have learned based on each of the expectations.
   - For example, the student may grab the tactile cue for area to state, "I have learned that the area is all the space measured within a figure."

3. Students refer back to the photographs or models/tactile representations of examples of real-life situations in which these concepts are used to share when they could use these new skills.
   - For example, the student could touch the tactile representations for area and table top to state, "I can use area to determine if an object will fit on my desk."

B. Exit Assessment

1. Students are given a new word problem to solve that includes mixed measurements within the same system.
2. Students work independently to find perimeter and area of rectangles and solve for the situation.
3. Students should use a similar table as that used during practice.

New Problem Example: Josh is designing a display for his Science Fair project. His display must have a perimeter of 120 inches or 10 feet. He found two display boards: board one measures 24 in x 3 ft and board two measures 30 in x 30 in.

- Which display meets the criteria?
- Is it display 1?
- Is it display 2?
- Both?
- None?
- Which display gives Josh the largest display area?

Multiple means of representation: Provide students with a copy of the word problem and the table. Have drawings and manipulatives available for students to use.

Multiple means of expression: Allow students to solve the problem using formulas and/or models.

Multiple means of engagement: Ensure students are actively involved in their small groups. Present different problems related to students' interests. Use questioning to encourage students to explain their strategies.

Additional Considerations for Emerging Readers and Emerging Communicators

1. Use the same supports as used in the practice section to solve for the given problem.
2. Use the supports described in Lesson Body, Step 4: Converting Units of Measure.

Lesson 1: Resources

The following pages are examples of activities and exercises from Lesson Body, page 8.

Area and Perimeter of Similar Figures:

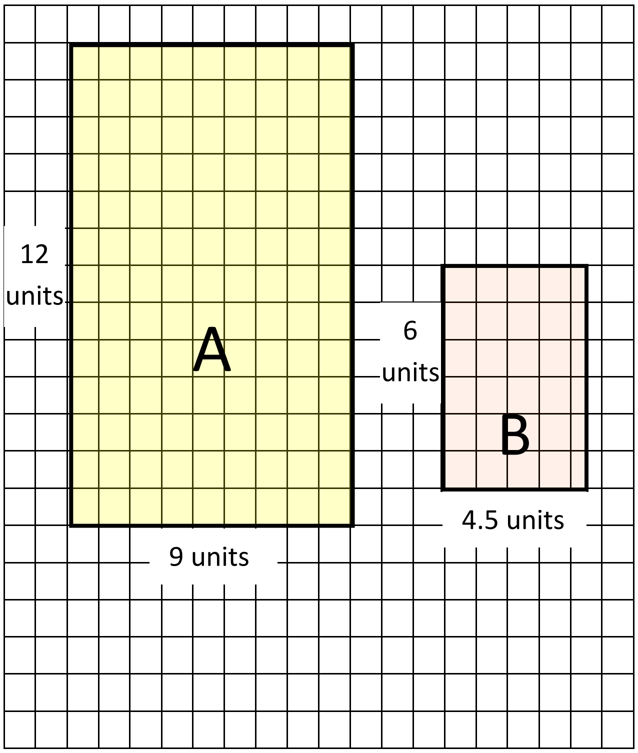


Figure A
A = 12 units x 9 units
A = 108 units^2

P = 12 units +12 units + 9 units + 9 units
P = 42 units

Figure B
A = 6 units x 4.5 units
A = 27 units^2

P = 6 units + 6 units + 4.5 units + 4.5 units
P = 21 units

Compare Area of figures A and B

$$\frac{AA}{AB} = \frac{108}{27}$$

$$\frac{AA}{AB} = \frac{4}{1}$$

Compare Perimeter of figures A and B

$$\frac{PA}{PB} = \frac{42}{21}$$

$$\frac{PA}{PB} = \frac{2}{1}$$

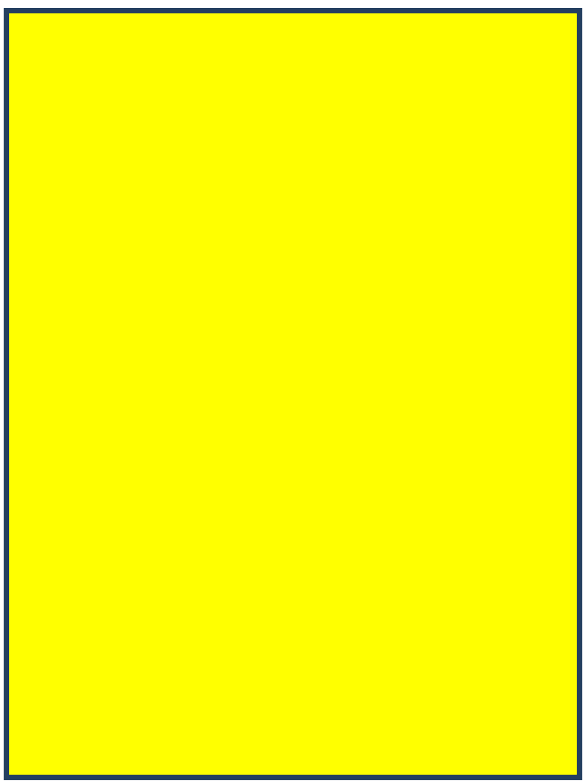
Lesson 1: Resources – continued

The student can use manipulatives (next 2 images) to compare the two figures by laying rectangle B over rectangle A until completely covered to determine how many times bigger area of A is than B. The student can compare perimeter by laying rectangle B over rectangle A to determine how many are needed to create the same length (2), and how many are needed to create the same width (2).

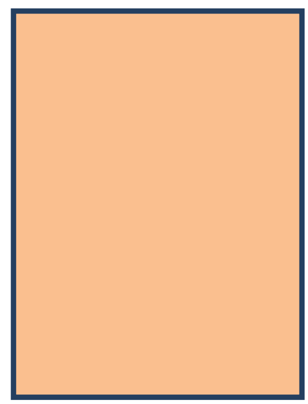
The area of A is __4__times the area of B.   The perimeter of A is __2__times the perimeter of B.

Tactile representations of similar figures: Cut out figures using construction paper, poster board, card board, sand paper, etc. Representations can also be cut out as templates or frames to lay over grid paper or cut out of transparencies to see grid lines.

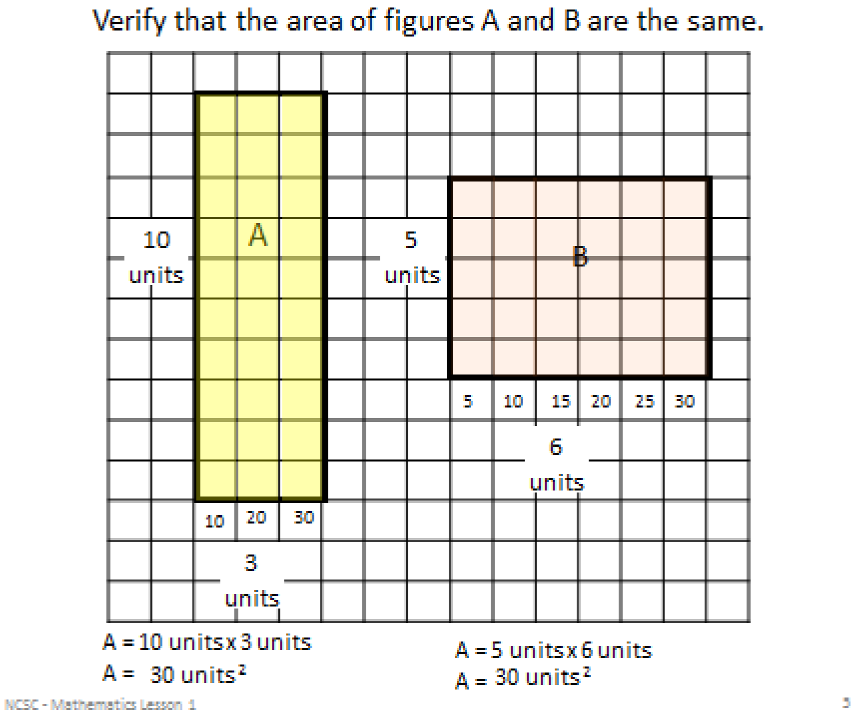
Rectangle A



Rectangle B



Lesson 1: Resources – continued



Is the area of Figure A the same as the area of Figure B?

Figure A:	Figure B:
A= 10 units x 3 units	A= 6 units x 5 units
A= 30 units (squared)	A= 30 units (squared)

Is the perimeter of Figure A the same as the perimeter of Figure B?

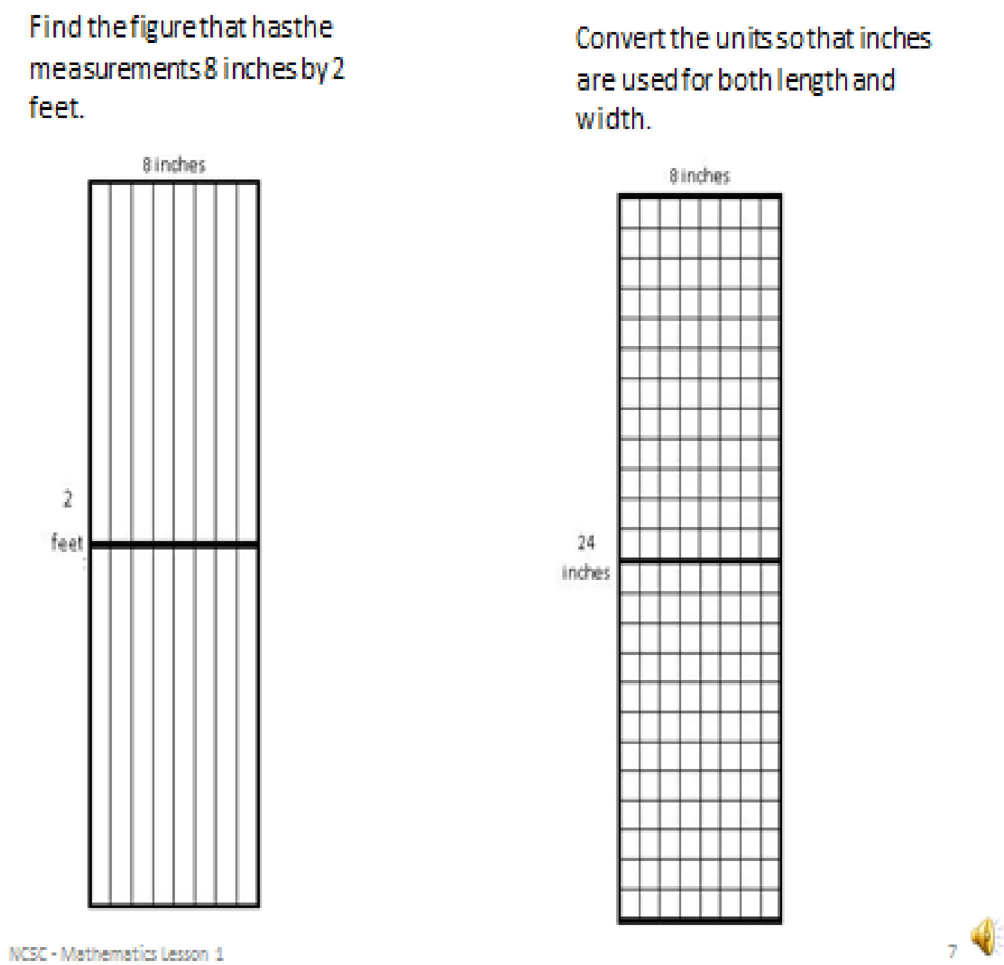
Figure A:	Figure B:
P= 10 units + 10 units + 3 units + 3 units	P= 6 units + 6 units + 5 units + 5 units
P= 26 units	P= 22 units

The perimeter of Figure A and Figure B is:

The same Different

The perimeter of Figure A is <u>more than / less than / the same as</u> the perimeter of Figure B.

Lesson 1: Resources – continued

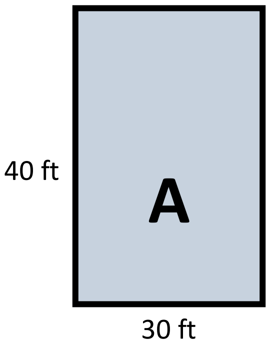


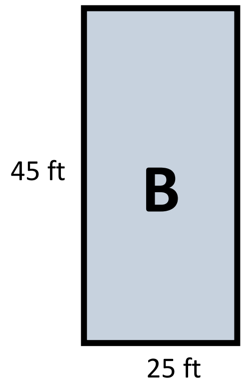
These figures can be represented tactilely with raised, thick, exterior and center lines to represent the height measured in feet. They can also be represented with thinner, raised lines to represent width and height in inches.

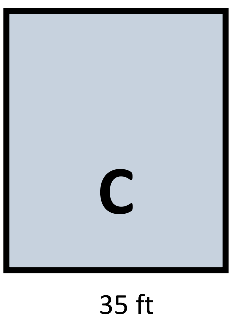
Lesson 1: Resources – continued

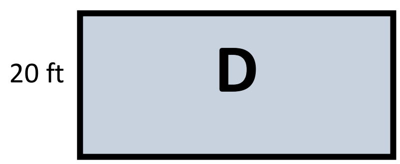
The following pages are examples of activities /exercises from Lesson Practice, page 11.

Model	Length	Width	Perimeter	Area
A	40 ft	30 ft	140 ft	1200 ft^2
B	45 ft	25 ft	140 ft	
C	35	35	140 ft	
D	20	50	140 ft	

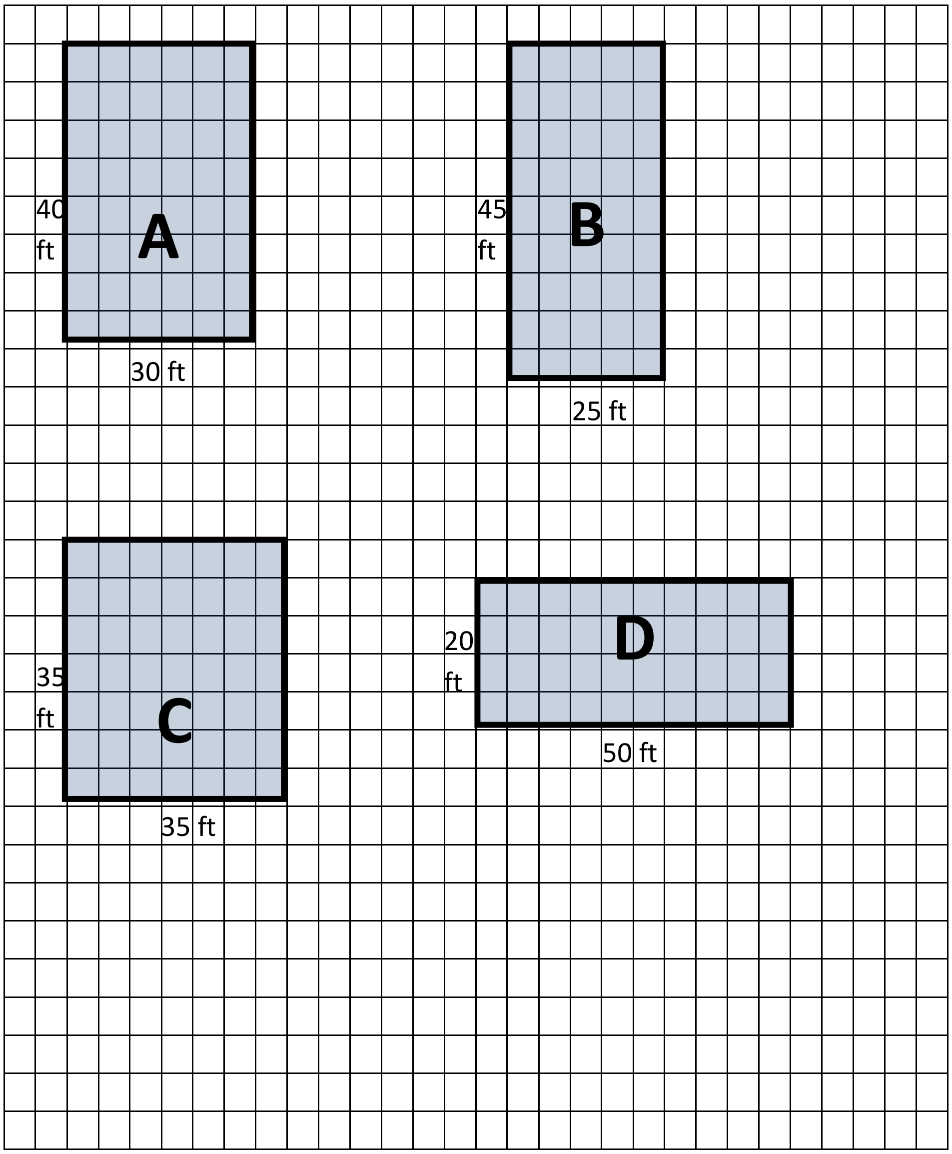








Lesson 1: Resources– continued



Lesson

2

Grade Span: 9 - 10	**Content Area:** Mathematics - Measurement
Lesson 2 of the Unit	**Approximate Time Needed:** 50 minutes

Objective: Students will make decisions about units and scales that are appropriate for problem solving situations involving mathematics or across disciplines or contexts.

Essential Question(s):

1. How does the area of a figure change when one of its dimensions is doubled, tripled, etc.?
2. How can we use ratios and proportions to determine how the area of figures change when at least one of the dimensions change?

Materials Set Up:

- Provide practice and review worksheets or class discussion about ratio and proportion.
- Activities will be varied and should include individual and group worksheets, text exercises, and hands-on problem solving activities.

Materials Needed:

- Worksheets
- Grid paper
    - Grid paper with raised lines
    - Grid paper copied on transparencies for light boxes
- Square tiles
- Geoboard
- Pencils
- Overhead projector
- Transparencies
- 12-inch ruler
- Yardstick
- Poster board
- Chalk board, white board or Smart Board

Lesson Vocabulary

Area
Dimensions
Length
Perimeter
Proportion

Ratio
Width

Lesson 2: Introduction – 10 minutes

A. Activate Previous Knowledge

1. Lead discussion on meaning of ratio and proportion.
2. Provide practice worksheets on finding ratios and proportions.

Multiple means of representation: Provide definitions of ratio and proportion. Provide drawings or models of ratios and proportions. Worksheets can be presented with fewer items per page. Worksheets may be printed on various color sheets. Worksheets may be presented using a computer.

Multiple means of expression: Students may express the meaning of ratio and proportion by describing a situation that represents a ratio or proportion. Students may create a model by drawing or using manipulatives to express the meaning of ratio and proportion. Students may use drawings or models to complete worksheets. Students may complete work on a computer.

Multiple means of engagement: Provide examples of ratios and proportions based on students' interests or real life situations in which they would be used.

Additional Considerations for Emerging Readers and Emerging Communicators

1. Provide picture/tactile/object representation of vocabulary words as they are used throughout the discussion.
    - Provide concrete examples when discussing the meaning.
    - During the discussion, students match a picture representation to the example provided which students use as a reference throughout the lesson.
    - Examples can be provided by using a computer program and adaptive software with a talking word processor.

2. Include picture/tactile/object representations of key words in directions and word problems.
    - Provide graphic or manipulative representations of proportional relationships or use a computer program with alternate keyboard or switch access and talking word processor of proportional relationships.

Lesson 2: Introduction – continued

B. Establish Goals/Objectives for the Lesson

Inform students that in this lesson the will make decisions about units and scales that are appropriate for problem solving situations within mathematics or across disciplines or contexts.

Explain that they will:

1. Set up and solve proportions.
2. Convert units of measurement using standard/known conversions.
3. Recognize when to multiply and when to divide in converting measurements.
4. Use ratio and proportion to convert measurements.
5. Use appropriate known formulas for area.

Multiple means of representation: Along with posting lesson objectives in the classroom, provide individual copies for students.

Multiple means of expression: Allow students to record lesson objectives in different formats: mathematics journals, computer, premade or original graphic organizers, etc.

Multiple means of engagement: Brainstorm ideas of how and when these skills might be relevant to "me."

Additional Considerations for Emerging Readers and Emerging Communicators

1. Provide students with keys words paired with symbols/images/tactile representations.

2. Provide the key words in the lesson objectives paired with images or tactile representations, record into mathematics journals. Students may use an electronic picture writer to record the lesson objectives.

3. Provide students with visual and/or tactile representations of situations when these concepts are used.

Lesson 2: Body – 20 minutes

Direct Instruction and/or Facilitation of the Lesson

1. Students draw rectangles of various dimensions on grid paper or form on Geoboards or with square tiles and to determine the area of each rectangle. Discuss why the area of the rectangles are the same or different.

2. Students draw or form a rectangle A with given dimensions (e.g., 10”x5”). Then have students draw or form rectangle B by doubling the length of first one side and both sides of rectangle A. Find the ratio of the area of rectangle A to the area of rectangle B.

Multiple means of representation: Use models and/or drawings during large group instruction. Allow students to have a copy of a drawing or a model at their desks.

Multiple means of expression: Provide a list of formulas to determine area and perimeter, or provide options for using manipulatives and/or computer models.

Multiple means of engagement: Allow students to use paper/pencil, manipulatives, computer, etc. to complete exercises.

Additional Considerations for Emerging Readers and Emerging Communicators

1. Students should determine the area of various given rectangles using the same strategies and supports as used in lesson 1.

2. Draw a model of rectangle A with given dimensions (e.g., 5” x 4”) or provide a manipulative model or virtual template of rectangle A. Students place the model on grid paper and verify the length and width by counting the units along the length of the figure and the width of the figure. Students determine the area by counting the units within the figure. Students should keep a record of the length, width, and area of rectangle A.

Rectangle	Length	Width	Area
A	5 in	4 in	20 in^2
B			
C			
D			

3. Once students have determined the area of rectangle A, provide the rule and/or model for setting up a ratio of area to width.
   - The letters representing length, width, and area can be paired with a picture representation of each measurement.

See Example: PowerPoint Lesson 2, Slide 1.

4. The ratio of area to width for rectangle A is $\frac{A}{W}$.

5. Provide a template of rectangle B with given dimensions as the width twice as wide as rectangle A. Students should keep a record of the length and width of rectangle B (e.g., 5” x 8”).

6. Once students have recorded the length and width of rectangle B, provide the rule and/or model for setting up the ratios of rectangles A and B to determine the area of B.
   - The letters representing length, width, and area can be paired with a picture representation of each measurement.

$\boxed{A}$ $\boxed{B}$

$$\frac{A}{W} = \frac{A}{W}$$

See Example: PowerPoint Lesson 2, Slide 2.

7. Using the information from the chart, input the numbers into the proportions using the proportions for area and width.

$\boxed{A}$ $\boxed{B}$

$$\frac{20in^2}{4in} = \frac{A}{8in}$$

8. Using the templates, students demonstrate that it takes 2 of rectangle B to make the same width of rectangle A.
   - Students should conclude that the width of rectangle B is 2 times the width of rectangle A.
   - Allow students to use a calculator to verify that multiplying the width of A by 2 will give students the width of B (4 x 2=8)

$\boxed{A}$ $\boxed{B}$

$$\frac{20in^2}{4in\ \boldsymbol{x2}} = \frac{A}{8in}$$

9. Explain: If you multiply the bottom number by 2, you must multiply the top number by 2.
   - Allow students to use a calculator to determine the area of rectangle B.

$\boxed{A}$ $\boxed{B}$

$$\frac{20in^2\ \boldsymbol{x2}}{4in\ x2} = \frac{A}{8in}$$

$$\frac{20in^2\ \boldsymbol{x2}}{4in\ x2} = \frac{40in^2}{8in}$$

10. Students place the template on grid paper to determine the area by counting the units within the figure.

See Example: PowerPoint Lesson 2, Slide 3.

11. Repeat by providing students with a template for rectangle C (10” x 4”), the length of which is double that of rectangle A.
    - Use the proportional formulas for area and length.

Lesson 2: Practice – 10 minutes

Repeat the exercise by tripling the dimensions and/or by reducing the dimensions by one-half.

Multiple means of representation: Use models and/or drawings during large group instruction. Allow students to have a copy of a drawing or a model at their desks.

Multiple means of expression: Provide a list of formulas to determine area and perimeter, or provide options for using manipulatives and/or computer models.

Multiple means of engagement: Allow students to use paper/pencil, manipulatives, computer, etc., to complete exercises.

Additional Considerations for Emerging Readers and Emerging Communicators

- Use the same supports as used throughout the lesson body.

Lesson 2: Closure – 10 minutes

A. Revisit/Review Lesson and Objectives

Remind students that they were to make decisions about units and scales that are appropriate for problem solving situations within mathematics or across disciplines or contexts, and:

1. Set up and solve proportions.
2. Convert units of measurement using standard/known conversions.
3. Recognize when to multiply and when to divide in converting measurements.
4. Use ratio and proportion to convert measurements.
5. Use appropriate known formulas for area.

Multiple means of representation: Along with posted lesson objectives in the classroom, students may refer to their individual copies.

Multiple means of expression: Students share what they have learned in different formats: through writing, drawing, creative expression, etc.

Multiple means of engagement: Brainstorm ideas of how and when these skills might be relevant to "me."

Additional Considerations for Emerging Readers and Emerging Communicators

1. When reviewing the expected outcomes, have students refer to the lesson objectives they recorded in their mathematics journals or in their electronic picture versions.

2. Students use the information recorded in their journals to refer back to the lesson objectives' key words paired with images or tactile/objects representations so they can share what they have learned based on each of the expectations.
   - For example, a student may touch the tactile cues for ratio and area to state, "I have learned how to compare areas of similar figures by using ratios."

3. Students refer back to the photographs or tactile representations of examples of real-life situations in which these concepts are used to share when they could use these new skills.

B. Exit Assessment

1. Generalize the effect on the area of the rectangle when one and/or both dimensions are multiplied by a factor of "n."

2. Students review their work from the lesson and conclude that when only one dimension is changed, the area is affected by the same amount of change, but if both dimensions are changed, the area is affected by the change of width times the change of length.

Multiple means of representation: Allow students to refer back to their work samples, models, drawing, notes, etc.

Multiple means of expression: Students share what they have learned in different formats: writing, drawing, creative expression, etc.

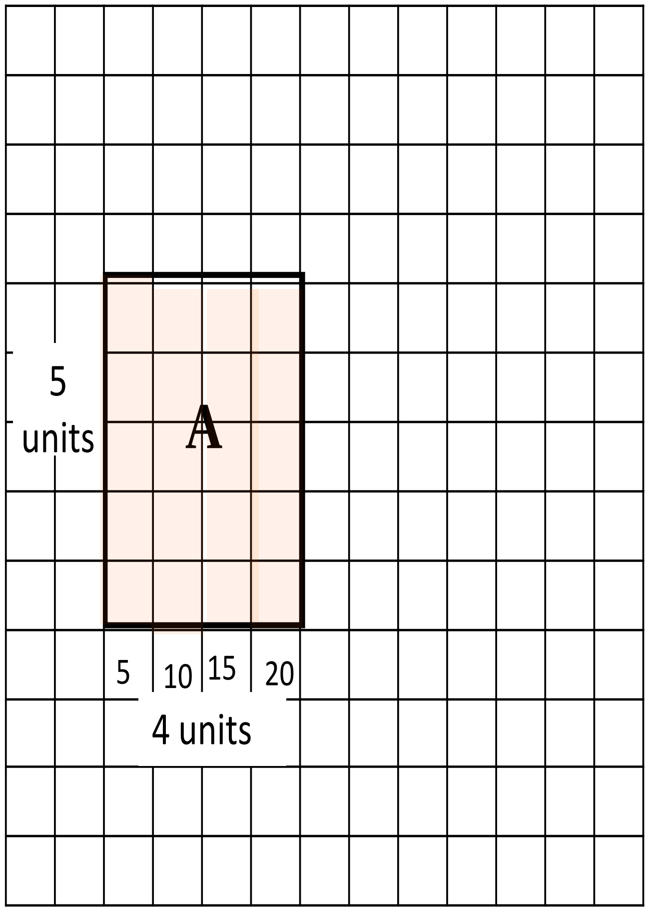
Multiple means of engagement: Allow students to review work independently or to review with a partner or small group.

Additional Considerations for Emerging Readers and Emerging Communicators

1. Students show the effect on the area of the rectangle when one and/or both dimensions are multiplied by a factor of "n" by drawing models or using manipulatives to demonstrate how many of one figure is needed to cover the other.

2. Use ratios and proportions to show the relationship between the areas of the original rectangle and the new rectangle that was created by changing dimensions by a factor of "n."

Lesson 2: Resources



What is the ratio of Area to Width of rectangle A?

$$\frac{A}{W} = \frac{20\ u^2}{4\ u}$$

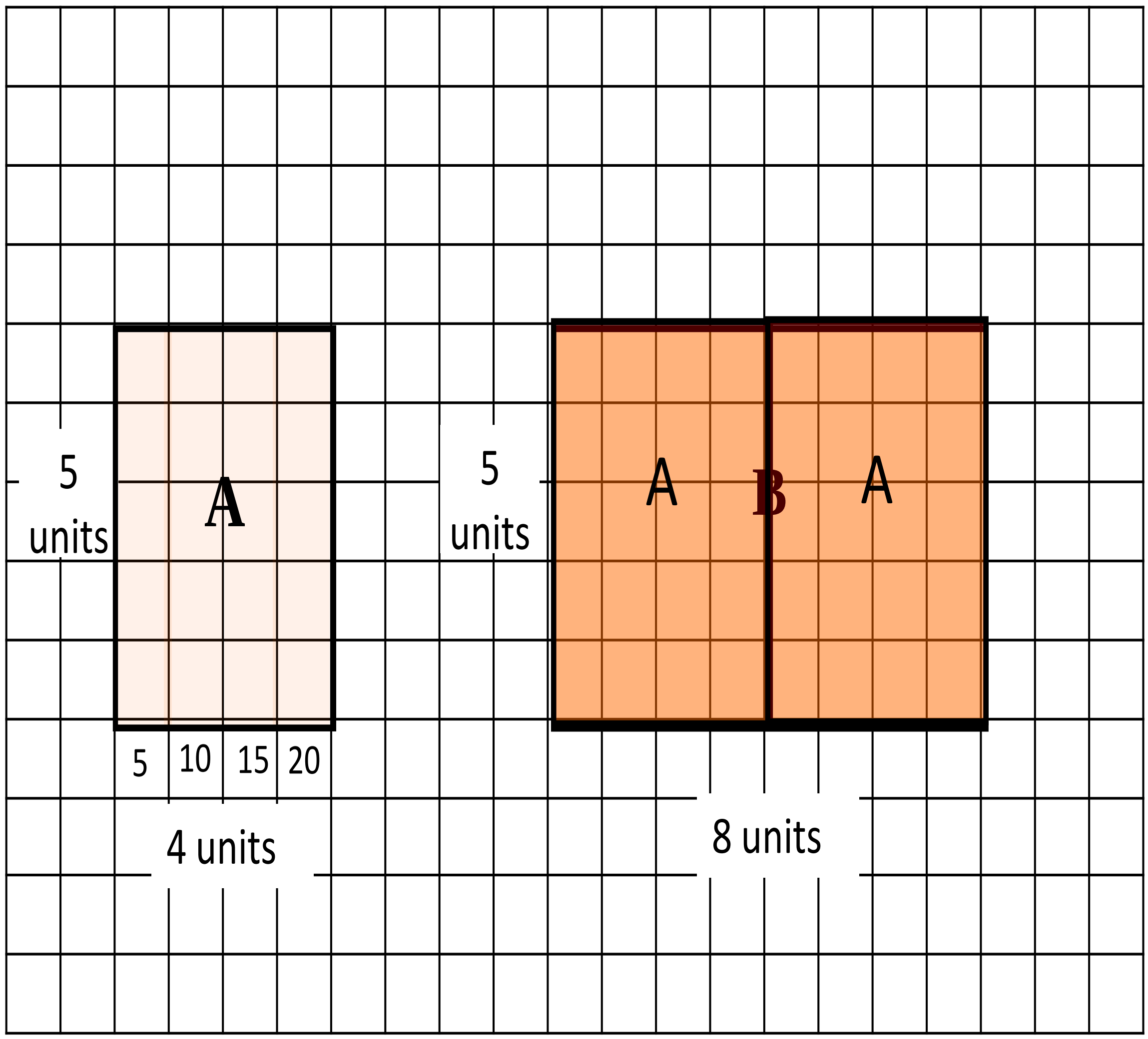
Determine the area of rectangle A.

$\boxed{A}$ $5u \times 4u = \quad 20\ u^2$

NCSC – Mathematics Lesson 2

1

Lesson 2: Resources– continued



What happens to the area of rectangle A if we double the width?

If the ratio of Area to Width of rectangle A is $\frac{A}{W} = \frac{20\ u^2}{4\ u}$

Use the ratio $\boxed{A}\ \frac{A}{W} = \boxed{B}\ \frac{A}{W}$ to figure the Area of rectangle B?

NCSC – Mathematics Lesson 2 2

Lesson 3

Grade Span: 9 - 10	**Content Area:** Mathematics – Measurement Investigating Measurement in the Real World
Lesson 3 of the Unit	**Approximate Time Needed:** 90 minutes

Objective: Students will make decisions about units and scales that are appropriate for problem solving situations within mathematics or across disciplines or contexts.

Essential Question: How can we use proportion to convert measurements from one unit to another in the same system?

Materials Set Up:

- Provide practice and review worksheets or class discussion about ratio and proportion.
- Activities will be varied and should include individual and group worksheets, text exercises, and hands-on problem solving activities.

Materials Needed:

- Worksheets
- Grid paper
- Grid paper with raised lines
- Grid paper copied on transparencies for light boxes
- Square tiles
- Geoboard
- Pencils
- Overhead projector
- Transparencies
- 12-inch ruler
- Yardstick
- Poster board
- Chalk board, white board, or Smart Board

Lesson Vocabulary:
Area
Centimeter
Conversion
Foot

Inch
Length
Meter
Proportion
Ratio
Width
Yards

Lesson 3: Introduction – 15 minutes

A. Activate Previous Knowledge

1. Lead a discussion on the meaning of ratio and proportion.

2. Provide a practice worksheet(s) on finding ratios and proportions.

Multiple means of representation: Present illustrations or models of ratio and proportions during discussions.

Multiple means of expression: Allow students to use paper and pencil, models, computers, etc. to practice the concepts of ratios and proportions.

Multiple means of engagement: Present real life uses for ratios and proportions related to students' interests.

Additional Considerations for Emerging Readers and Emerging Communicators

1. Provide picture and/or tactile representations of ratio and proportion as well as concrete examples throughout the discussion.
    - Examples can be provided using a computer program and adaptive software with a talking word processor.
    - Refer to ratios and examples used in Lesson 2.

2. Include picture representations of key words in directions and word problems.
    - Provide graphic or manipulative representations of proportional relationships or use a computer program with a switch or alternate keyboard access and talking word processor.

B. Establish Goals/Objectives for the Lesson

Inform students that they will make decisions about units and scales that are appropriate for problem solving situations within mathematics or across disciplines or contexts and:

1. Set up and solve proportions.
2. Convert units of measurement using standard/known conversions.
3. Recognize when to multiply and when to divide in converting measurements.
4. Use ratio and proportion to convert measurements.
5. Use appropriate known formulas for area.
6. Solve problems requiring calculations that involve different units of measure within a

measurement system.

Multiple means of representation: Along with posting lesson objectives in the classroom, provide individual copies for students.

Multiple means of expression: Allow students to record lesson objectives in different formats: mathematics journals, computer, premade or original graphic organizers, etc.

Multiple means of engagement: Brainstorm ideas of how and when these skills might be relevant to “me.”

Additional Considerations for Emerging Readers and Emerging Communicators

1. Provide students with keys words paired with symbols/images/tactile representations.
2. Provide the key words in the lesson objectives paired with images/symbols/tactile representations, record into mathematics journals or use an electronic picture writer to record the lesson objectives.
3. Provide students with photographs/objects/tactile representations of examples of situations when these concepts are used.

Lesson 3: Body – 30 minutes

Direct Instruction and/or Facilitation of the Lesson

1. Lead the discussion on the types of customary units used to measure length: inches, feet, and yards.

2. List the common conversions: 12 in = 1 ft; 3 ft = 1 yd; 36 in = 1 yd.

3. Show that when converting a larger unit to a smaller unit, we multiply (e.g., 4 ft = 4 x 12 = 48 in) or set up and solve a proportion:
   a. $\frac{1\ ft}{12\ in} = \frac{4\ ft}{x\ in}$
   b. $\frac{1\ yd}{36\ in} = \frac{2\ yd}{x\ in}$
   c. $\frac{1\ yd}{3\ ft} = \frac{2\ yd}{x\ ft}$

4. Show that when converting a smaller unit to a larger unit, we divide: (e.g., 6 in = 6/12 =1/2 ft):
   a. Convert feet to yards: Number of feet ÷ 3= # of yds
      e.g., 2 ft ÷ 3 = 2/3 yds
   b. Convert inches to yards: Number of inches ÷ 36= # of yds
      20 in ÷ 36 = 20/36 or 5/9 yd
   c. Use ratio and proportion:
      $\frac{1\ ft}{12\ in} = \frac{x\ ft}{6\ in}$

      $\frac{3\ ft}{1\ yd} = \frac{2\ ft}{x\ yd}$

5. Convert units of measure for area:
   a. Show using grid paper, tiles, or Geoboard that a square measuring 12 in on a side is the same size as a square measuring 1 ft on a side.
   b. The area of a 12-in by 12-in square = 144 sq. in. or in^2
   c. The area of a 1 ft square = 1 sq ft or ft^2
   d. Therefore, 144 sq in = 1 sq ft or ft^2
   e. In like manner, show 9 sq ft = 1 sq yd or yd^2

6. Discuss how measurements and area can be used in real world situations.
   a. For example, area of floor for tile or carpet, length and width of pictures for frames, area of table tops for tile or tablecloth, length and width of windows for curtains, etc.
   b. Provide students an opportunity to communicate ideas with the class.

Multiple means of representation: Use models and/or drawings on grid paper during large group instruction. Allow students to have a copy of a drawing or a model at their desks. Provide examples of measuring tools. Provide a list of conversion formulas to convert between measurements within the same system.

Multiple means of expression: Allow students to use paper/pencil, manipulatives, computer, etc., to complete exercises.

Multiple means of engagement: Allow students to brainstorm ideas by writing descriptions of examples, drawing examples, acting out examples, etc.

Additional Considerations for Emerging Readers

1. Provide picture representations of inch, foot, and yard as well as concrete examples throughout discussion.

2. Remind students that when the measurements increase, use multiplication.
    - Remind students how to make sure the ratio remains balanced: Whatever is multiplied on the top must be multiplied on the bottom.
    - Refer back to supports used in the introduction.
    - $\frac{1\,ft}{12\,in}$ $\boxed{\text{x 4}}$ $= \frac{4\,ft}{x\,in}$

3. Remind students that when the measurements decrease, use division.
    - Remind students how to make sure the ratio remains balanced: Whatever is divided on the top must be divided on the bottom.
    - Refer back to supports used in the introduction.

    $\frac{1\,ft}{12\,in}$ $\boxed{\div 2}$ $= \frac{x\,ft}{6\,in}$

4. Students measure and draw a square that is 12 inches by 12 inches.
    - Using the formula for area, students determine the area in square inches.
    - Students measure the square again using the measurement of foot.
    - Students should see that it is 1ft x 1ft and equals 1 ft^2.
    - Students should conclude that $144in^2 = 1ft^2$.

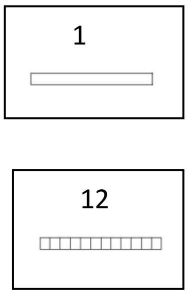
5. Students measure and create a square that is 3 ft x 3ft by taping it out on the floor and determine the area.
    - Students measure the same square using a yardstick.
    - Students should determine the square is also 1yd x 1 yd and equals 1 yd^2.
    - Therefore, $9ft^2$ and $1yd^2$ are equal.

See Example: Lesson 3 Conversions.

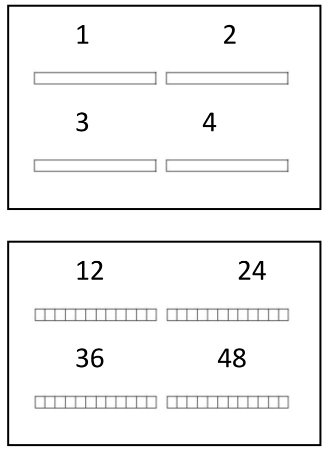
Additional Considerations for Emerging Communicators

1. Provide picture and tactile/object representations of inch, foot, and yard as well as concrete examples of measurement tools for the different units throughout discussion.

2. Provide students with concrete examples of the ratios when increasing measurements.



$$\frac{1\ ft}{12\ in} = \frac{4\ ft}{x\ in}$$



3. Provide students with concrete examples of the ratios when converting to smaller units.

$$\frac{1\ ft}{12\ in} = \frac{x\ ft}{6\ in}$$

See Resources: Lesson 3 tools to use, pages 47- 49.

Lesson 3: Practice – 30 minutes

1. Model a problem with the class that involves making decisions about units and scales, and determine various ways to solve it.
    a. Problem 1: A floor is 9 ft wide and 12 ft long. How many tiles (12" on a side) are needed to completely cover the floor?
    b. Draw a rectangle to represent dimensions 9 ft by 12 ft or make a scale drawing of it.

2. Model a second problem for students.
    a. Problem 2: A floor is 9 ft wide and 12 ft long. How many tiles (18" on a side) are needed to completely cover the floor?
    b. Students recommend whether to convert the floor plan to inches or the tiles to feet.
    c. If students recommend converting 9 ft and 12 ft to inches, then find the area of the rectangle in square inches (108in x 144in) and divide by the area of a tile (12in x 12in).

For example:

- 9 x12in = 108 in 12 x 12in = 144 inches
- Area of the floor in inches: 108 in x 144 in = 15,522 in^2
- Area of the tile in inches: 18 in x 18 in = 324 in^2
- 1552 $in^2 \div 324\ in^2 = 48\ in^2$, so 48 tiles are needed

Multiple means of representation: Allow students to have a written copy of the problem, drawn models of the situation, and/or conversion formulas as needed/requested.

Multiple means of expression: Students may draw or use manipulatives to model solutions or use the computer.

Multiple means of engagement: Create situations that include areas of interest to students.

Additional Considerations for Emerging Readers

1. Provide the written problem to include picture representations of relevant words so students can follow along as the problem is introduced.
    - Provide students a scale drawing of the rectangle (floor) on grid paper measuring 9 units by 12 units with each unit representing a foot.
        - Label the rectangle "floor" using word and picture representation.
        - Each square in the grid represents 1 inch in length and 1 inch in width.
        - Highlight around 12 in x 12 in to represent 1 ft x 1 ft.
        - Provide students with paper or object squares that equal the size of a square foot on the graph paper.

- Use these to represent the 12” tiles.
- Allow students to use the conversion chart from previous lessons to determine how to best convert the units.
- Convert the floor to inches or convert the tiles to feet.

- Once students have made the conversion, determine the area of the tile using the equation length x width.
  - Students should discover that the area of the tile is 1ft^2.
  - Use the ratio $\frac{1\ tile}{1\ ft^2}$ to determine how many tiles are needed to cover the area of the floor: $\frac{1\ tile}{1\ ft^2} = \frac{?\ tiles}{108\ ft^2}$
- Students use a calculator to determine and solve the proportions.
  - Or students determine how many tiles are needed to complete the length of one side of the floor by placing a manipulative tile on the floor plan and count 9 tiles needed and repeat for the width, counting 12 tiles.
  - Students multiply 9x12 to determine the number of tiles needed to complete the floor.

2. Provide the written problem to students to include picture representations of relevant words so students can follow along as the problem is introduced.

- Be sure students have the picture representations of inches, feet, tile, and floor so they can give an opinion as to which rectangle should be converted to which unit of measure.
- Students should still have the scale drawing of the rectangle (floor) on grid paper measuring 9 units by 12 units with each unit representing a foot and the rectangle labeled ”floor” using word and picture representations.
- Provide students with paper or object squares that represent 18” scaled to one and a half the size of the grid paper unit.
- Given a model of the multiplication problem for converting feet to inches

  (___ft x 12 inches) and a calculator, students convert the length and the width of the floor from feet to inches.
  - 9 x 12in = 108 in
  - 12 x 12in = 144 inches
- Using the formula for area, students use the calculator to determine the area of the floor (108 in x 144 in) and the area of the tile (18 in x 18 in).

• Students create a ratio of the area of the floor to the area of the tile then divide: $\frac{15522\ in^2}{324\ in^2} = 48.$

Lesson 3: Practice – continued

Additional Considerations for Emerging Communicators
1. Provide students with relevant picture or tactile/object representations of relevant words/concepts as the problem is introduced. • Use real or replicas of 12-inch tiles and create a 9 ft by 12 ft rectangle on the floor using colored tape or a computer program to model the floor plan. • Using the foldable ruler, review that 12 inches is the same as one foot. • Convert scale representation of tiles from 12 inches to one foot. • Place manipulative tiles down the length of one side of the taped floor plan and count how many tiles are needed to cover the length of that side. • Repeat for one side the width of the floor. • Multiply the two numbers to determine the number of tiles needed to cover the floor or use a computer program to input the tiles and input the numbers into a multiplication problem to solve the problem. 2. Provide students with picture or tactile/object representations of relevant words/concepts as the problem is introduced. • Use real or replicas of 12-inch tiles and create a 9 ft by 12 ft rectangle on the floor using colored tape or a computer program to model the floor plan. • Using one foot rulers, show that it takes 9 rulers to cover the length of the floor. • Using the foldable ruler, review that there are 12 inches in a foot. • Convert the length and width of the floor from feet to inches by inputting the correct number to complete the multiplication problem (___ft x 12 inches) using a calculator or computer program. • Place manipulative tiles down the length of one side of the taped floor plan and count how many tiles are needed to cover the length of that side. • Repeat for one side the width of the floor. • Multiply the two numbers to determine the number of tiles needed to cover the floor or use a computer program to input the tiles and input the numbers into a multiplication problem to solve the problem. **Important Note for Communicators Considered Pre Symbolic**: The number load may need to be reduced.

Lesson 3: Closure – 15 minutes

A. Revisit/Review Lesson and Objectives

Remind students that they were to make decisions about units and scales that are appropriate for problem solving situations involving mathematics or across disciplines or contexts, and:

1. Set up and solve proportions.
2. Convert units of measurement using standard/known conversions.
3. Recognize when to multiply and when to divide in converting measurements.
4. Use ratio and proportion to convert measurements.
5. Use appropriate known formulas for area.
6. Solve problems requiring calculations that involve different units of measure within a measurement system.

Multiple means of representation: Along with posting lesson objectives in the classroom, students may refer to their individual copies.

Multiple means of expression: Students share what they have learned in different formats: writing, drawing, creative expression, etc.

Multiple means of engagement: Share ideas of how and when these skills might be relevant to "me."

Additional Considerations for Emerging Readers and Emerging Communicators

1. When reviewing the expected outcomes, students refer to the lesson objectives they recorded or collected in their mathematics journals or in their electronic picture versions.

2. Students use the information recorded in their journals to refer back to the lesson objectives key words paired with images. From that information they share what they have learned based on each of the expectations.
   - For example, students may grab the tactile cue for area to state, "I have learned that the area is all the space measured within a figure."

3. Students should refer back to the photographs and/or tactile representations of examples of real-life situations when these concepts are used to share when they could use these new skills.
   - For example, students could touch the tactile representations for area and tabletop to state, "I can use area to determine if an object will fit on my desk."

Lesson 3: Closure – continued

B. Exit Assessment

1. Students solve a third problem using models and paper and pencil.
    - For Example, How many 6 in square tiles are needed to cover a 3 ft. by 5 ½ ft. counter top?
        - Discuss the results showing more than one strategy.

Multiple means of representation: Allow students to have a written copy of the problem, drawn models of the situation, and/or conversion formulas as needed/requested.

Multiple means of expression: Students may draw or use manipulatives to model solutions or use the computer.

Multiple means of engagement: Create situations that include areas of interest to students.

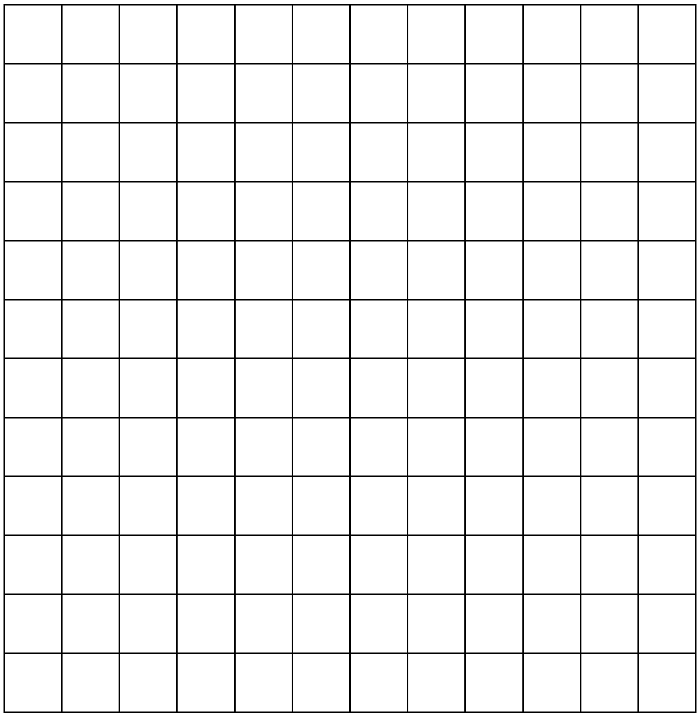
Additional Considerations for Emerging Readers and Emerging Communicators

- Use the same supports as used in the practice section.

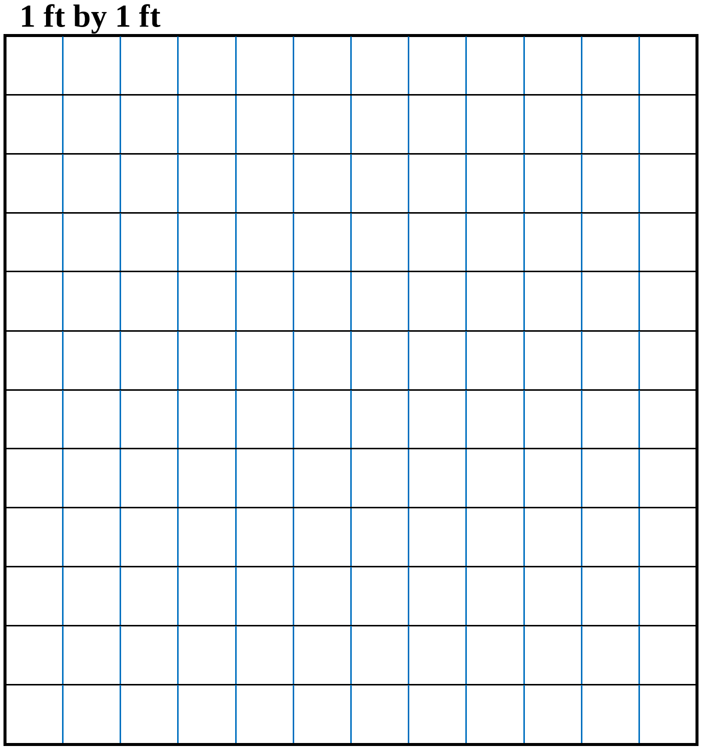
Lesson 3: Resources

12 in x 12 in

12 inches

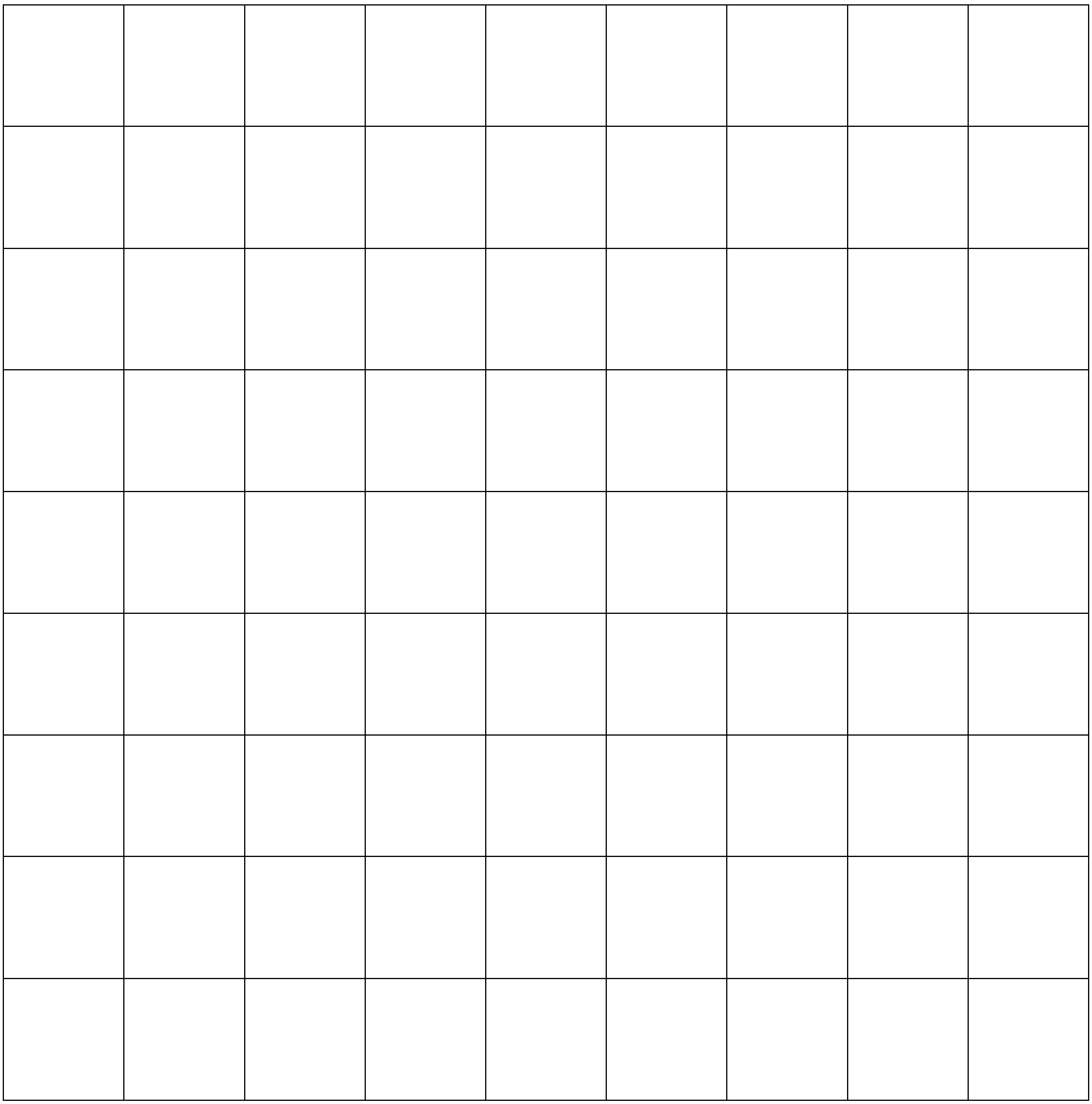


1 ft by 1 ft



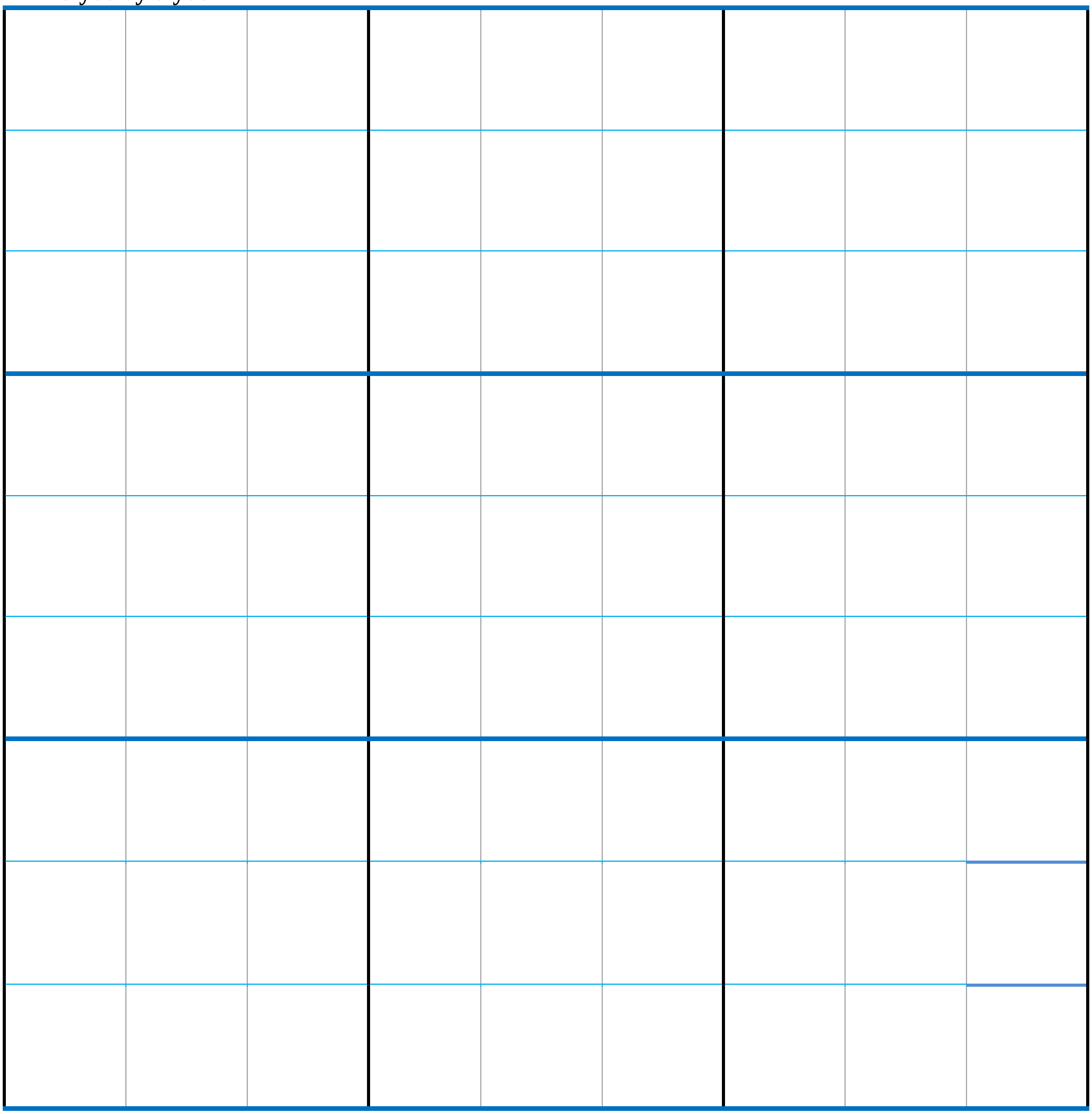
Lesson 3: Resources– continued

9 ft by 9 ft **9 feet**



Lesson 3: Resources– continued

3 yds by 3 yds



Lesson 4

Grade Span: 9 - 10	**Content Area:** Math - Geometry
Lesson 4 of the Unit	**Approximate Time Needed:** 55 minutes

Objective: Students will make decisions about units and scales that are appropriate for problem solving situations within mathematics or across disciplines or contexts.

Essential Questions:

1. What are the relationships among the measurements of dimensions, area, and perimeter in problem solving situations?
2. How does changing the sides of a square affect the area?

Materials Set Up:

- Set up problem and question on overhead/whiteboard.
- Mark off floor in varying size squares up to 25 square feet. (e.g., 1’x1’, 2’x2’, 3’x3’, 4’x4’, 5’x5’) to demonstrate size needed for dancing.
- Create or select manipulatives to represent area and people.

Materials Needed:

- Calculator
- Pencils, paper, graph paper, masking tape
- Manipulatives
- CD player with various types of music
- Blank table with columns for name and dance space
- Measuring tape, yardstick, ruler

Lesson Vocabulary

Area
Length, Width
Ratio
Unit of Measure
Unit Rate

Lesson 4: Introduction – 10 minutes

A. Activate Previous Knowledge

1. Remind students that they have been working on the concepts of perimeter and area of shapes of different sizes.

2. Review ratios.

3. Discuss how they can use this knowledge to solve problems they may encounter in the real world.

4. Present the following idea/problem:
    - Introduce the idea of planning a dance party in the classroom.
    - How many people would you expect to attend?
    - How large will the dance floor need to be?

5. Students use previous knowledge to brainstorm ways to solve this problem.

Multiple means of representation: Present real life problems using drawings, models, and video representations of people dancing on a dance floor.

Multiple means of expression: Allow students to present ideas for problem solving using computer models, demonstrations, visuals, etc. Record problem solving ideas in different formats: mathematics journals, computer, premade or original graphic organizers, etc.

Multiple means of engagement: Use student-chosen dance styles and music when presenting problem. Allow students to work individually or in small groups based on learning style.

Additional Considerations for Emerging Readers and Emerging Communicators

1. During the review, be sure students have graphic and/or tactile representations of relevant vocabulary (area, perimeter, length, width) as well as related materials/drawings/objects representations from previous lessons.

2. Provide examples through pictures, tactile cues and/or videos of couples on various dance floors that are very crowded, average, and very empty, etc.
    - Point out how much space people have to dance.
    - Ask guiding questions:
        - “Do the dancers have enough space?”
        - “Is there room for more people?” etc.
    - Students attempt to add more people to an object representation or virtual representation of the dance floor.
    - Create a list of possible dance styles by using words paired with pictures.

3. Students use their math journals to refer to previous strategies used for solving problems. Ensure students have a way to contribute to the brainstorming.

Lesson 4: Introduction – continued

B. Establish Goals/Objectives for the Lesson

Inform students that they will make decisions about units and scales that are appropriate for problem solving situations within mathematics or across disciplines or contexts, and:

1. Identify, quantify, and compare the attributes of the objects, situations, and/or events that need to be measured to solve the problem/situation.
2. Use appropriate units of measure to identify, quantify, and compare objects, situations, and/or events to solve a real world problem.
3. Convert units when necessary.
4. Represent data in various forms

Multiple means of representation: Along with posting lesson objectives in the classroom, provide individual copies for students.

Multiple means of expression: Allow students to record lesson objectives in different formats: mathematics journals, computer, premade or original graphic organizers, etc.

Multiple means of engagement: Brainstorm ideas of how and when these skills might be relevant to "me."

Additional Considerations for Emerging Readers and Emerging Communicators

1. Provide students with keys words paired with symbols/images/tactile representations (i.e., units of measure, inch, foot, yard, centimeter, and meter).

2. Provide the key words in the lesson objective paired with symbols/images/tactile representations to paste into their mathematics journals or collect in a mathematics basket or bag. Students may use an electronic picture writer to record the lesson objectives.

3. Provide students with photographs, models, or tactile representations of examples of situations when these concepts are used.

Lesson 4: Body – 15 minutes

Direct Instruction and/or Facilitation of the Lesson

Whole Group Discussion:

1. Review students' ideas on how to solve the dance floor problem. Pull out relevant ideas to try making sure the following are included:
   a. Measure a square/rectangular area in the classroom that could be used as a dance floor.
   b. Choose the best unit of measurement to measure the space.
   c. Explore strategies for determining how much space each person needs to dance depending on type of music.
      - What is the best unit of measurement to use?
      - Introduce unit rate, square feet per couple.

2. In small groups, students list at least three types of dance/music they will model and measure to determine how much floor space is needed per person.
   - Allow students to volunteer to dance.
   - Students demonstrate different styles of dancing, slow or fast.
   - Students who did not want to dance should measure and record the dance space needed per person and per style of dance in a table using appropriate unit of measurement, square footage.
   - Be sure the amount of dance space needed for students in wheelchairs is considered.
   - Display the information in the table at the front of the classroom.

Dance Style	Space Needed for a Person		
	Length	Width	Area
Slow dance	*2 ft*	*2ft*	*4ft^2*

Multiple means of representation: Allow students to refer to their brainstorming notes during discussion. When discussing unit rate, provide familiar examples (e.g., miles per hour). Provide students with a copy of the word problem and the table above. Have drawings and manipulatives available for students to use. Provide options for demonstrating different dance styles (e.g. volunteers demonstrate, bring dancers into the classroom to demonstrate, watch video demonstrations, etc.)

Multiple means of expression: Allow students to solve the problem using formulas and/or models and record information into the tables using various formats paper and pencil, computer, etc.

Multiple means of engagement: Ensure all students are actively involved in their small groups. Use music and dance styles related to students' interests. Use questioning to encourage students to explain their strategies.

Additional Considerations for Emerging Readers

1. Introduce the concept of unit rate as area per person or the amount of space needed per person to have enough room to dance.
   - Provide a list of brainstorming ideas for solving the problem.
   - Include picture and/or tactile representations of different dance styles as needed.

2. Be sure all students have a job in their small groups that relates to the mathematical concept of measuring.
   - When students are measuring, be sure they have the needed supports for reading the ruler/yardstick to the nearest foot and inch.
   - Provide students with a copy of the table as well as word/picture representations of the different dance styles that can be used to complete column one.

Additional Considerations for Emerging Communicators

1. Introduce the concept of unit rate as area per person or the amount of space needed per person to have enough room to dance.

2. Represent some of the brainstorming ideas they came up with in the introductory lesson using tactile and object representations to review the ideas with the class.
   - Provide some new ideas in tactile representation that students share with the class.

3. Be sure all students have a job in their small groups that relates to the mathematical concept of measuring.
   - When students are measuring, they keep track of the number of feet by working with a peer.
     - Each time the peer lays the ruler and measures a foot, the other student places a tactile representation of a foot long in a basket for length and a tactile representation of a foot wide for width.
     - When finished, students count how many feet were collected/measured for length and width.

4. Observe how students communicate within their groups.
   - Ensure they have a means for sharing ideas and gaining peers' attention.
   - If not, allow opportunities to practice within the group.
   - Be sure activities are engaging to encourage communication.

5. Provide students with a copy of the table as well as word/picture/tactile representations of the different dance styles that can be used to complete column one.
   - If students are completing the table, they place a tactile representation for each dance style in column one.
   - Record the number of feet for length and width by tallying each time a foot is measured.

Lesson 4: Practice – 15 minute

1. Break into small groups to solve the problem.
   - Pose the problem: Your class is having a party and wants a dance floor. The biggest dance floor in the classroom is 8ft x 8ft.
   - Based on each dancer's estimation of the dance space needed for one person preforming one style of dance, how many people can dance at one time on the dance floor?
   - Calculate each style separately by using unit rate.

Dance Style	Space Needed for One Person			Number of People Who Can Dance at One Time
	Length	Width	Area	
Slow dance	*2 ft*	*2ft*	*4ft^2*	*16*

   - Based on the information in the table, what style of music would you play?
   - Explain why.

2. Bring the whole group back together.
   a. Fill in the table to indicate the number of dancers who can dance at one time based on the style of dance.
      - Unit rate of couple per square feet needed based on different dance styles.
   b. Discuss the style of music the class would choose.

Multiple means of representation: Provide students with a copy of the word problem, a template of the formulas for the unit rate/ratios and the table. Have drawings and manipulatives available for students to use.

Multiple means of expression: Allow students to solve the problem using the formulas, drawings, computer graphics, and/or models, etc. Record the number of people into the tables using various formats: paper and pencil, Smart Board, computer, etc.

Multiple means of engagement: Ensure all students are actively involved in their small groups, and use music and dance styles related to students' interests. Use questioning to encourage students to explain their strategies.

Lesson 4: Practice – continued

Additional Considerations for Emerging Readers
1. Present students with the problem written with words paired with picture and/or object representations of the most salient vocabulary from the problem. • Determine the area of the dance floor, using previous learned strategies (e.g., 8ft x 8ft = $64ft^2$). • Using information from the table, use the unit rate and equivalent ratios to determine how many people can dance on the floor at one time: o Slow dance $\frac{1\ person}{4\ ft^2} = \frac{x\ person}{64\ ft^2}$ • Repeat for each style of music. 2. Individuals from groups share their results to complete the class table. Students should have their own copies of the table for reference. Provide picture/number representations for students to use to communicate results if needed.
Additional Considerations for Emerging Communicators
1. Present the problem to students written with words paired with pictures or object representations. • Have a section of the classroom floor measured out and taped in an 8ft x 8ft square. • Students move around the perimeter of the dance floor and within the area. • If the floor has one foot square tiles, students skip count the tiles by 8, hitting a preprogrammed switch or placing a representation of each long foot (1x8) in a basket and determine the total of 64. • Students stop hitting the switch at the end of the length or stop adding a long foot to indicate understanding/performance of area. • Repeat process for each style of music. • Students should collect a representation of their group's results as the group collects information on each style. 2. Individuals from groups share the results to complete the class table. • Students should use their own copies of the group results, which they collected in their mode of communication, to contribute to the whole class table.

Lesson 4: Closure – 15 minutes

A. Revisit/Review Lesson and Objectives

Remind students that they were to make decisions about units and scales that are appropriate for problem solving situations within mathematics or across disciplines or contexts, and:

1. Identify, quantify, and compare the attributes of the objects, situations, and/or events that need to be measured to solve the problem/situation.
2. Use appropriate units of measure to identify, quantify, and compare objects, situations, and/or events to solve a real world problem.
3. Convert units when necessary.

Multiple means of representation: Along with posting lesson objectives in the classroom, students may refer to their individual copies.

Multiple means of expression: Students can share what they have learned in different formats: writing, drawing, creative expression, etc.

Multiple means of engagement: Share ideas of how and when these skills might be relevant to "me."

Additional Considerations for Emerging Readers and Emerging Communicators

1. When reviewing the expected outcomes, have students refer to the lesson objectives they recorded in their mathematics journals or their electronic picture versions.

2. Students use the information recorded in their journals to refer back to the lesson objectives key words paired with images. From that information, they share what they have learned based on each of the expectations.
    - For example, students grab the tactile cue for area to state, "I have learned that the area is all the space measured within a figure."

3. Students should also refer back to the photographs or tactile representations of real-life situations in which these concepts are used to share when they could use these new skills.
    - For example, students touch the tactile representations for area and tabletop to state, "I can use area to determine if an object will fit on my desk."

Lesson 4: Closure – continued

B. Exit Assessment

1. Tell students: “This will be your ticket out the door”:
    - If you have a party at your house and have a 10ft x 10ft dance floor, determine how many people could dance at the same.
    - Use the information from the table created during the lesson body.

2. Observe how students solve the problems.
    - Take anecdotal notes or assess students using a rubric.
    - The situation/problem dictates the type of measurement to use.
        - Did students determine the area of the dance floor correctly?
        - Could students use the area to compute the answer to the question?
        - Did students use the rate and equivalent ratios to determine how many people could dance a certain style at one time?

3. Students return to whole group.
    - Small groups present their solutions to the class and explain their process for determining their answers.

Multiple means of representation: Provide students with a copy of the word problem, a template of the formulas for the unit rate/ratios and the table. Have drawings and manipulatives available for students to use.

Multiple means of expression: Allow students to solve the problem using formulas, drawings, computer graphics, and/or models, etc. Record the number of dancers into the tables using various formats: paper and pencil, Smart Board, computer, etc.

Multiple means of engagement: Ensure all students are actively involved in their small groups, and use music and dance styles related to students’ interests. As you observe group work, use questioning to encourage students to explain their strategies.

Lesson 4: Closure – continued

Additional Considerations for Emerging Readers
1. Present students with the problem written with words paired with picture symbols of the most salient vocabulary from the problem. • Determine the area of the dance floor, using previous learned strategies (e.g., 8ft x 8ft = $64ft^2$). • Using information from the table, use the unit rate and equivalent ratios to determine how many people can dance on the floor at one time: o Slow dance $\frac{1\ person}{4\ ft^2} = \frac{x\ person}{64\ ft^2}$ • Repeat for each style of music. 2. Provide students with a copy of a modified rubric and review expectations. 3. Allow students to use all supports provided throughout the unit lesson so far. 4. Students should have their own copies of the table for reference. 5. Provide picture/number representations for students to use to communicate results if needed.
Additional Considerations for Emerging Communicators
1. Present students with the problem written with words paired with pictures or an object representation of the problem. • Have a section of the classroom floor measured out and taped in an 8ft x 8 ft square. • Students move around the perimeter of the dance floor and within the area. • If the floor has one foot square tiles, students skip count the tiles by 8, hitting a preprogrammed switch or placing a representation of each long foot (1 x 8) in a basket and to determine the total of 64. • Students stop hitting the switch at the end of the length or stop adding a long foot to indicate understanding/performance of area. • Repeat for each style of music. 2. Provide students with a copy of a modified rubric in picture and/or tactile representation and review expectations. 3. Allow students to use all supports provided throughout the unit lesson so far. 4. Students have their own copies of the group results to the dances in the form of visual and/or tactile representations of the dance style paired with the number of people. 5. Students contribute to the class table by handing the paired representations to the teacher.

Lesson 4: Resources

Dance Style	Space Needed for a Person		
	Length	Width	Area
Slow dance	*2 ft*	*2ft*	*4ft^2*

Lesson 4: Resources– continued

Dance Style	Space Needed for One Person			Number of People Who Can Dance at One Time
	Length	Width	Area	
Slow dance	*2 ft*	*2ft*	*4ft²*	*16*

Lesson

5

Grade Span: 9 - 10	**Content Area:** Math - Geometry
Lesson 5 of the Unit	**Approximate Time Needed:** 90 minutes or two 45 minute blocks

Objectives:

- Identify and quantify attributes of the problem that need to be measured.
- Determine a pattern.
- Generalize relationships.
- Determine the percent of increase/decrease.
- Determine the precision of measurement.

Essential Question(s):

1. What are the relationships among the measurements of dimensions, area, and perimeter in problem solving situations?
2. How does changing the sides of a square affect the area?
3. How can we use variable expressions to reflect relationships?

Materials Needed:

- Large and small grid graph paper
- Worksheets

Lesson Vocabulary:

Area
Centimeter
Foot
Inch
Length
Meter
Ratio
Unit of Measure
Unit Rate
Width
Yard

Lesson 5: Introduction – 15 minutes

A. Activate Previous Knowledge (unit rate)

1. Review the dance floor problem from the previous lesson with the whole group.

2. Pose a new problem and discuss vocabulary with the class.
   - Alex and Aldo planted several square apple orchards.
   - The table below shows the number of trees and the size of the orchards.

3. Review the information presented in each part of the table. Ask students to analyze the relationship between the side lengths and number of trees. No calculations needed. Keep it general (e.g., in orchard 1, the number of trees to length in feet is 2:1; in orchard 4, the number of trees to feet is 1:1).

Orchard number	Length of each side (feet)	Number of apple trees
1st		
2nd	8 feet	4 trees
3rd	12 feet	9 trees
4th	16 feet	16 trees

Multiple means of representation: Present real life problems using drawings, models, and video representations of orchards of various sizes.

Multiple means of expression: Allow students to present ideas for problem solving using computer models, demonstrations, visuals, etc. Record problem solving ideas in different formats: mathematics journals, computer, premade or original graphic organizers, etc.

Multiple means of engagement: Students may choose the type of orchard when presenting problem. Allow students to work individually or in small groups based on learning style.

Additional Considerations for Emerging Readers and Emerging Communicators

1. During review, students should refer to their math journals or notes. Be sure students have graphic and/or tactile representations of relevant vocabulary (area, perimeter, length, width) as well as related materials/drawings/object representations from previous lessons.

2. Provide examples through pictures, videos, or tactile representations of orchards of various sizes
   - Point out how much space trees need for maximum growth and production.
   - Ask guiding questions:
     - "Do they have enough space?"
     - "Is there room for more trees?"
   - Students attempt to add more trees to an object representation or virtual representation of the orchard.

B. Establish Goals/Objectives for the Lesson

Inform students that they will make decisions about units and scales that are appropriate for problem solving situations involving mathematics within mathematics or across disciplines or contexts and:

1. Identify and quantify attributes of the problem that need to be measured.
2. Determine a pattern.
3. Generalize relationships.
4. Percent of Increase/ Decrease.
5. Determine the precision of measurement.

Multiple means of representation: Along with posting lesson objectives in the classroom, provide individual copies for students.

Multiple means of expression: Allow students to record lesson objectives in different formats: mathematics journals, computer, premade or original graphic organizers, etc.

Multiple means of engagement: Brainstorm ideas of how and when these skills might be relevant to “me.”

Additional Considerations for Emerging Readers and Emerging Communicators

1. Provide students with keys words paired with symbols/images/ tactile representations (i.e., units of measure, inch, foot, yard, centimeter, and meter).

2. Provide key words in lesson objective paired with symbols/images/tactile representations to record into mathematics journals. Students may use an electronic picture writer to record the lesson objectives.

3. Provide students with photographs, models, or tactile representations of examples of situations in which these concepts are used.

Lesson 5: Body – 30 minutes

Direct Instruction and/or Facilitation of the Lesson

During this portion of the lesson, students will generalize relationships and determine the appropriate scale to express the relationship between two quantities.

1. Review students' ideas on how to solve the orchard problem.
    a. Alex and Aldo planted several square apple orchards.
    b. The table below shows the number of trees and the size of the orchards.

Orchard Number	Length of Each Side (feet)	Area of Each Orchard (ft^2)	Number of Apple Trees
1st	x	?	y
2nd	8 ft	64 ft^2	4 trees
3rd	12 ft	144 ft^2	9 trees
4th	16 ft	256 ft^2	16 trees
5th	x	?	y
n			

2. Students determine the area of each orchard (e.g., The 2nd orchard has an area of 64 ft^2 because 8 x 8 = 64).
3. Given the number of apple trees in each orchard, students determine the square footage needed for each tree using ratios and proportions (e.g., $\frac{area\ of\ orchard}{number\ of\ tree} = \frac{area}{1\ tree}$ or the unit rate of area per tree).
4. Using the ratio from orchard 2, students determine the unit rate (e.g., $\frac{64^2}{4\ trees} = \frac{?^2}{1\ tree}$ or 64 $ft^2 \div$ 4 trees = 16 ft^2 needed for each tree) and confirm that measurement is true for each orchard (i.e., 144 $ft^2 \div$ 9 trees = 16 ft^2 and 256 $ft^2 \div$ 16 trees = 16 ft^2).
5. Given the measurements in the length of each side column, students determine the rate of change in the length of each orchard (i.e., ____, 8, 12, 16 is a +4 pattern).

Note: Students work in pairs to answer parts 1 - 5 of the problem.

6. Using the rate of change +4, students determine the length of each side (x) for orchards 1 and 5, and fill in the column of the table.

Note: Use whole group discussion for part 6.

7. Students use that information to determine the area of the 1st and 5th orchards (i.e., the 1st orchard has an area of 16 ft^2 because 4ft x 4ft = 16ft^2 and the 5th orchard has an area of

400 ft^2 because 20ft x 20ft = 400ft^2).

8. Students use the fact that each tree needs 16 ft^2 to determine how many trees can be planted in the 1st and 5th orchards using $\frac{area\ of\ orchard}{area\ per\ tree}$ = # of trees (i.e., for the 1st orchard, $\frac{16^2}{16^2}$ = 1 tree and for the 5th orchard, $\frac{400^2}{16^2}$ = 25 trees).

9. Students graph the rate of change in the length of each side and the consequent number of trees for each orchard (i.e., (x, y) where x = length of each side and y = the number of apple trees.

Multiple means of representation: Allow students to refer to their brainstorming notes during discussion. When discussing unit rate, provide familiar examples (e.g., miles per hour). Provide students with a copy of the word problem and the table under #1. Have drawings and manipulatives available for students to use.

Multiple means of expression: Allow students to solve the problem by using formulas and/or models and record information into the tables using various formats: computer, premade or original graphic organizer, etc. Allow students to use a reference of formulas.

Multiple means of engagement: Ensure all students are actively involved in their partnerships. Use scenarios related to students' interests. For example, if a student is interested in animals instead of orchard trees, the scenario could involve the rate of grazing area per horse. Use questioning to encourage students to explain their strategies.

Lesson 5: Body – continued

Additional Considerations for Emerging Readers and Emerging Communicators
1. Refer to the brainstorming ideas for solving the problem. Include picture and/or tactile representations as needed. • Provide students with copies of the table as well as word/picture/tactile representations of the words orchard and apple trees. • Knowing that the orchards are square, students should determine that the length and width are the same. • Students determine the area of orchard #2 by using the formula length x length = area or 8ft x 8ft = 64 ft^2 and/or students may also draw the orchard on grid paper to determine the area. • Students can also be provided with a manipulative model or virtual template of orchard #2 so they can determine the area by counting the units. • Since the numbers will be quite large, provide students with a means to skip count to determine area. o Students can be given units grouped by 8 and a calculator set up to add 8 so each time students place a row of units into the template, they hit enter on the calculator to add 8. o Students stop when the template is filled and indicates the final number for area from the calculator. o If students are using a computer program, the program would be set up in the same way as lessons 1 and 2. **See Example:** Manipulative worksheets or PowerPoint Lesson 5, Slide 1. 2. Review the concept of unit rate (area/tree) or the amount of space needed per tree. • If orchard #2 is 64 ft^2 and has 4 trees, how many square feet is needed for one tree? Students should set up the ratio as $\frac{area\ of\ orchard}{number\ of\ trees}$ or $\frac{64^2}{4\ trees} = \frac{?^2}{1\ tree}$. • Allow students to review strategies used in lesson 3 for using ratios and proportions to solve problems. • Students should remember that the equation must remain balanced and that whatever was done to the top portion must be done to the bottom. • Since the numbers decrease, students would use division. $\frac{64^2}{4\ trees} = \frac{?^2}{1\ tree}$ or $\frac{64\ ft^2}{4\ trees}$ $\div 4$ $\div 4$ $= \frac{6\ ft^2}{tree}$ The unit rate is 16ft^2 per tree. • Students use manipulatives to determine how to divide the trees evenly to have a group of one tree (divide by 4). • Students should divide the orchard by four as well. **See Example:** PowerPoint Lesson 5, Slide 2.

3. Instruct students to determine the rate of change in the unit length of each orchard.
    - Students determine the pattern (x, 8, 12, 16, x; pattern is +4).
    - Students draw each orchard and lay them on top of each other to determine how the side lengths change or add/subtract the difference between unit lengths of the consecutive orchards to determine that each orchard changes by 4 ft.
    - Students use that information to determine the unit length of orchard #1 and orchard #5.
        - For example: length of orchard #1 +4ft = length of orchard #22

$$x+4ft = 8ft$$
$$x + 4ft - 4ft = 8ft – 4ft$$
$$x=4ft$$

        - For example: length of orchard #4 + 4 ft = length of orchard #5

$$16 \text{ ft} + 4 \text{ ft} = x$$
$$20 \text{ ft} = x$$

See Example: PowerPoint lesson 5, Slide 3.

4. Students use the information of unit length to determine the area of orchards #1 and #5.
    - Students use given formula for a square (length x length = area) and/or draw the orchards on grid paper and count the squares to determine the area.
    - Students use manipulatives of the rate of change by placing a unit length of four, starting at the end of the length and width on orchard #2 to determine the unit length for orchard #1.
    - Students draw or model the orchards on grid paper and count the squares to determine the area.

See Example: PowerPoint Lesson 5, Slide 4.

- Students use the rate of change by adding it onto the length of orchard #4 to determine the length of orchard #6.

See Example: PowerPoint Lesson 5, Slide 5.

5. Now that students have determined the unit rate, they determine the number of trees per orchard for orchard #5.
    - Using unit length determined in step 3, students use the formulas and ratios to determine the area of the orchard and the number of trees that can be planted in the orchard.
    - Students can also use grid paper to draw orchard #5 (based on the dimensions of 20 ft x 20 ft) and by using a cut out of the unit rate, determine how many trees can be planted.

See Example: Lesson 5 or students use virtual manipulatives as in PowerPoint Lesson 5 Slide 2.

6. Tell students they will be graphing the relationship between the size of the garden and the number of trees that can be planted.
    - Provide students with a coordinate grid with the x- and y- axes labeled.
    - Students must use the information from their tables to create ordered pairs and complete the graph.
    - The columns from which the ordered pairs are created are already labeled as x and y, but they can also be highlighted as needed.
    - Students count over (the run) for x and up (the rise) for y, or students find the matching number for x and move the point up to the matching number for y.

See Example: PowerPoint Lesson 5, Slide 6.

Important Consideration: For some students, the difficulty/complexity can be reduced by using only the first quadrant of the coordinate grid.

Lesson 5: Practice – 30 minutes

1.Provide similar problems and additional practice questions based on students' responses.

- For example, Casey and Liz want to plant their own square apple orchard.
- They decide to increase the sides of Alex and Aldo's 3rd orchard by 25%.
- If they keep the same area per tree, how many trees can they plant in their square orchard?

Multiple means of representation: Provide students with a copy of the word problem and the table. Have drawings and manipulatives available for students to use.

Multiple means of expression: Allow students to solve the problem by using formulas and/or models and record information into the tables using various formats: computer, premade or original graphic organizer, etc. Allow students to use a reference of formulas.

Multiple means of engagement: Ensure all students are actively involved in their partnerships. Use scenarios related to students' interests. For example, if a student is interested in animals instead of orchard trees, the scenario could involve a rate of grazing area per horse. Use questioning to encourage students to explain their strategies.

Additional Considerations for Emerging Readers and Emerging Communicators
1.Provide students with copies of the problem paired with picture and/or tactile representations. Ensure students have the table used during instruction to refer to for solving this problem.

2. Students determine the area of Casey's and Liz's orchard by multiplying the area of orchard #3 by 1.25.

3. Students may also use a drawing of orchard #3 on grid paper and divide it evenly in fourths (or quarters) to determine what 25% more would be and combine the quarter representation with orchard #3.

See Example: Manipulatives or PowerPoint Lesson 5, Slides 7 & 8.

4. Once students have determined the area of Casey and Liz's orchard, they can determine how many trees can be planted based on the unit rate of $\frac{16\,ft^2}{1\,tree}$.

5. Students should use the same strategies and supports as they used previously.

Lesson 5: Closure - 15 minutes

a. Revisit/Review Lesson and Objectives

Remind students that they were to make decisions about units and scales that are appropriate for problem solving situations within mathematics or across disciplines or contexts and:

1. Identify and quantify attributes of the problem that need to be measured.
2. Determine a pattern.
3. Generalize relationships.
4. Percent of increase/ decrease.
5. Determine the precision of measurement.

Multiple means of representation: Along with posting lesson objectives in the classroom, students may refer to their individual copies.

Multiple means of expression: Students can share what they have learned in different formats: writing, drawing, creative expression, etc.

Multiple means of engagement: Share ideas of how and when these skills might be relevant to "me."

Additional Considerations for Emerging Readers and Emerging Communicators
1. When reviewing the expected outcomes, have students refer to the lesson objectives they recorded in their mathematics journals or their electronic picture versions. 2. Students use the information recorded in their journals to refer back to the lesson objectives' key words paired with images. From that information, they share what they have learned based on each of the expectations.   a. For example, students may grab the tactile cue for area to state "I have learned

that the area is all the space measured within a figure."

3. Students should also refer back to the photographs of examples of real-life situations when these concepts are used to share when they could use these new skills.
    - For example, students could touch the tactile representations for area and orchard to state, "I can use the unit rate of area to tree to determine the size of the orchard."

B. Exit Assessment

1. Students work either in pairs or individually to produce their own word problems similar to the ones presented in this lesson. Once the problems are written, students identify the unit rate of their problem (e.g., area per tree or area per person). If time permits, students can trade problems and solve them as a review for another in-class activity.

Multiple means of representation: Ensure students have the previous word problems from this lesson and/or lesson 4 to review and model. Have previous drawings, models, and manipulatives available for students to use.

Multiple means of expression: Allow students to create the problem using various formats: computer, premade or original graphic organizer, models, etc. Allow students to use a reference of formulas.

Multiple means of engagement: Ensure all students are actively involved in creating their problems. Encourage students to use scenarios related to their interests. For example, if a student is interested in animals instead of orchard trees, the scenario could involve a rate of grazing area per horse. Use questioning to encourage students to explain their strategies.

Additional Considerations for Emerging Readers and Emerging Communicators

1. Students should have access to a variety of picture representations and models as they brainstorm ideas for their problems. Provide choices of interest to students in picture/tactile format.

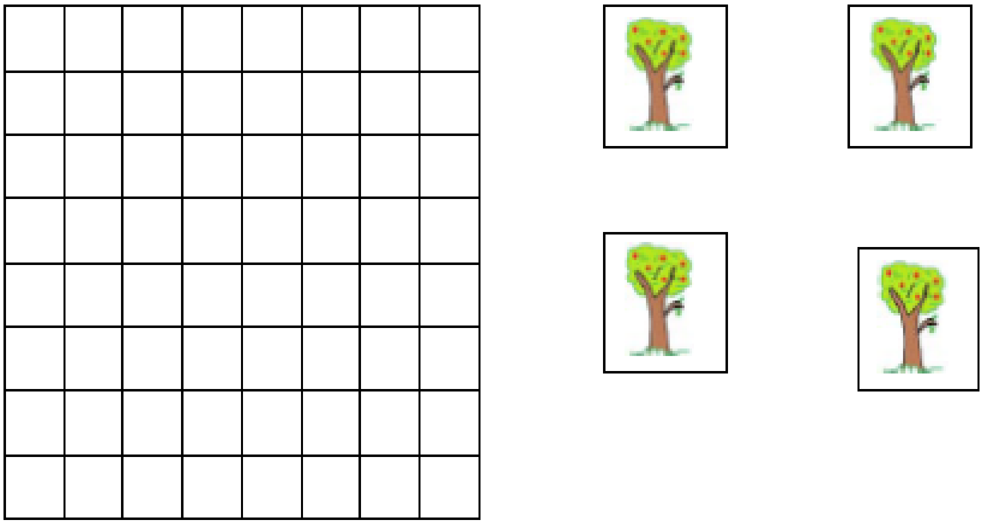
2. Students review the information in previous problems and choose the key words paired with images to use when creating their problems or provide students with a template of a word problem that they can complete with key words and unit rate concepts.

Lesson 5: Resources

Alex and Aldo planted several square apple orchards. The table below shows the number of trees and the size of the orchards (See ppt demonstration).

Orchard Number	Length of Each Side (feet)	Area of Each Orchard (ft^2)	Number of Apple Trees
1^{st}	x	?	y
2^{nd}	8 ft	64 ft^2	4 trees
3^{rd}	12 ft	144 ft^2	9 trees
4^{th}	16 ft	256 ft^2	16 trees
5^{th}	x	?	y
n			

Orchard #2 is 8ft x 8ft = 64ft² and has 4 trees.

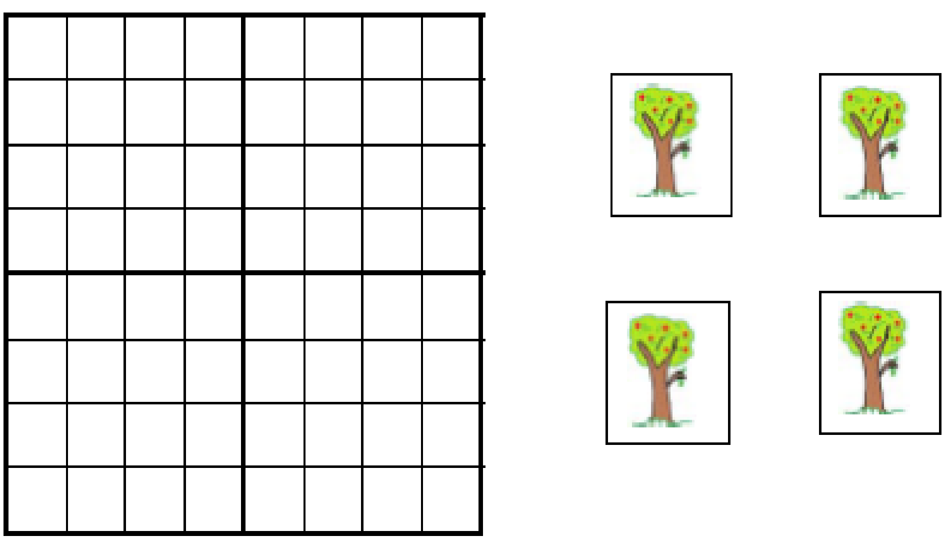


How much space is needed for one tree?

$\frac{64^2}{4\ trees} = \frac{?^2}{1\ tree}$ to get one tree, student should divide the group of trees by four $\frac{64^2}{4\ trees \div 4} = \frac{?^2}{1\ tree}$.

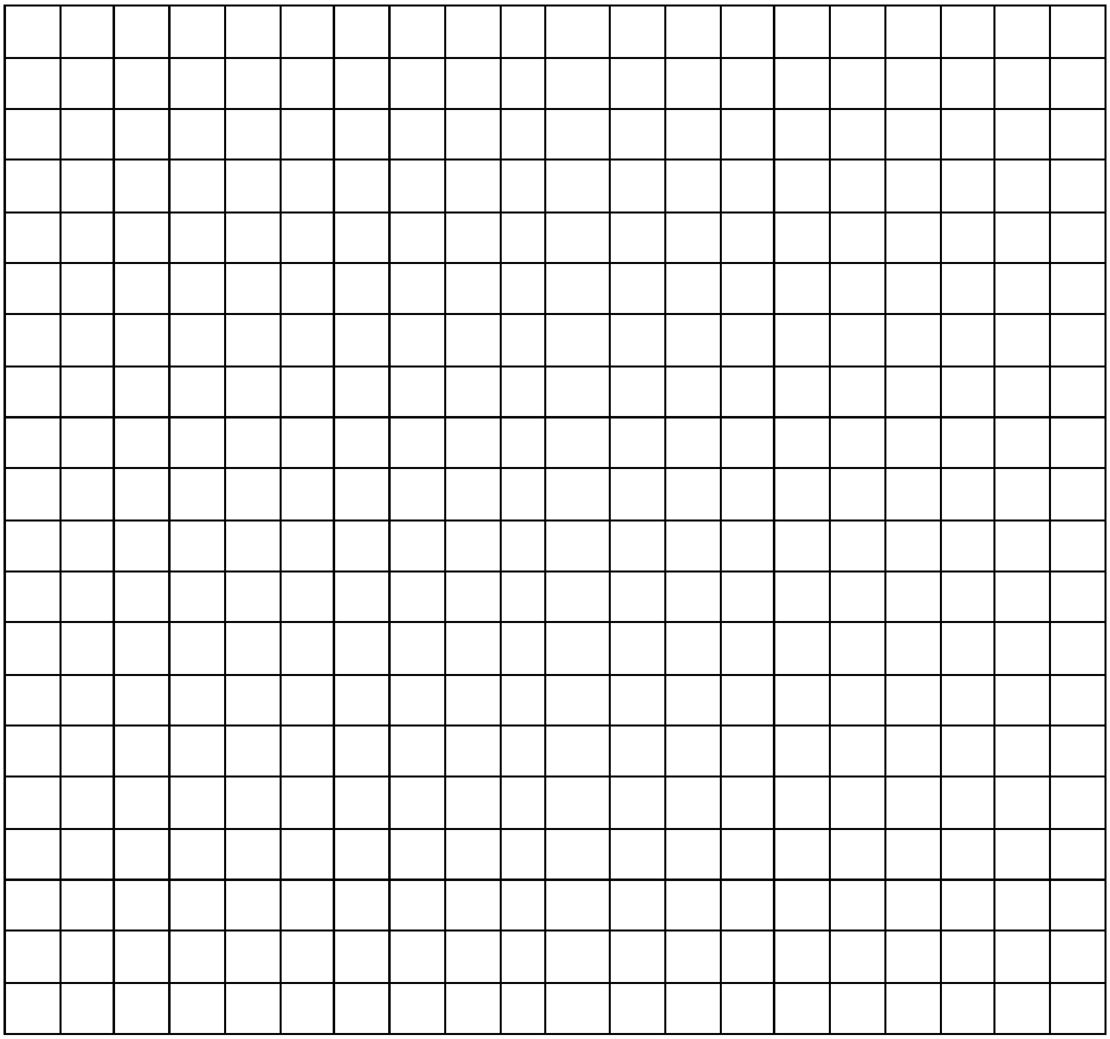
If the student divides the trees by four, she/he must divide the area of the orchard by four

$$\frac{64^2 \div 4}{4\ trees \div 4} = \frac{?^2}{1\ tree}$$

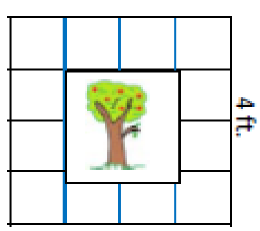


Orchard #5 is 20ft x 20ft

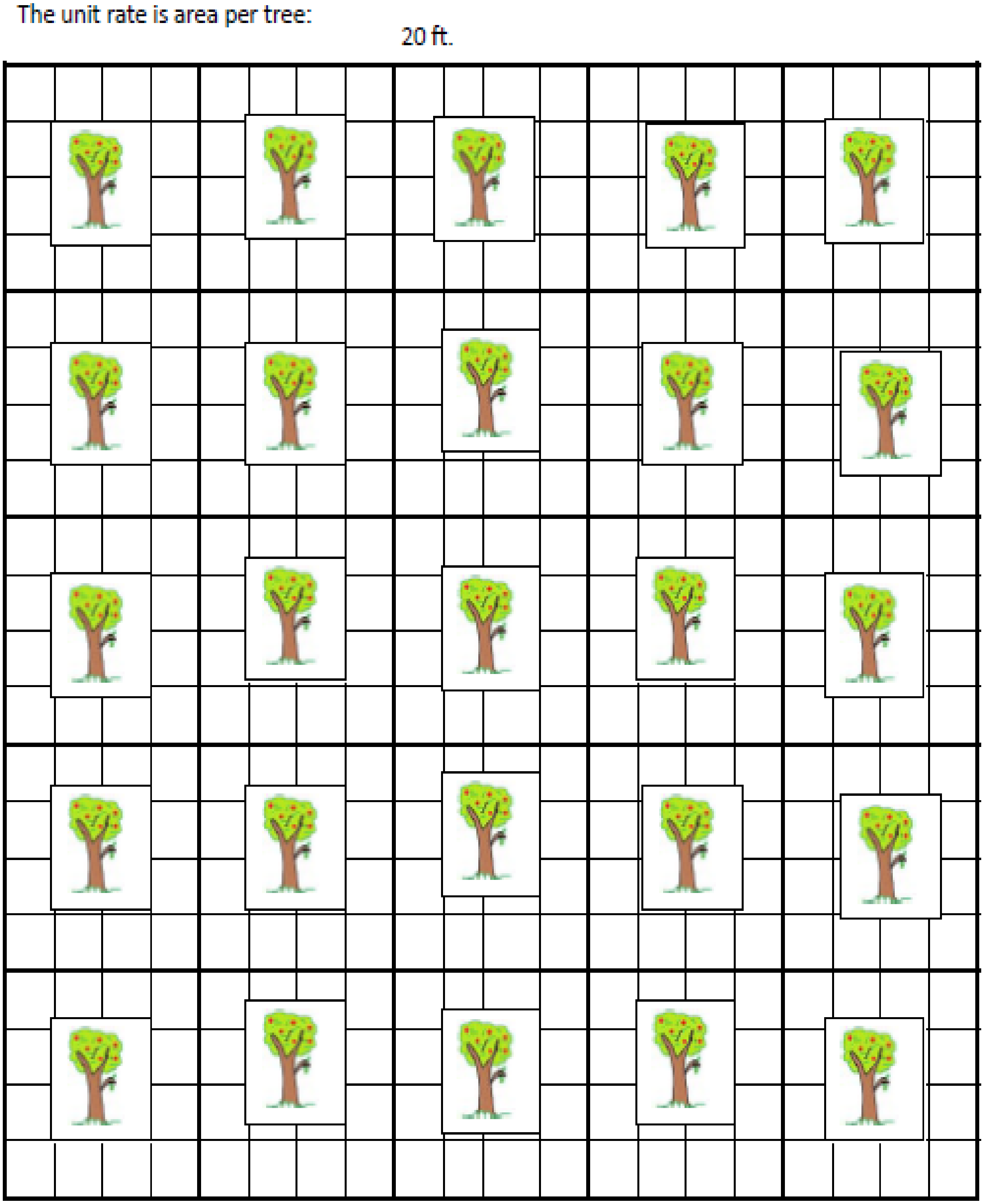
20 ft.

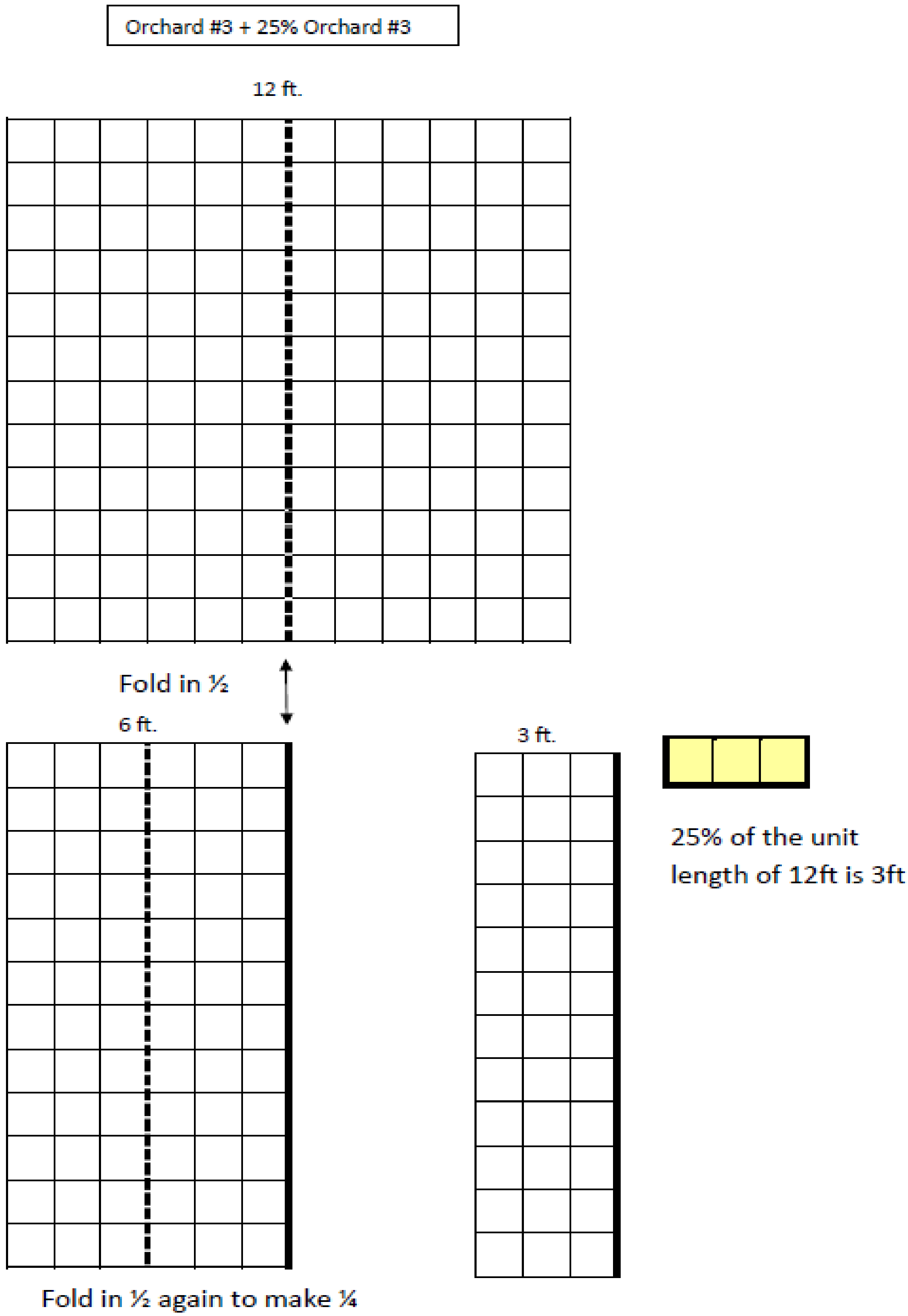


Unit rate 16ft^2 per tree

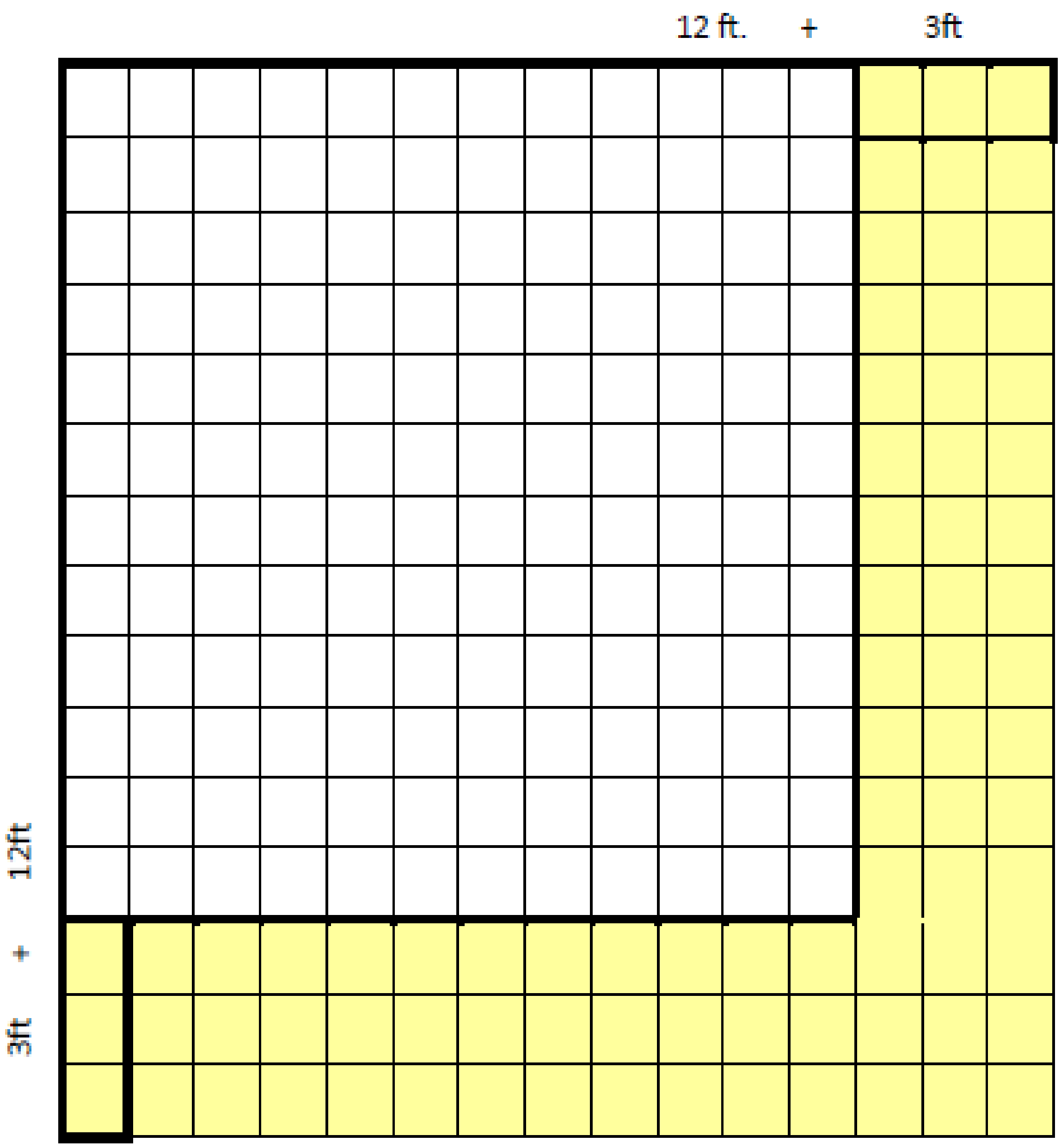


The unit rate is area per tree:




Orchard #3 + 25% Orchard #3
12 ft.
Fold in ½
6 ft.
3 ft.
25% of the unit
length of 12ft is 3ft
Fold in ½ again to make ¼

Orchard #3 + 25% Orchard #3



Lesson

6

Grade Span: 9 - 10	**Content Area:** Investigating Measurement in the Real World – *Culminating Activity*
Lesson 6 of the Unit	**Approximate Time Needed:** 45 minutes

Objectives:

- Identify and quantify attributes of the problem that need to be measured.
- Determine a pattern.
- Generalize relationships.
- Determine percent of increase/decrease.
- Determine the precision of measurement.

Essential Questions:

1. What are the relationships among the measurements of dimensions, area, and perimeter in problem solving situations?
2. How can we use variable expressions to reflect relationships?
3. How do we determine limitations of measurement

Materials Needed:

- Large and small grid graph paper
- Worksheets

Lesson Vocabulary

Area
Centimeter
Conversion
Foot
Inch
Length
Meter
Perimeter
Proportion
Ratio
Rectangles
Similar Rectangles
Unit of Measure
Unit Rate
Width
Yards

Lesson 6: Culminating Activity - 45 minutes

CULMINATING ACTIVITY FOR THE UNIT.

A. Revisit/Review Unit and Lesson Objectives

Remind students that throughout these lessons they were to make decisions about units and scales that are appropriate for problem solving situations within mathematics or across disciplines or contexts, and:

1. Convert units using standard/known conversion units.
2. Use appropriate known formulas for the area.
3. Solve multistep problems involving one unit of measure.
4. Set up and solve proportions.
5. Convert units of measurement using standard/known conversions.
6. Recognize when to multiply and when to divide in converting measurements.
7. Use ratio and proportion to convert measurements.
8. Use appropriate known formulas for area.
9. Identify, quantify, and compare the attributes of the objects, situations, and/or events that need to be measured to solve the problem/situation.
10. Use appropriate units of measure to identify, quantify, and compare objects, situations, and/or events to solve a real world problem.
11. Convert units when necessary.

Conduct a class discussion on which skills were used to solve different types of problems. Discuss the additional strategies they used to implement the skills and solve the problems.

Multiple means of representation: Along with posted lesson objectives in the classroom, students may refer to their individual copies of the objectives and their mathematics journals.

Multiple means of expression: Students share what they have learned or strategies they have used by showing different models, pictures, drawings, etc. used throughout the lessons.

Multiple means of engagement: Share ideas of how these skills have been useful in solving the problems from previous lessons and what strategies were the most helpful.

Additional Considerations for Emerging Readers and Emerging Communicators

1. When reviewing the expected outcomes, have students refer to the lesson objectives they recorded in their mathematics journals or their electronic picture versions.
2. Students use the information recorded in their journals to refer back to the lesson objectives' key words paired with images. From that information, they share what they have learned based on each of the expectations.
   - For example, the student may grab the tactile cue for area to state, "I have learned that the area is all the space measured within a figure."

3. Students refer to examples of their work to demonstrate how the skills and strategies were used.
    - For example, a student could touch the tactile representations for area and orchard to state, “I used the concept of area to determine how much space a tree needs to grow.”

B. Exit Assessment

Scenario:

The freshman class officers at Riverside High School are planning the annual Freshman Class Winter Dance. They have decided to hold the dance at the White Oak Country Club. The room they have reserved for the dance is carpeted, but for events such as this dance, the country club places parquet flooring over the carpet to make a dance floor.

The parquet flooring is laid in interlocking sections, and so the dance floor can be arranged to be various sizes to accommodate the number of dancers attending the event. However, the class officers need to let the country club managers know a month in advance of the dance how big to make the dance floor so the club workers will have enough time to get the flooring laid out appropriately. The class officers decide to base that decision on their latest ticket sales.

Things to Consider:

- The minimum size of the dance floor being considered by the class officers is 30 ft x 30 ft.
- The class officers assume 30 ft x 30 ft will be enough room for 100 couples to be on the dance floor at once.
- The country club workers can increase the sides of the dance floor in 5 ft increments in either direction, but the class officers want to maintain a square dance floor.
- The maximum the 30 ft sides of the dance floor can be increased is by 50%.

Tasks:

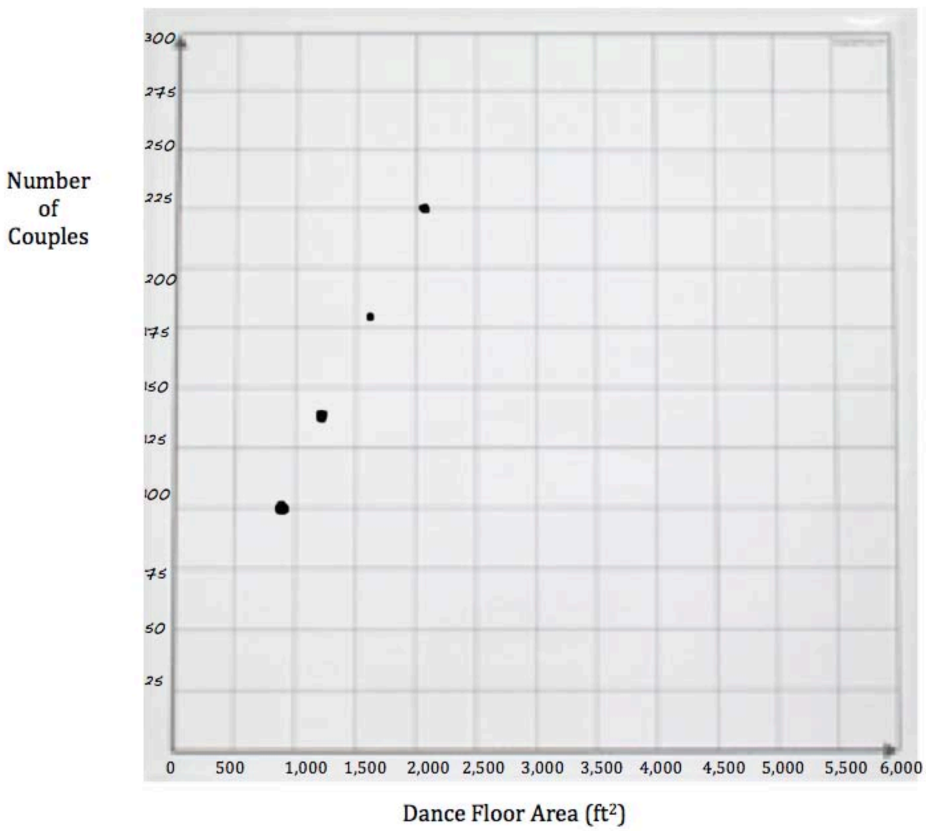
1. Determine how many square feet of dancing space each couple would have if the dance floor is 30 ft x 30 ft (i.e., 30 ft x 30 ft = 900 ft^2 and 900 ft^2 ÷ 100 couples = 9 ft^2 for each couple).
2. Determine the maximum length of the sides of the dance floor (i.e., 30 ft x 50% = 15 ft and 30 ft + 15 ft = 45 ft as the maximum length for the dance floor).
3. Using *n* to represent the number of possible increases, solve for *n* having determined that the 50% increase to 30 ft would be an additional 15 ft and that the increments are made 5 ft at a time (i.e., n = 15 ft ÷ 5 ft increments = 3 increases [35, 40, and 45]).
4. Determine the area of the dance floor for each of the 3 increases, and determine the number of couples that each would accommodate (i.e., 45 ft x 45 ft = 2,025 ft^2 and 2,025 ft^2 ÷ 9 ft^2 needed per couple = enough space for 225 couples).

5. Make a chart showing all of the dance floor size possibilities as well as the number of couples that could be accommodated by each.

	Side Lengths	Area of the Dance Floor	Number of Couples
	30 ft x 30 ft	900 ft^2	100 couples
1st increase	35 ft x 35 ft	1,225 ft^2	136 couples*
2nd increase	40 ft x 40 ft	1,600 ft^2	177 couples*
maximum increase	45 ft x 45 ft	2,025 ft^2	225 couples

* 1,225 ft^2 ÷ 9 ft^2 actually = 136.1 and 1,600 ft^2 ÷ 9 ft^2 actually = 177.777778 so, there would be a little space left over but not enough for another couple.

6. Create a graph that shows the relationship between the increase in the size of the dance floor and the number of couples who can attend the dance. Make sure to scale and label your axes (i.e., (x,y) where x = dance floor area and y = number of couples).



7. Given that 165 couples' tickets have been sold, suggest how large the class officers should tell the country club managers the floor will need to be and explain why it should be 40' x 40' because that area can accommodate up to 177 couples whereas the 35' x 35' could only accommodate 136 couples.

Multiple means of representation: Allow students to refer to their mathematics journals and other notes as they solve the problem. Provide students with a copy of the word problem and the table. Have drawings and manipulatives available for students to use.

Multiple means of expression: Allow students to solve the problem using formulas and/or models and record information into the tables using various formats: computer, paper pencil, drawings, etc.

Multiple means of engagement: Ensure all students are actively involved in solving the problem. Encourage students to consider options for solving the problem that will engage them. Use questioning to encourage students to explain their strategies.

Additional Considerations for Emerging Readers

1. Provide the written problem to students paired with picture symbols and/or tactile cues as well as the things to consider list.
2. Allow students to use a text reader to initially access the problem and to go back and review the problem as needed.
3. Provide the written questions (1-7) to students paired with picture symbols and/or tactile cues.
4. Provide students with a variety of formulas, including the ratio for the unit rate ($\frac{area\ of\ floor}{\#\ of\ couples}$), area, and perimeter as well as a copy of the table below.
5. Students should have grid paper to create a model of the area of the dance floor as well as manipulatives representing both variables (area of dance floor and number of couples).
6. Students first need to determine the area of the dance floor by choosing and using the formula for area.
7. Students need to determine the unit rate.

8. Students use the formula chosen from options ($\frac{area\ of\ floor}{\#\ of\ couples} = \frac{area\ of\ floor}{1\ couple}$; $\frac{900\ ft^2}{100} = \frac{area\ of\ floor}{1}$), or manipulatives, etc.

	Side Lengths	Area of the Dance Floor	Number of Couples	Unit Rate area/couple
	30 ft x 30 ft	(Question #1)	100 couples	
1st increase				
2nd increase				
3rd increase				

9. Determine the maximum length of the sides of the dance floor (i.e., 30 ft x 50% = 15 ft and 30 ft + 15 ft = 45 ft as the maximum length for the dance floor).
10. Be sure students have the following “things to consider” in picture format and or electronic text reader to refer to when answering question 2:
    - The country club workers can increase the sides of the dance floor in 5 ft increments in either direction, but the class officers want to maintain a square dance floor.
        - Students create a unit length representing 5 ft or are given choices of various unit lengths, one of which is 5 ft that can be used to extend the dance floor.
        - Students could also have a template that represents + 5 to use to complete the column in the chart on floor lengths.
    - The maximum the 30 ft sides of the dance floor can be increased is by 50%.
11. Provide students with a variety of strategies for determining half of 30 (divide by two, fold representation of 30 ft x 30 ft dance floor in half and count half side length, etc.)
12. Using *n* to represent the number of possible increases, solve for *n* having determined that the 50% increase to 30 ft would be an additional 15 ft and that the increments are made 5 ft at a time (i.e., n = 15 ft ÷ 5 ft increments = 3 increases [35, 40, and 45]).
13. Students may use a drawing of the 30 ft x 30 ft dance floor on grid paper and divide it evenly in half to determine what 50% more would be.
14. Students use the 5 ft unit length template to determine how many increments of 5 the floor can be increased and what the measurement would be for each increase (i.e. 30 ft + 5 ft = 35 ft; 35 ft + 5 ft = 40 ft, etc.). Students may also determine the increase by counting by 5s from 30.
15. Determine the area of the dance floor for each of the 3 increases and determine the number of couples that each would accommodate (i.e., 45 ft x 45 ft = 2,025 ft^2 and 2,025 ft^2 ÷ 9 ft^2 needed per couple = enough space for 225 couples).

a. Students must remember to increase both the length and width by 5 ft and use the formula and/or manipulatives to determine the area of each floor increase.

16. Using the unit rate determined in step 1, students choose the correct given formula to use to determine how many couples can be accommodated for each increase, and/or students use manipulatives of the different floor measurements and the unit rate to determine how many couples can dance per floor size.
17. Make a chart showing all of the dance floor size possibilities as well as the number of couples that could be accommodated by each.
18. Students can be given the above chart to complete from results found in steps 1, 2, and 3.
19. Create a graph that shows the relationship between the increase in the side length of the dance floor and the number of couples that can attend the dance.
    - Students should scale and label the coordinate grid (first quadrant), using the side length of each dance floor, to keep the numbers manageable. (i.e., (x,y) where x = side length of the dance floor and y = number of couples; where the scale of x increases by 5's to 50 and the scale of y increases by 25 to 250).
    - Students can be given choices of scaled and labeled coordinate grids and choose the best representation for the problem and then plot the points on the chosen grid.
    - Students should use the table to determine the ordered pairs needed to plot the points.
    - Students identify and highlight the two variables used for graphing (side length and number of couples).
20. Finally, given that 165 couples' tickets have been sold, suggest how large the class officers should tell the country club managers the floor will need to be and explain why the dance floor should be 40' x 40' because that area can accommodate up to 177 couples whereas the 35' x 35' could only accommodate 136 couples.
    - Students should use their table and graph to determine the appropriate floor size.
    - Students explain using cloze sentences (e.g., The dance floor should be ________________ because that area can accommodate up to _______ couples whereas the _____________ could only accommodate ________ couples.)

Additional Considerations for Emerging Communicators

. Provide the written problem to students paired with picture symbols and/or tactile cues.

- Allow students to use a text reader to initially access the problem and to go back and review the problem as needed.
- Provide models of the initial problem as well as for the things to consider list.
- Modify the things to consider section to use smaller numbers for the side lengths and number of couples to start the problem.

Things to Consider:

- The minimum size of the dance floor being considered by the class officers is 12 ft x 12 ft.
- The class officers assume 12 ft x 12 ft will be enough room for 16 couples to be on the dance floor at once.
- The country club workers can increase the sides of the dance floor in 2 ft increments in either direction, but the class officers want to maintain a square dance floor.
- The maximum the 12 ft sides of the dance floor can be increased by is 50%.

Tasks:

1. Provide students with a variety of formulas, including the ratio for the unit rate ($\frac{area\ of\ floor}{\#\ of\ couples}$), area, and perimeter as well as a copy of the table.
2. Students should have grid paper to create a model of the area of the dance floor in question one as well as manipulatives representing both variables (area of dance floor and number of couples).
3. Students first need to determine the area of the dance floor by choosing and using the formula for area.

	Side Lengths (x)	**Area of the Dance Floor**	**Number of Couples (y)**	**Unit Rate (Area/couple)**
	12 ft x 12 ft	(Question #1)	16 couples	
1st increase				
2nd increase				
3rd increase				

4. Determine the maximum length of the sides of the dance floor (i.e., 12 ft x 50% = 6 ft and 12 ft + 6 ft = 18 ft as the maximum length for the dance floor).
5. Be sure students have the following things to consider in picture format and/or electronic text reader to refer to when answering question 2:
   - The country club workers can increase the sides of the dance floor in 2 ft increments in either direction, but the class officers want to maintain a square dance floor.
   - Students can be given choices of various unit lengths, one of which is 2 ft that can

be used to extend the dance floor.

- The maximum the 12 ft sides of the dance floor can be increased by is 50%.
- Provide students with a variety of strategies for determining half of 12 (divide by two, fold representation of 12 ft x 12 ft dance floor in half and count half side length, etc.).

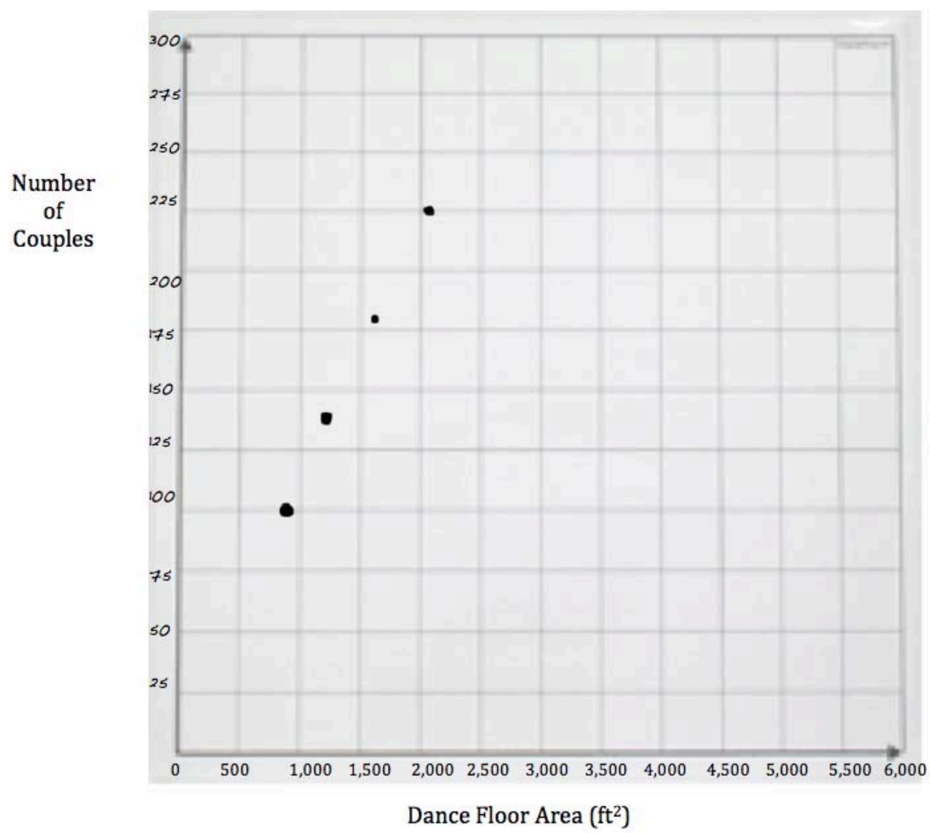
6. Using *n* to represent the number of possible increases, solve for *n* having determined that the 50% increase to 12 ft would be an additional 6 ft and that the increments are made 2 ft at a time (i.e., n = 6 ft ÷ 2 ft increments = 3 increases [14, 16, and 18]).
7. Determine the area of the dance floor for each of the 3 increases, and determine the number of couples that each would accommodate (i.e., 18 ft x 18 ft = 324 ft^2 and 324 ft^2 ÷ 9 ft^2 needed per couple = enough space for 36 couples).
8. Students must remember to increase both the length and width by 2 ft and use manipulatives and/or computer with virtual manipulatives to determine the area of each floor increase.
9. Using the unit rate determined in step 1, students should us the manipulative/template of the different floor measurements and the unit rate to determine how many couples can dance per floor size.
10. Make a chart showing all of the dance floor size possibilities as well as the number of couples that could be accommodated by each.
11. Students can be given the above chart to complete from results found in steps 1, 2, and 3
12. Students should refer to the representations used/created in each step above and multiple choice options to complete the table.
13. Create a graph that shows the relationship between the increase in the size of the dance floor and the number of couples that can attend the dance (make sure to scale and label axes) [i.e., (x,y) where x = dance floor area and y = number of couples).
14. The graph should be scaled and labeled using the side length of each dance floor, to keep the numbers manageable. [i.e., (x,y) where x = side length of the dance floor and y = number of couples; where the scale of x increases by 2's to 20 and the scale of y increases by 2 or 4 to 40).
15. Students can be given choices of scaled and labeled coordinate grids and choose the best representation for the problem and plot the points on the chosen grid.
16. Students should use their completed table to determine the ordered pairs needed to plot the points.
17. The x and y variables can be highlighted for graphing (side length and number of couples).
18. Students identify where the points should go by first identifying the number representing the independent variable (x) and the correct corresponding number representing the dependent variable (y), or students use a computer program to graph by indicating the numbers in an order pair.

See Example: PowerPoint Lesson 5, slide 6

19. Finally, given that 32 couples' tickets have been sold, suggest how large the class officers should tell the country club managers the floor will need to be and explain why it should be 18' x 18' because that area can accommodate up to 36 couples whereas the 16' x 16' could only accommodate 28 couples.
20. Students should use their table and graph to determine the appropriate floor size.
21. Students indicate which floor size by comparing the number of couples attending to the number of couples that can fit on the dance floor and choosing the floor with immediately higher/larger number.
22. Students explain using cloze sentences (e.g., The dance floor should be ______________ because that area can accommodate up to _______ couples whereas the _____________ could only accommodate ________ couples.)

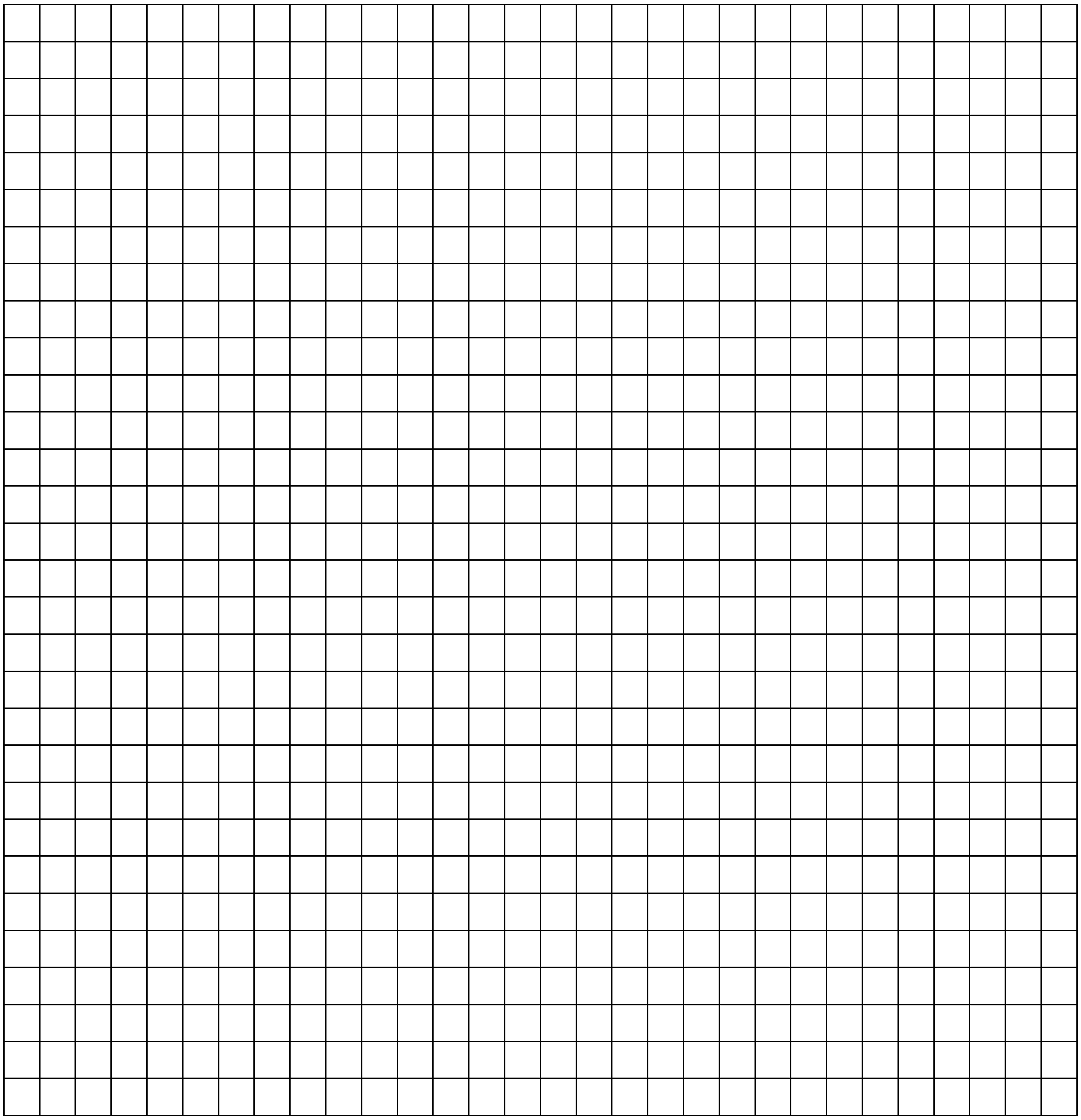
Lesson 6: Resources

	Side Lengths	Area of the Dance Floor	Number of Couples
	30 ft x 30 ft	900 ft^2	100 couples
1st increase	35 ft x 35 ft	1,225 ft^2	136 couples*
2nd increase	40 ft x 40 ft	1,600 ft^2	177 couples*
maximum increase	45 ft x 45 ft	2,025 ft^2	225 couples

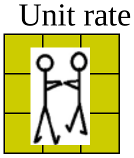


Lesson 6: Resources– continued

30 ft



Unit rate 9 ft^2 per couple



Lesson 6: Resources– continued

	Side Lengths	Area of the Dance Floor	Number of Couples	Unit Rate area/couple
	30 ft x 30 ft	(Question #1)	100 couples	
1st increase				
2nd increase				
3rd increase				

Lesson 6: Resources– continued

	Side Lengths (x)	Area of the Dance Floor	Number of Couples (y)	Unit Rate (Area/couple)
	12 ft x 12 ft	(Question #1)	16 couples	
1st increase				
2nd increase				
3rd increase				

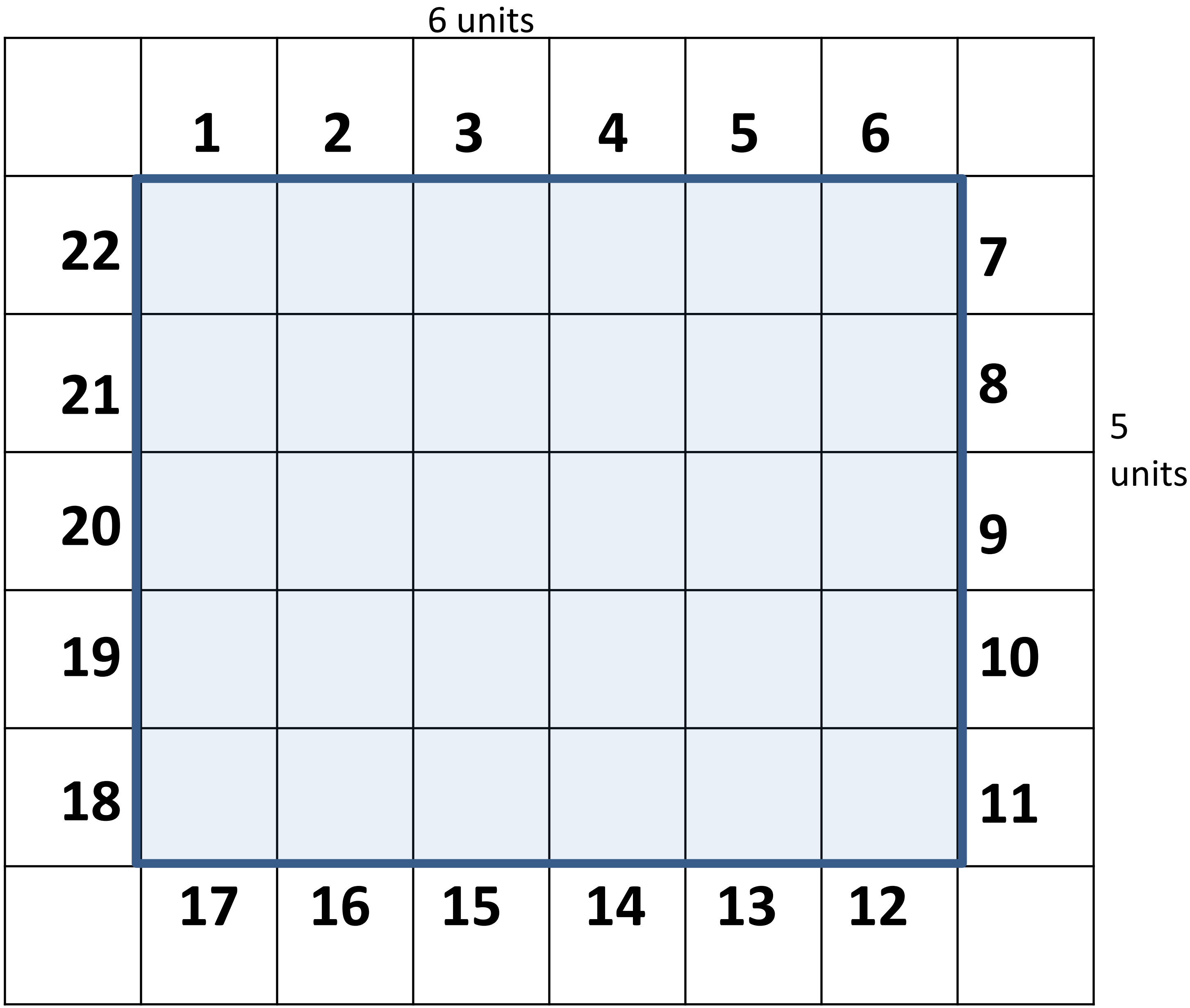
References:

National Governors Association Center for Best Practices, Council of Chief State School Officers (2010). *Common Core State Standards for Mathematics.* Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.

Hess, Karin K. (2010) Learning Progressions Frameworks Designed for Use with the Common Core State Standards in Mathematics K-12. Retrieved from http://www.nciea.org/publications/Math_LPF_KH11.pdf CCCs citation

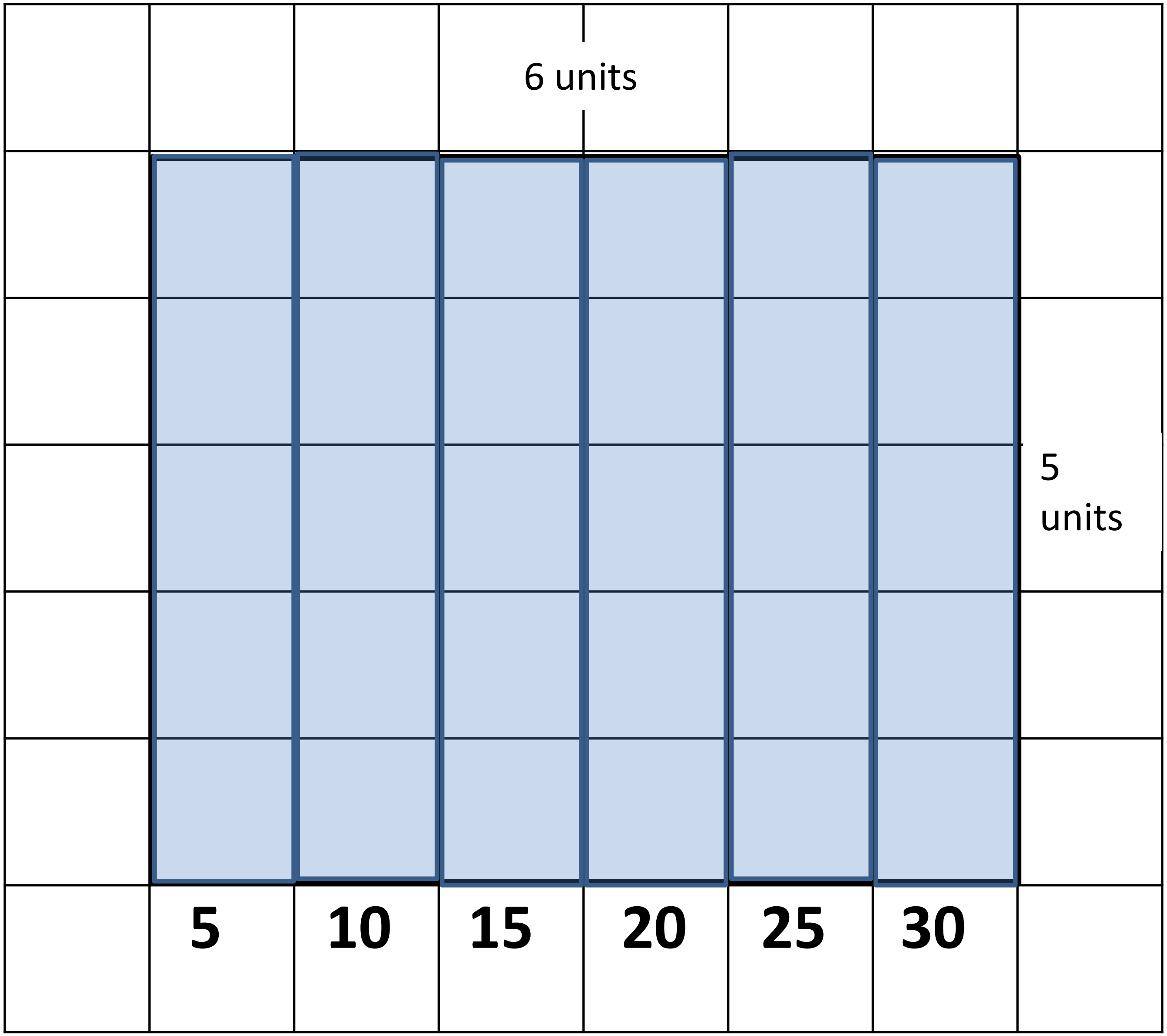
National Alternate Assessment Center. (2010). Mathematics: Measurement sample instructional unit for grades 9-10. Lexington, KY: University of Kentucky, Human Development Institute

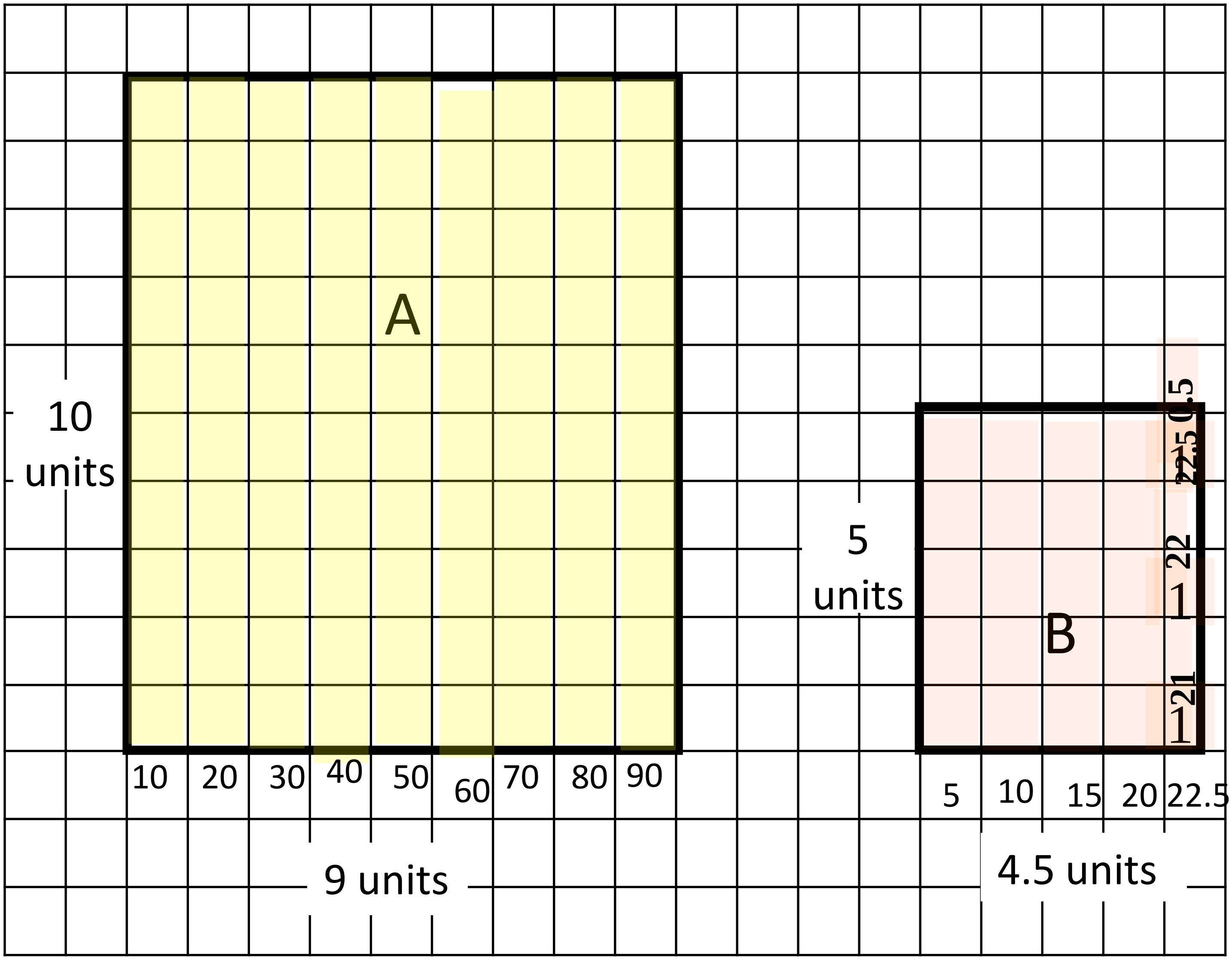
Find the perimeter of the figure below. $6u + 6u + 5u + 5u =$ **$22u$**



Find the area of the figure below.

$6u \times 5u =$ **$30u^2$**



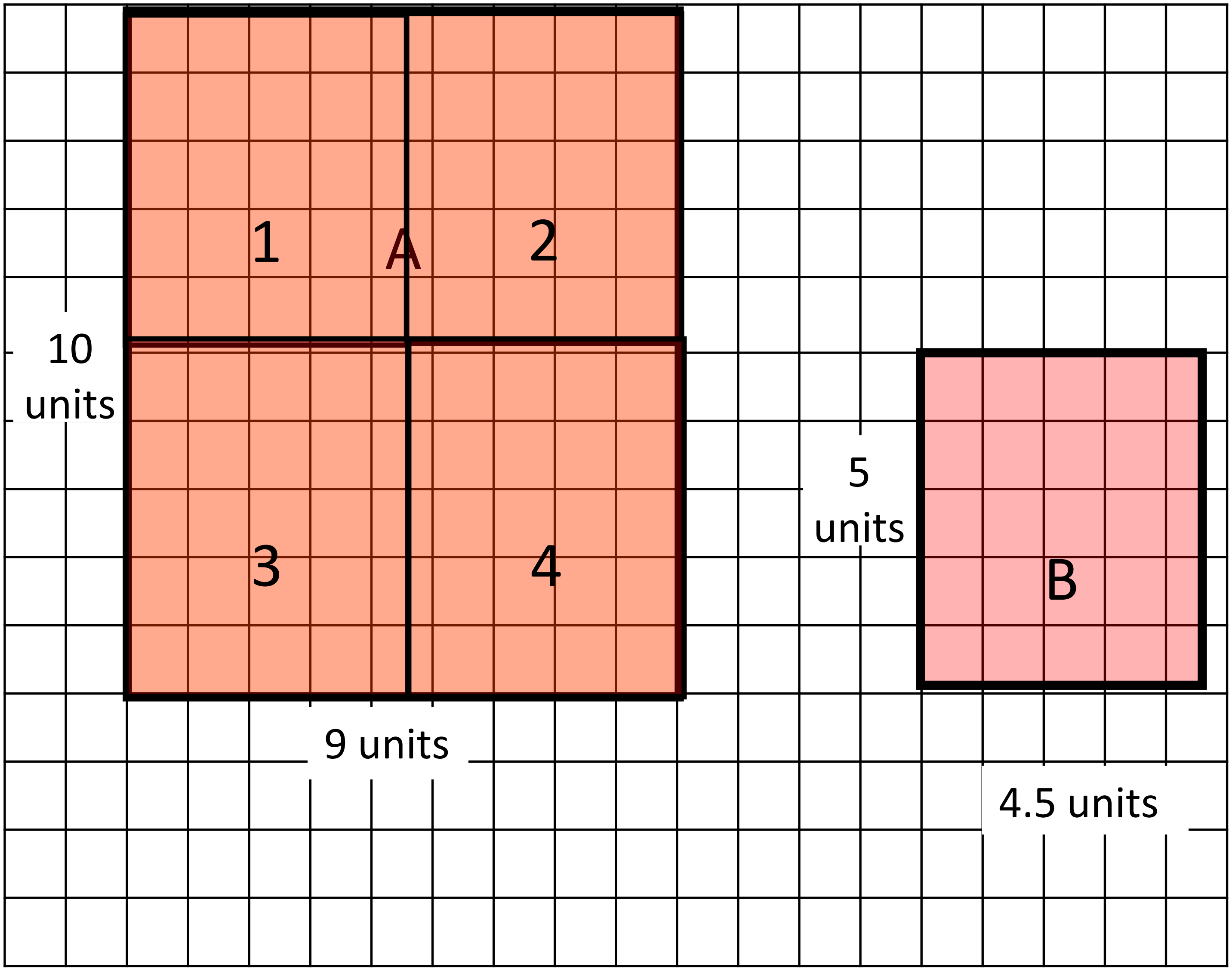


Determine the area of each rectangle.

A: $10u \times 9u = 90u^2$

B: $5u \times 4.5u = 22.5\ u^2$

Compare the area of figures A and B.

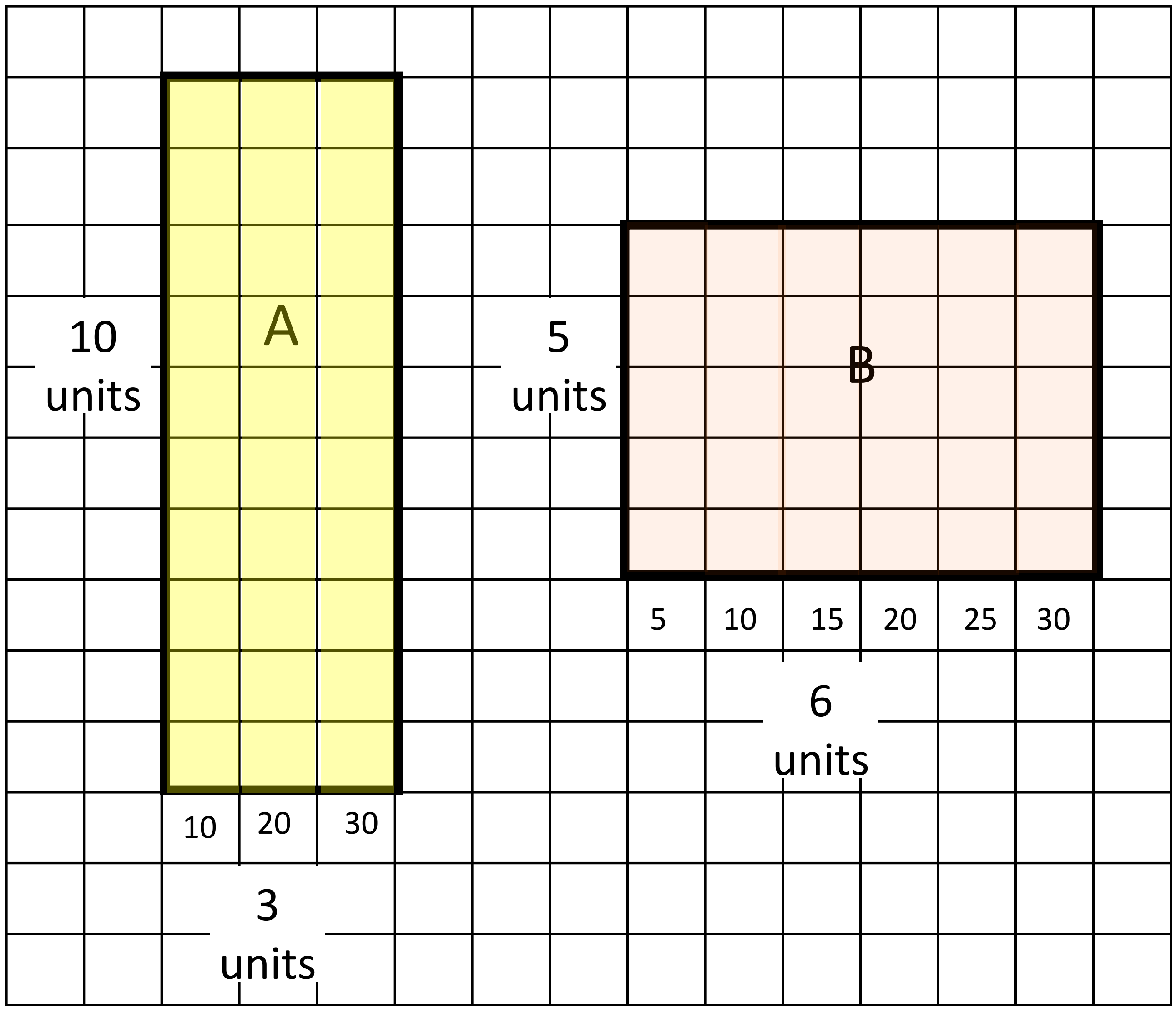


$10u \times 9u = 90u^2$

$5u \times 4.5u = 22.5\ u^2$

The area of figure A is four times as big as figure B.

Verify that the area of figures A and B are the same.



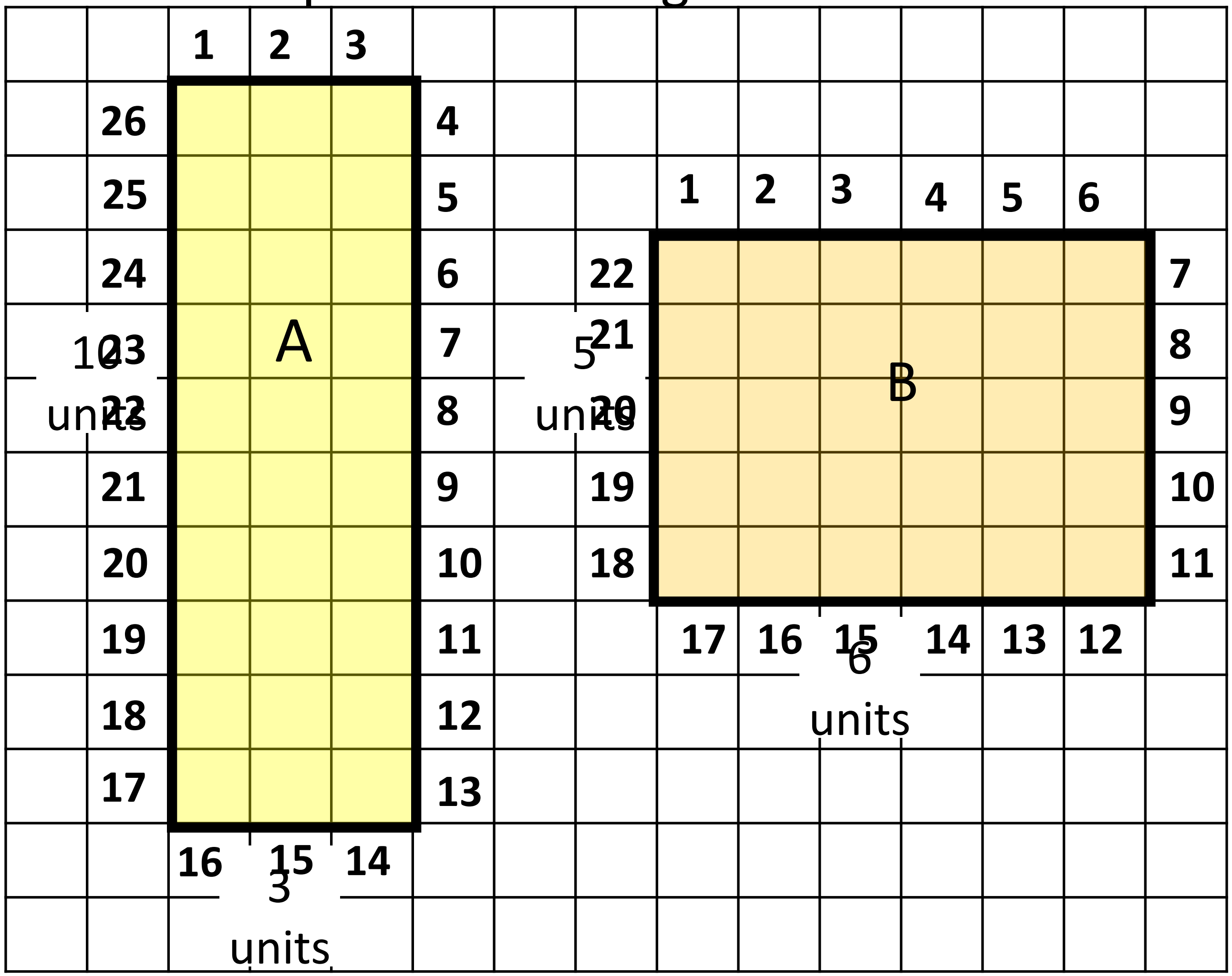
A = 10 units x 3 units

A = 30 units ^{2}

A = 5 units x 6 units

A = 30 units ^{2}

Determine if the perimeter of figures A and B are the same.



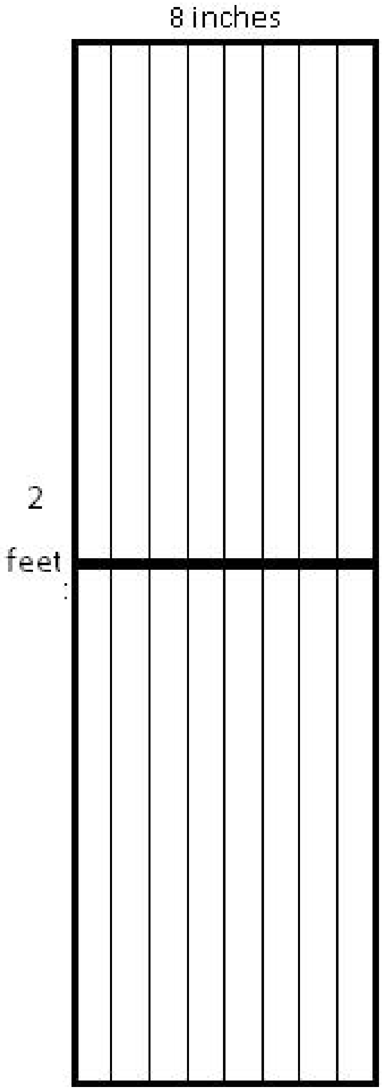
P = 10 units + 3 units + 10 units + 3 units
P = 26 units

P = 5 units + 6 units + 5 units + 6 units
P = 22 units

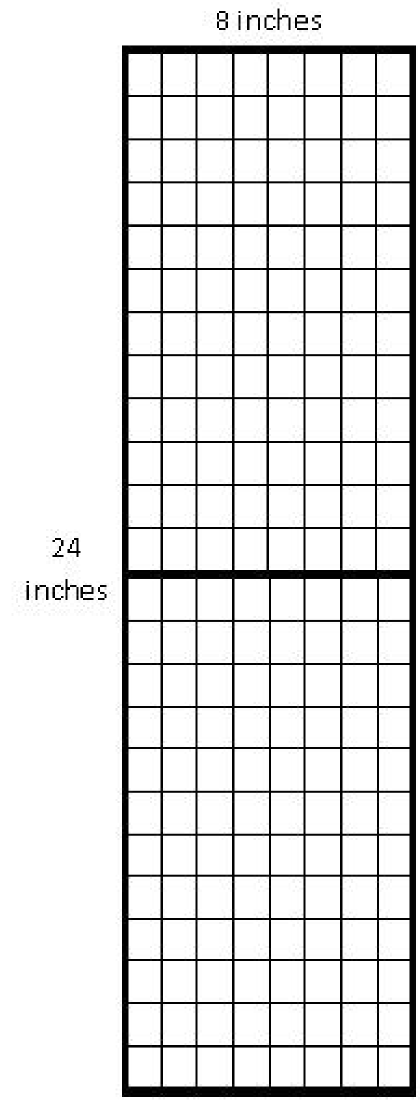
The perimeters of figures A and B are:

=	≠
same	different

Find the figure that has the measurements 8 inches by 2 feet.

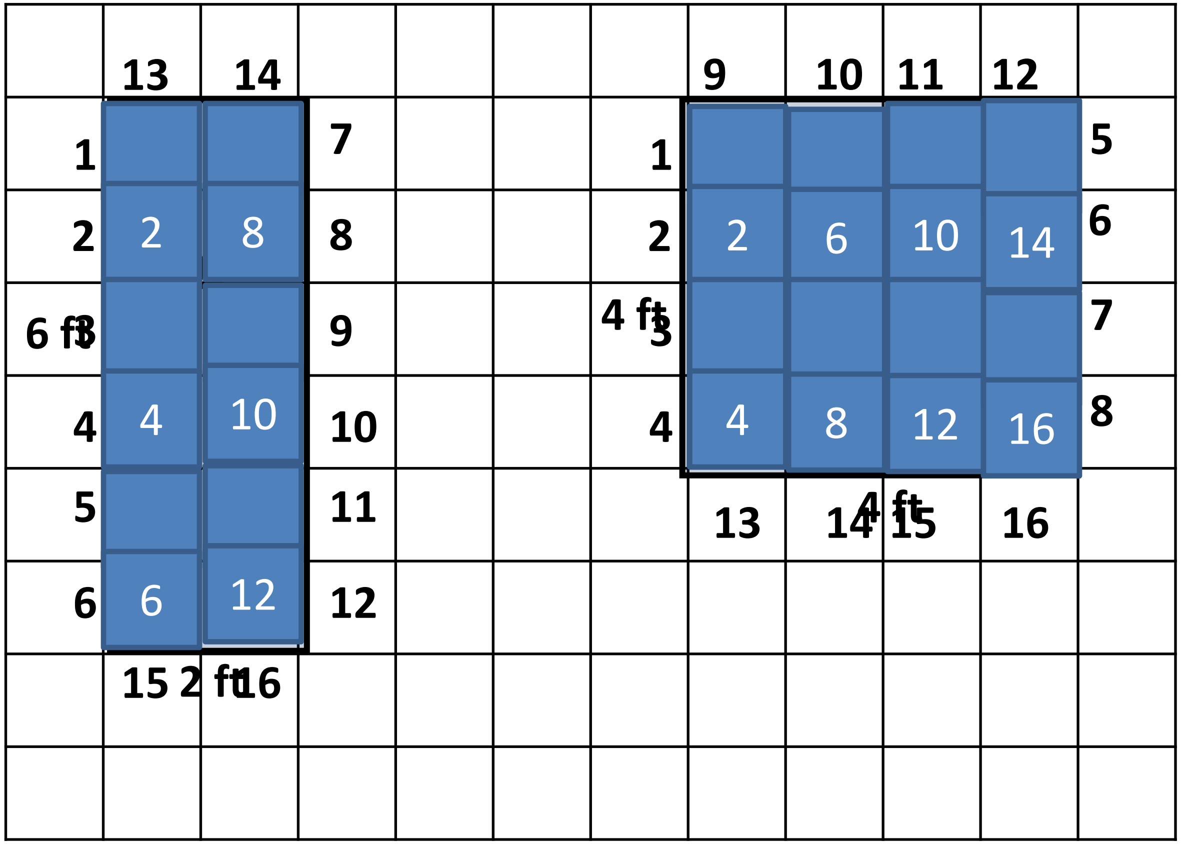


Convert the units so that inches are used for both length and width.

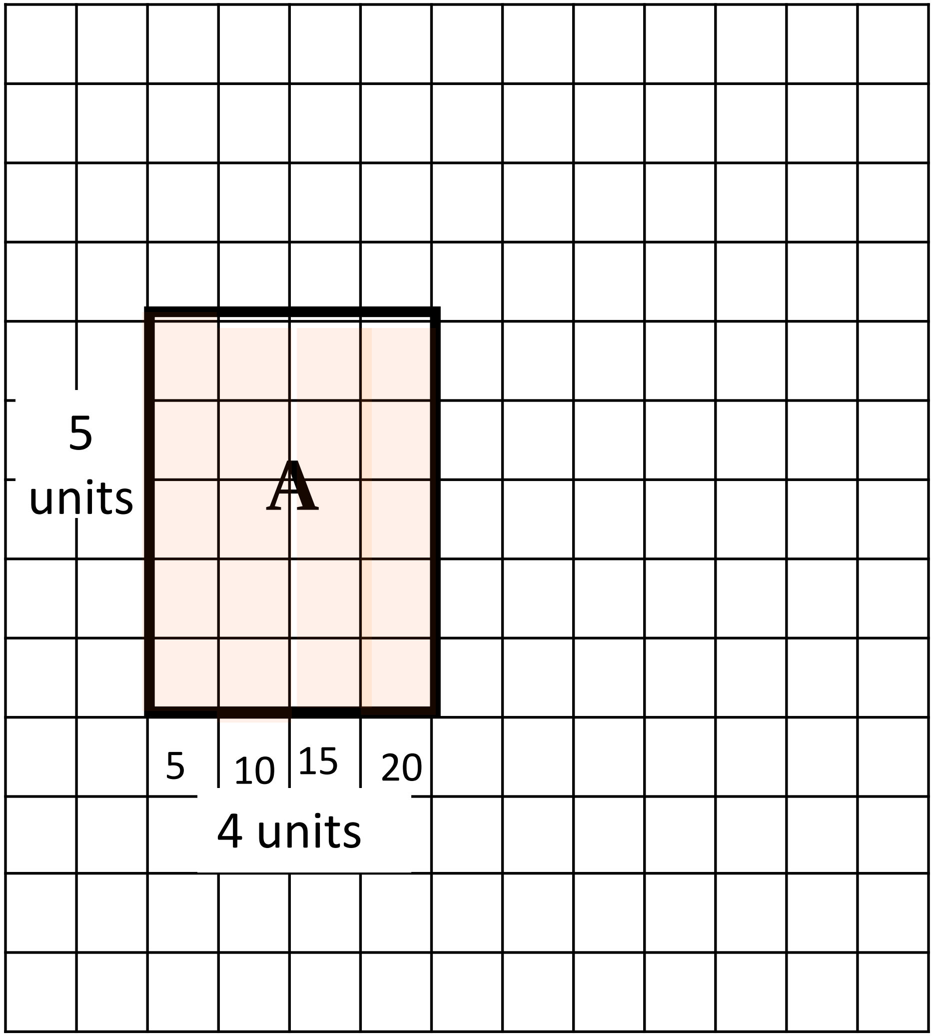


Verify the perimeter is 16 ft.

Then determine the area and record it in the table.



Model	Length	Width	Perimeter	Area
A	**6 ft.**	**2 ft.**	**16 ft.**	**12 ft.^{2}**
B	**4 ft.**	**4 ft.**	**16 ft.**	**16 ft.^{2}**

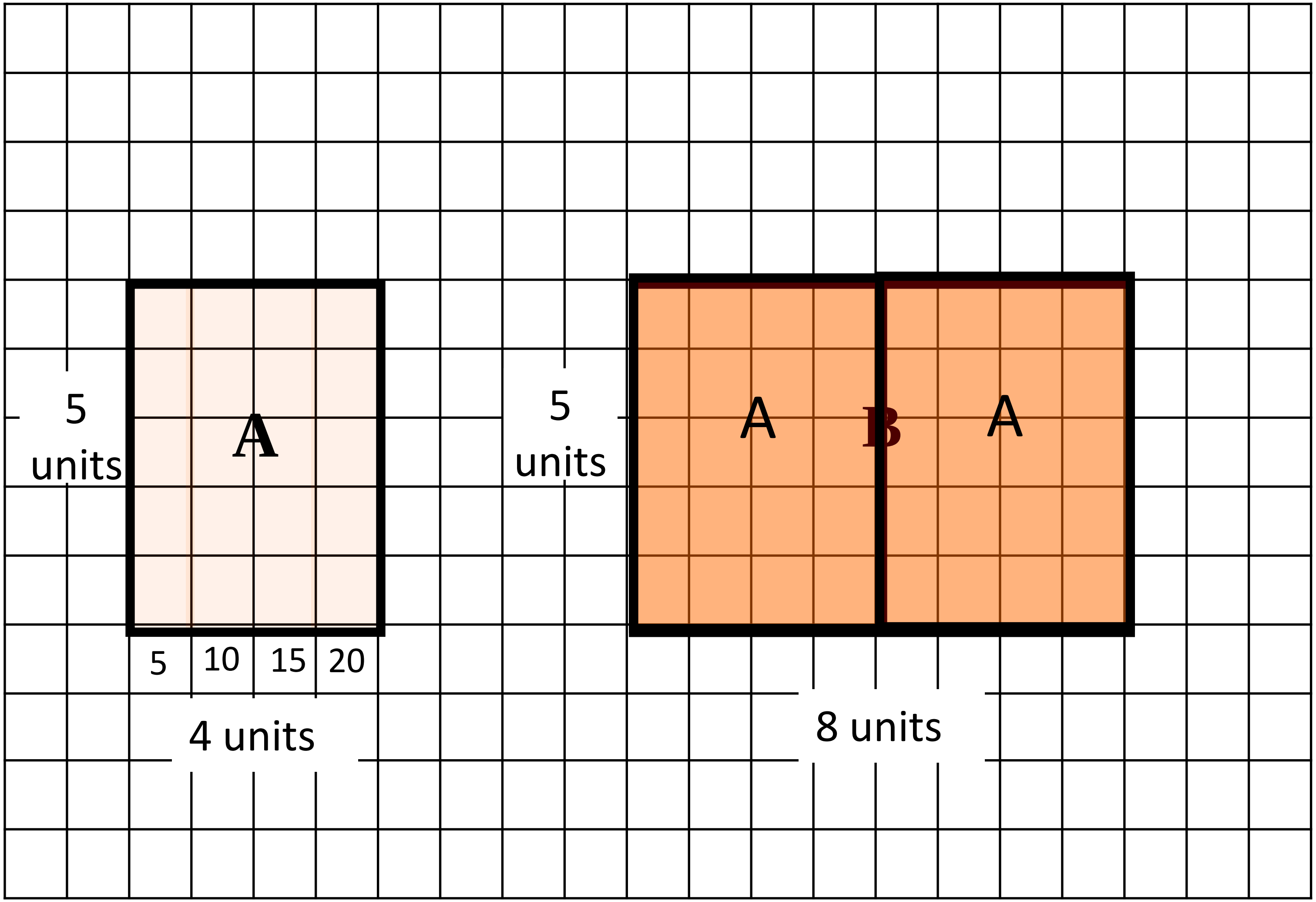


What is the ratio of Area to Width of rectangle A?

$$\frac{A}{W} = \frac{20\ u^2}{4\ u}$$

Determine the area of rectangle A.

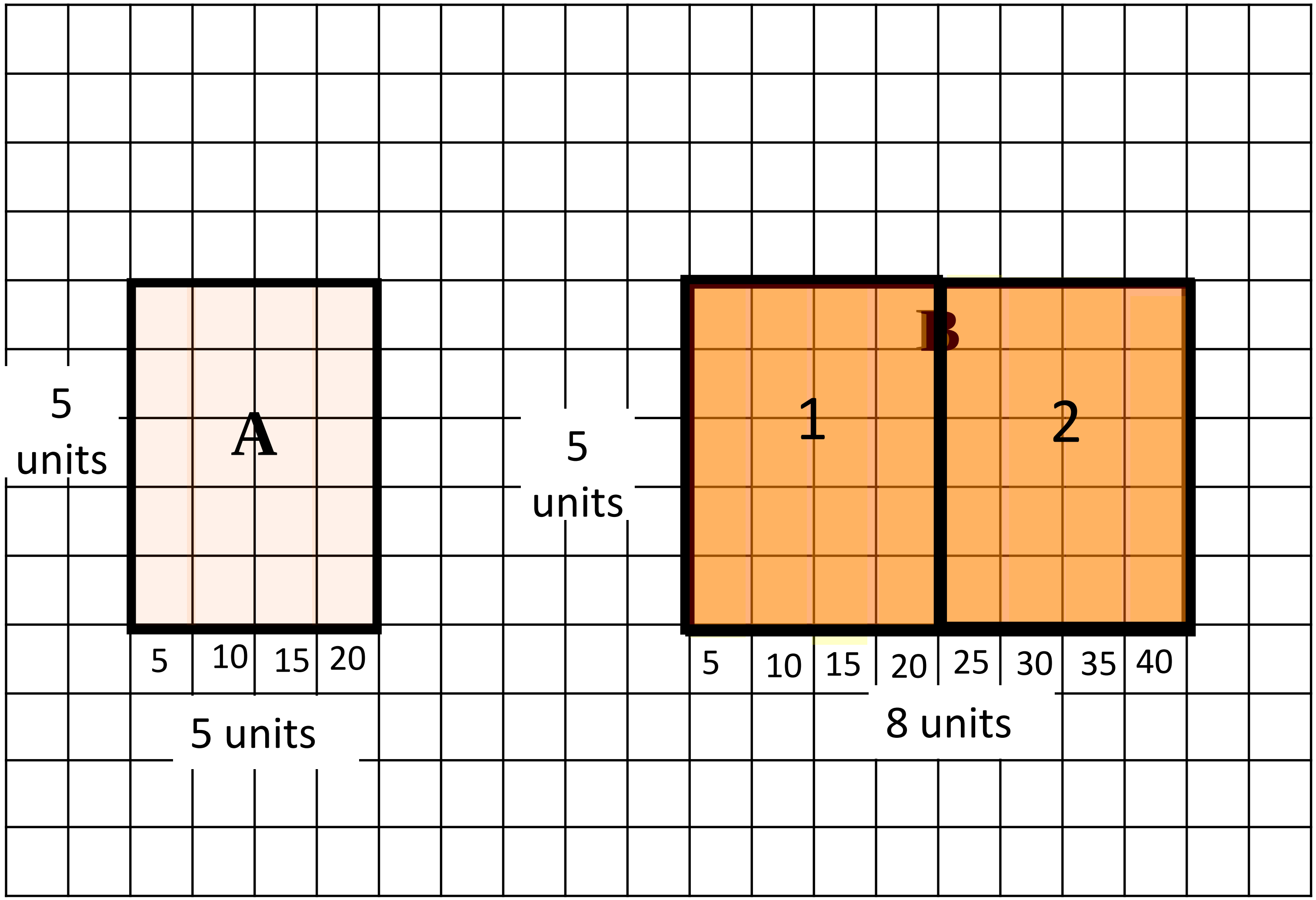
A 5u x 4u = 20 u^2



What happens to the area of rectangle A if we double the width?

If the ratio of Area to Width of rectangle A is $\frac{A}{W} = \frac{20\ u^2}{4\ u}$

Use the ratio $\boxed{A}\ \frac{A}{W} = \boxed{B}\ \frac{A}{W}$ to figure the Area of rectangle B?

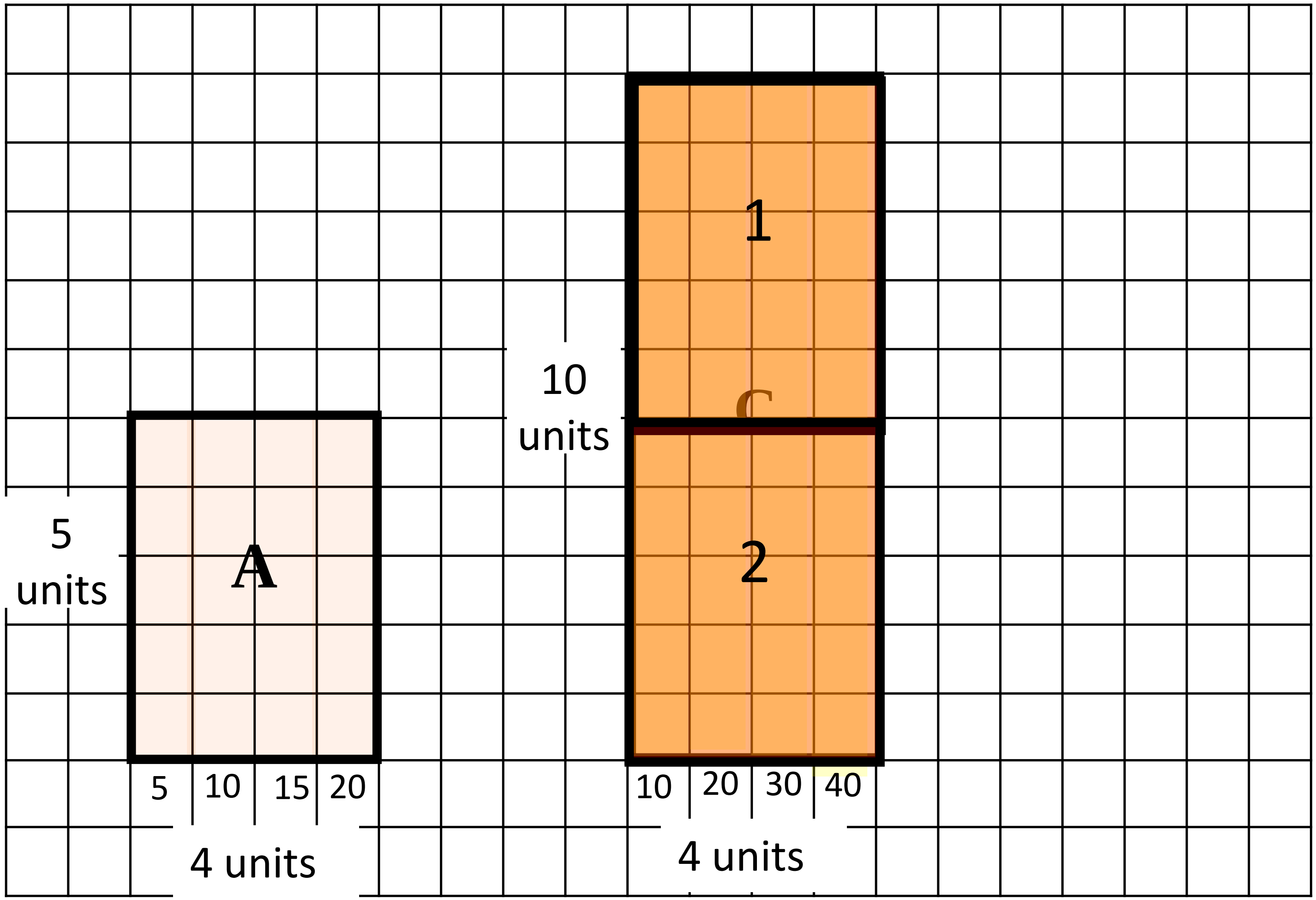


Use ratios and proportions to find the Area of rectangle B?

$$\boxed{A}\ \frac{A}{W} = \boxed{B}\ \frac{A}{W}$$

Check your answer by counting the units within rectangle B.

$$\boxed{A}\ \frac{20\ u^2 \ \times 2}{4\ u \ \times 2} = \boxed{B}\ \frac{40\ u^2}{8\ u}$$



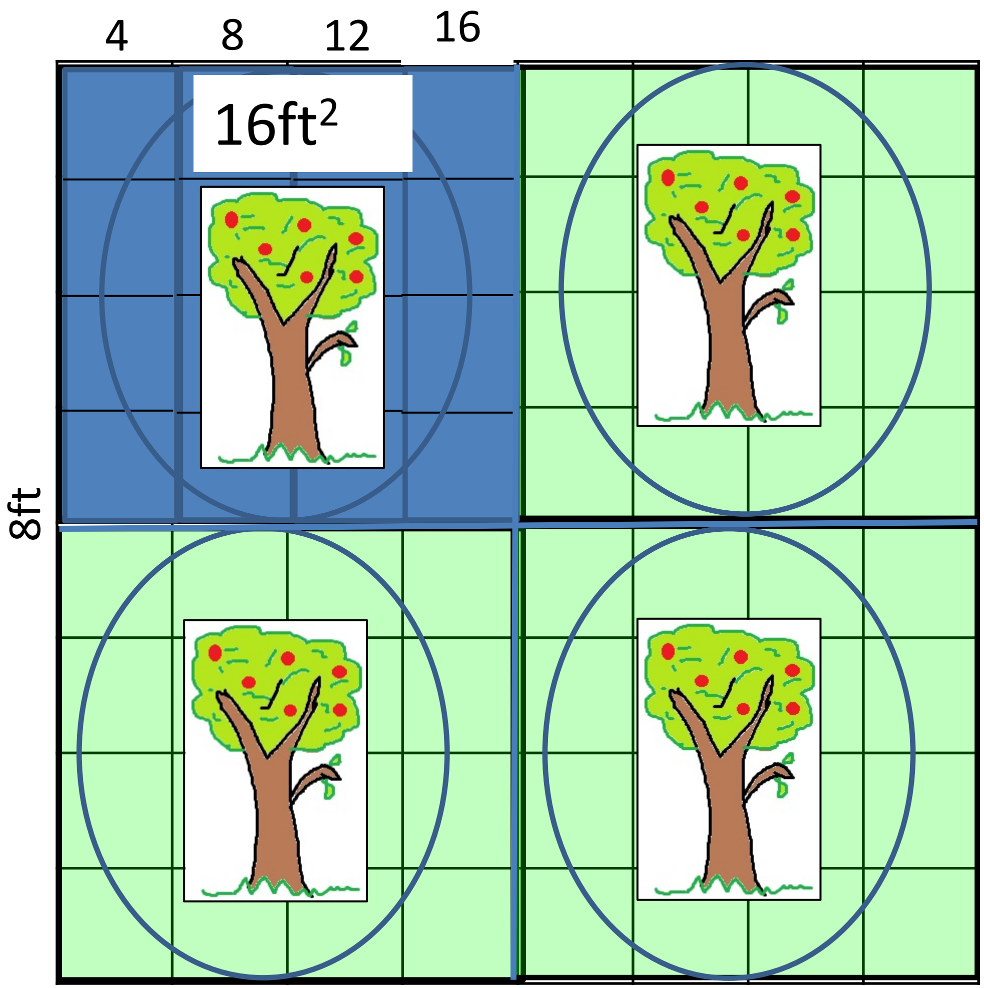
Use ratios and proportions to find the Area of rectangle B?

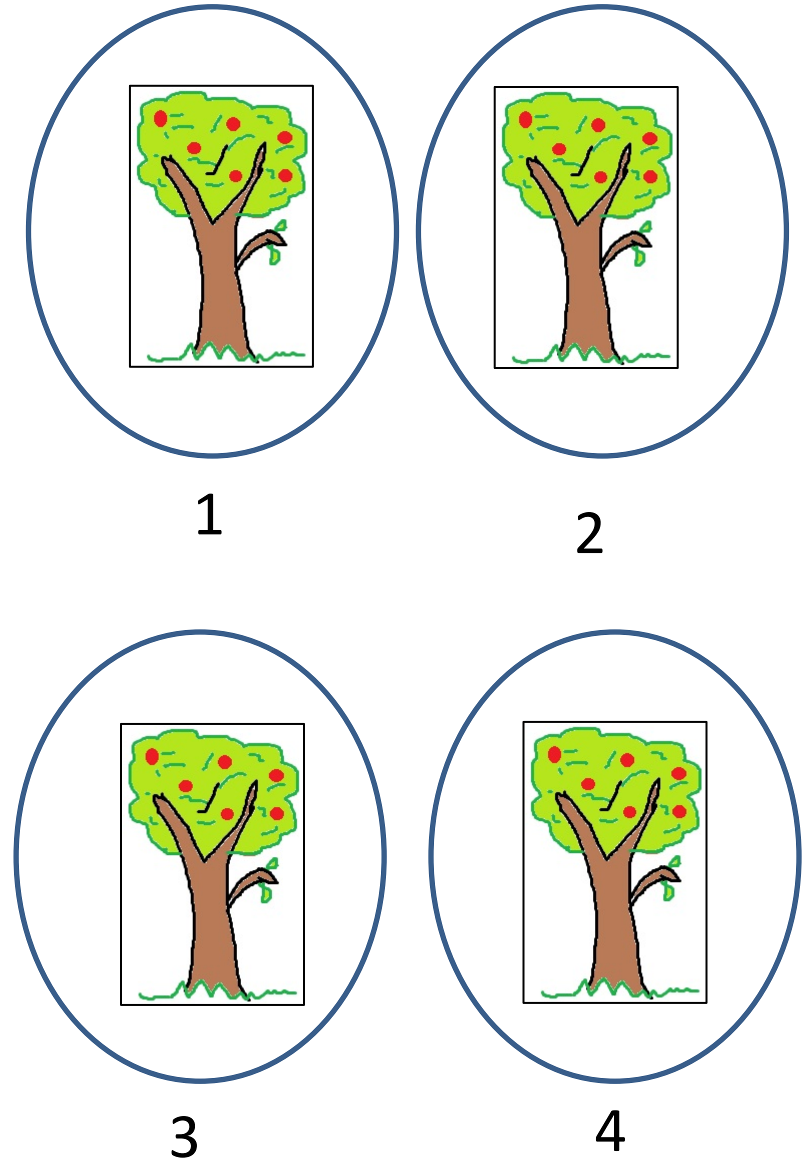
Check your answer by counting the units within rectangle B.

$$\boxed{A}\ \frac{A}{L} = \boxed{B}\ \frac{A}{L}$$

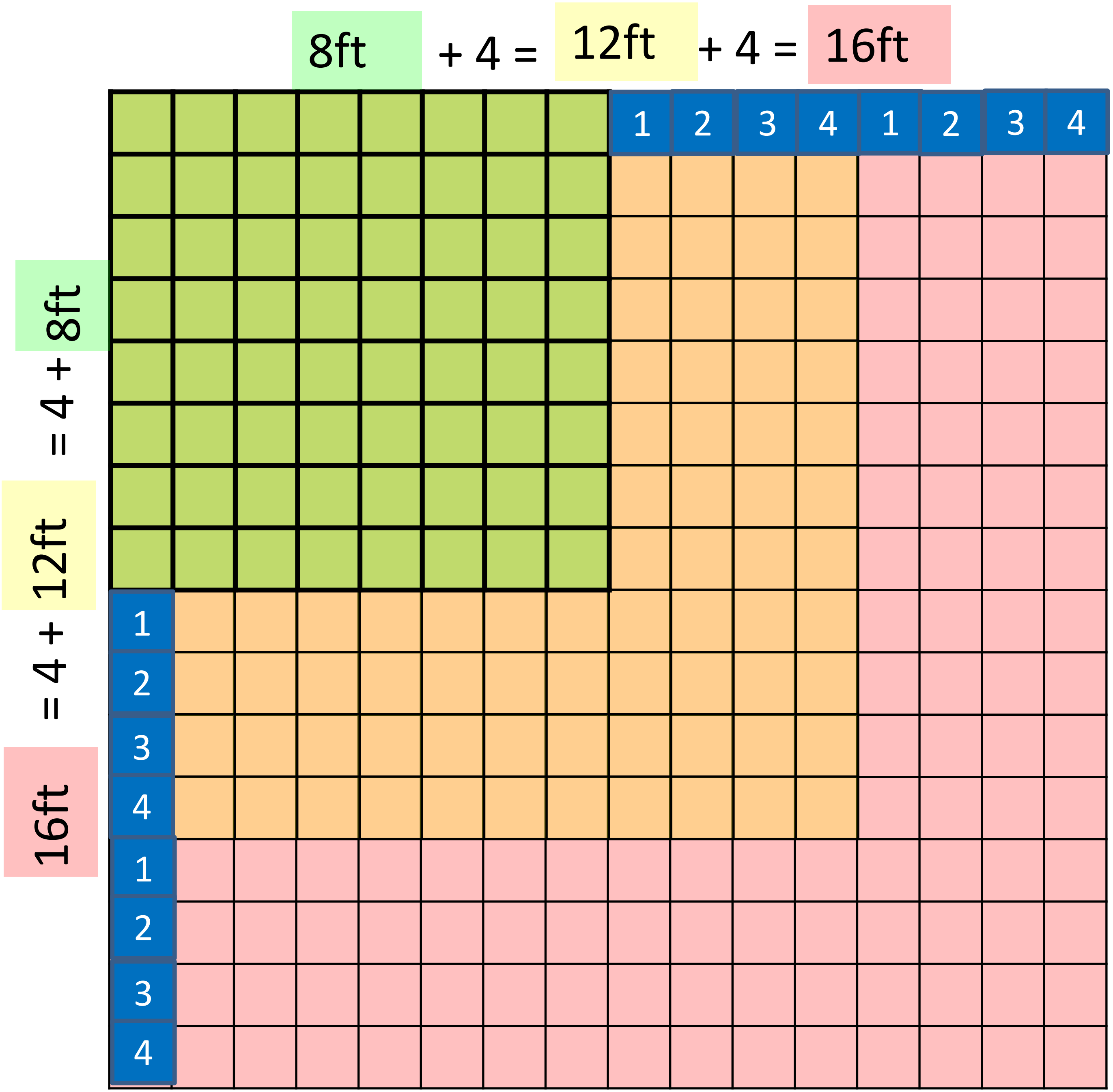
$$\boxed{A}\ \frac{20\ u^2\ \mathbf{x2}}{5\ u\ \mathbf{x2}} = \boxed{B}\ \frac{\mathbf{40\ u^2}}{10\ u}$$

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	8
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**	16
17	18	19	20	21	22	23	24	24
25	26	27	28	29	30	31	32	32
33	34	35	36	37	38	39	40	40
41	42	43	44	45	46	47	48	48
49	50	51	52	53	54	55	56	56
57	58	59	60	61	62	63	64	64





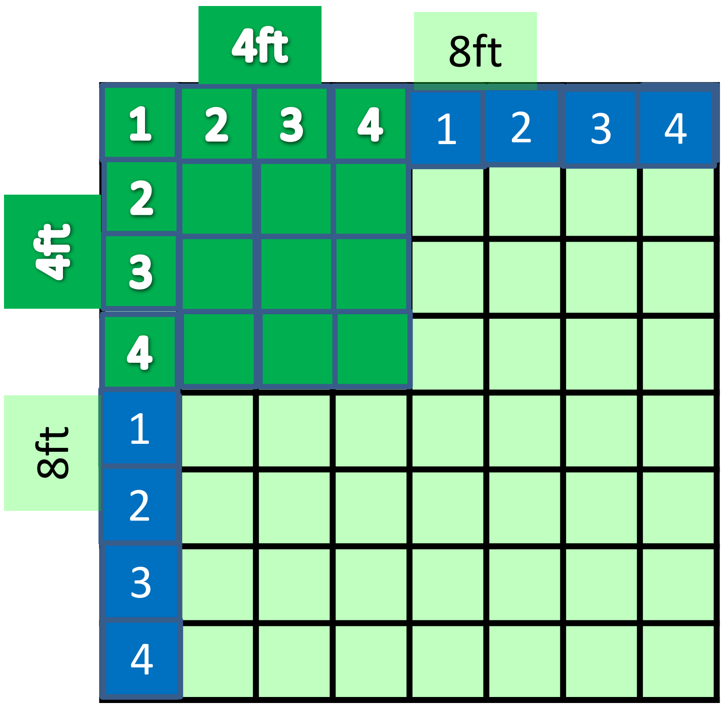
$$\frac{64^2}{4\ trees} = \frac{16ft^2}{1\ tree}$$


8ft + 4 = 12ft + 4 = 16ft
1 2 3 4 1 2 3 4
8ft + 4 = 12ft + 4 = 16ft
1 2 3 4 1 2 3 4

Length of Orchard One + 4 = Length of Orchard Two

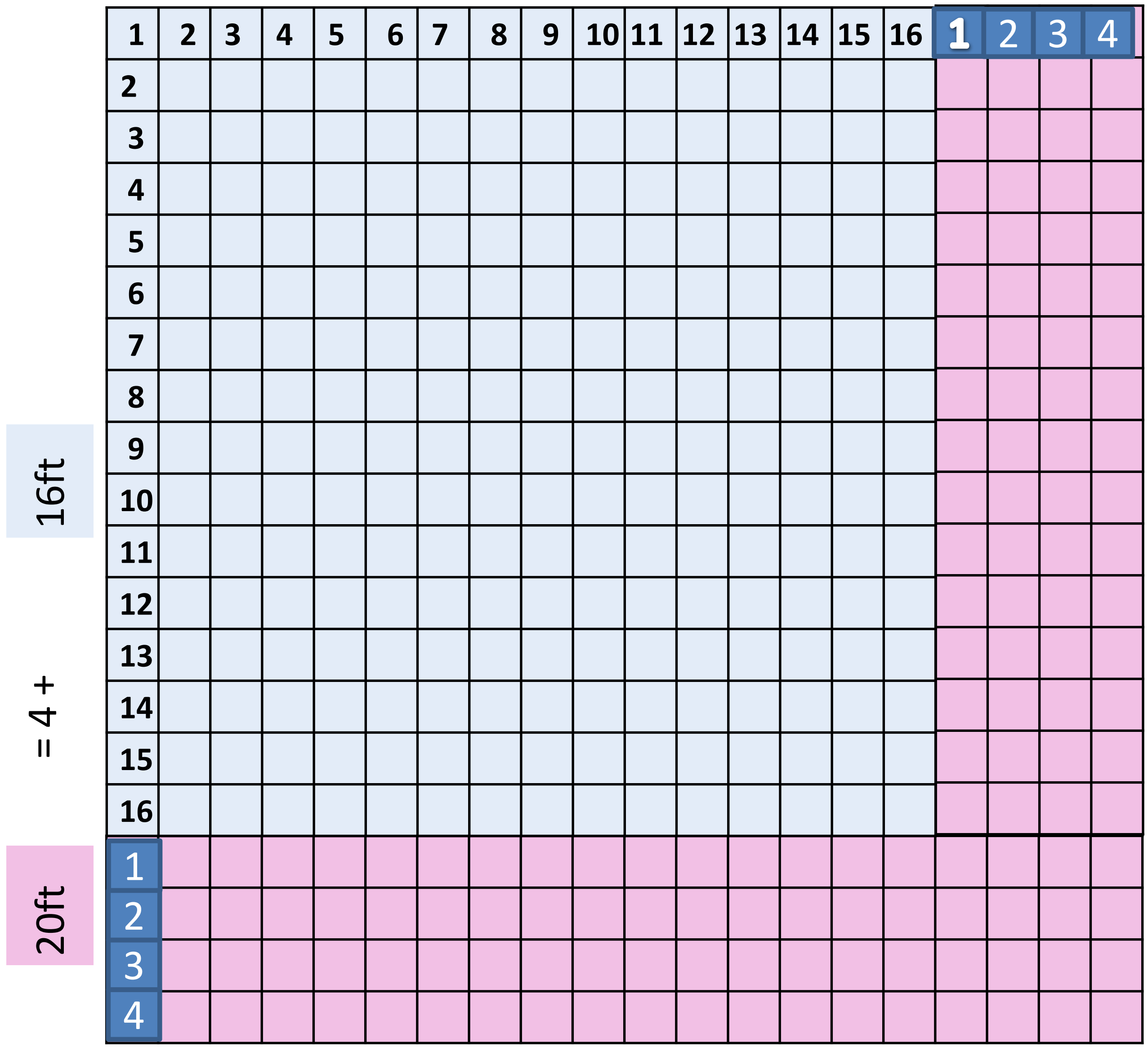
$x + 4 = 8$

Rate of change



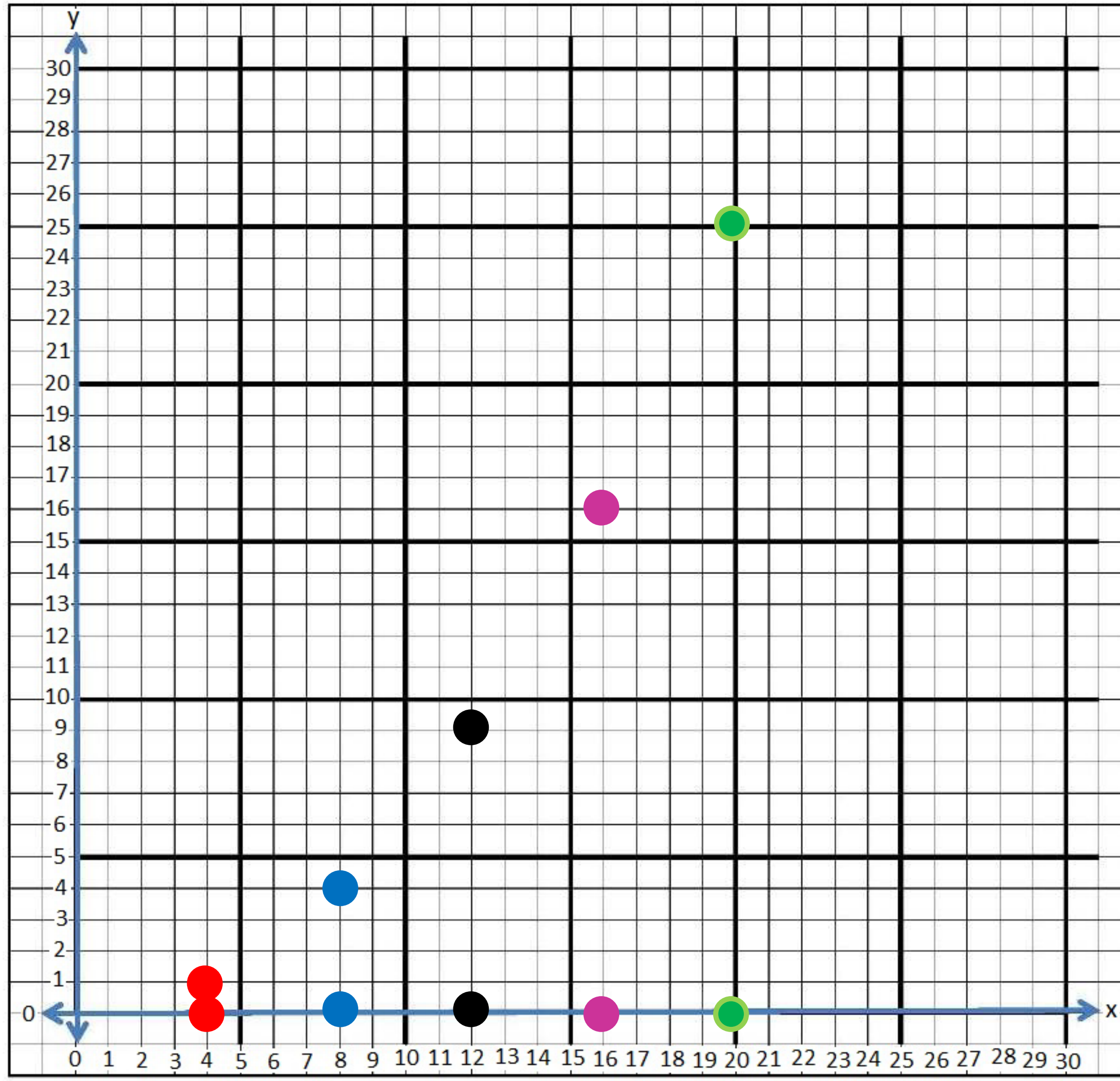
Length of Orchard Four + rate of change = Length of Orchard Five

16ft + 4 = 20ft

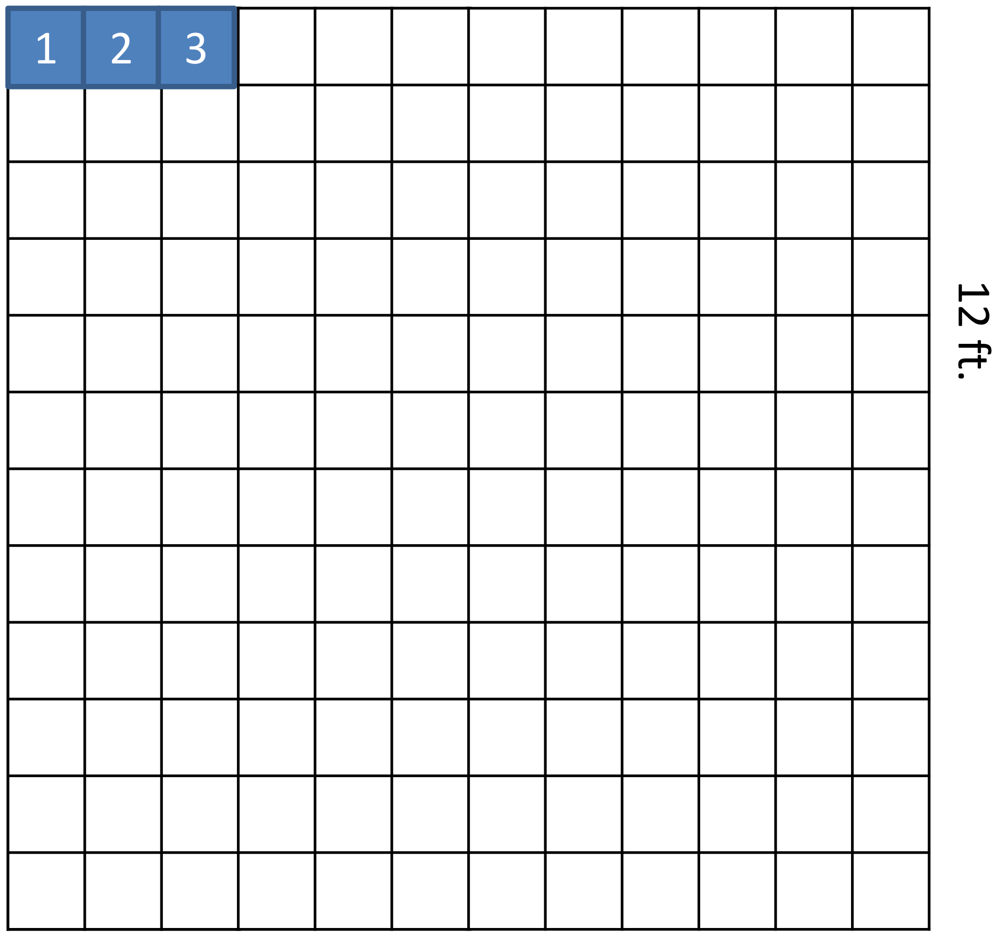


16ft + 4ft = 20ft

Orchard Number	Length of Each Side (feet) X	Area of Each Orchard (ft^2)	Number of Apple Trees y
1st	**4**	**16 ft^2**	**1**
2nd	**8**	**64 ft^2**	**4**
3rd	**12**	**144 ft^2**	**9**
4th	**16**	**256 ft^2**	**16**
5th	**20**	**400 ft^2**	**25**



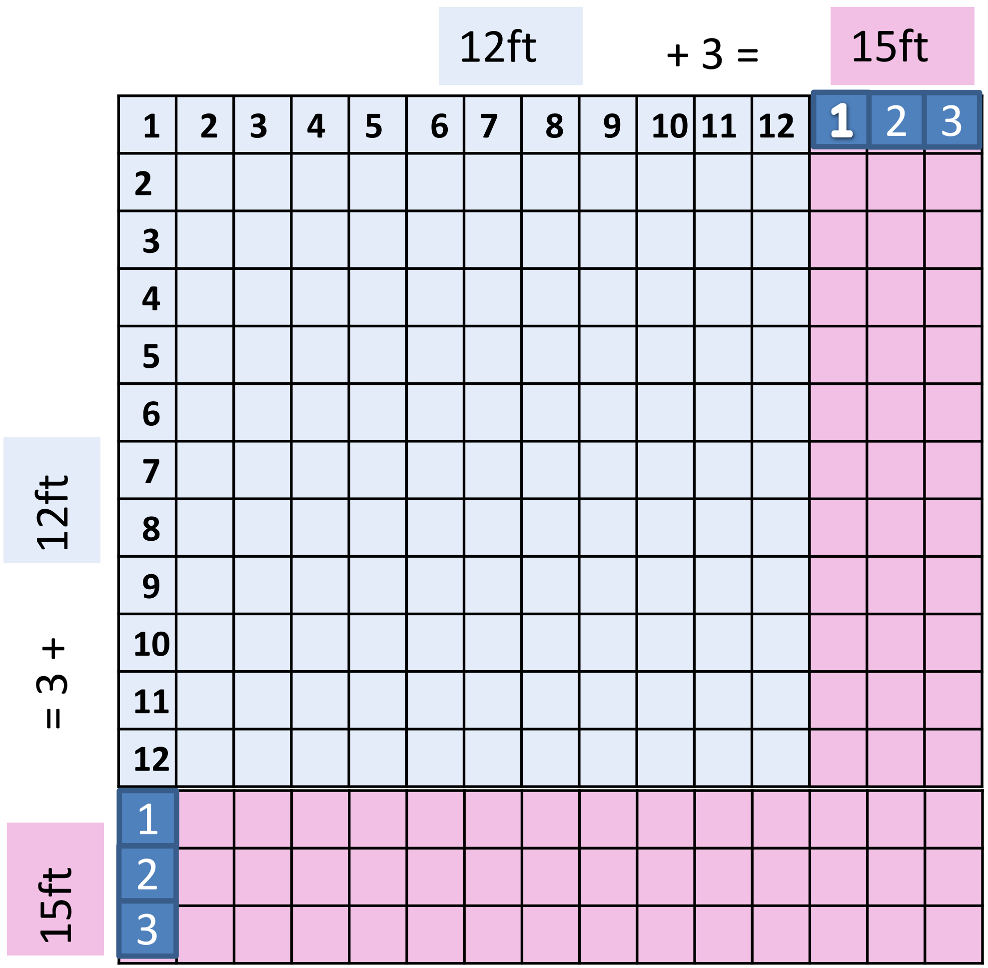
25% or ¼ of Orchard #2



Unit Length for
25% or ¼ of Orchard #2

1 2 3 4

Length of Orchard #3 + 25% of Orchard #3 = Length of Casey and Liz’s Orchard



12ft + 3ft = 15ft

National Center and State Collaborative

NCSC Math Activities with Scripted Systematic Instruction (MASSI): Elementary Equations

National Center and State Collaborative

The National Center and State Collaborative (NCSC) is applying the lessons learned from the past decade of research on alternate assessments based on alternate achievement standards (AA-AAS) to develop a multi-state comprehensive assessment system for students with significant cognitive disabilities. The project draws on a strong research base to develop an AA-AAS that is built from the ground up on powerful validity arguments linked to clear learning outcomes and defensible assessment results, to complement the work of the Race to the Top Common State Assessment Program (RTTA) consortia.

Our long-term goal is to ensure that students with significant cognitive disabilities achieve increasingly higher academic outcomes and leave high school ready for post-secondary options. A well-designed summative assessment alone is insufficient to achieve that goal. Thus, NCSC is developing a full system intended to support educators, which includes formative assessment tools and strategies, professional development on appropriate interim uses of data for progress monitoring, and management systems to ease the burdens of administration and documentation. All partners share a commitment to the research-to-practice focus of the project and the development of a comprehensive model of curriculum, instruction, assessment, and supportive professional development. These supports will improve the alignment of the entire system and strengthen the validity of inferences of the system of assessments.

IDEAs that Work
U.S. Office of Special Education Programs

The contents of this lesson were developed as part of the National Center and State Collaborative by Keri Bethune, Julie Thompson, Alicia Saunders, and Diane Browder at University of North Carolina at Charlotte and verified by Amy Lehew, math content expert, under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the U.S. Department of Education and no assumption of endorsement by the Federal government should be made.

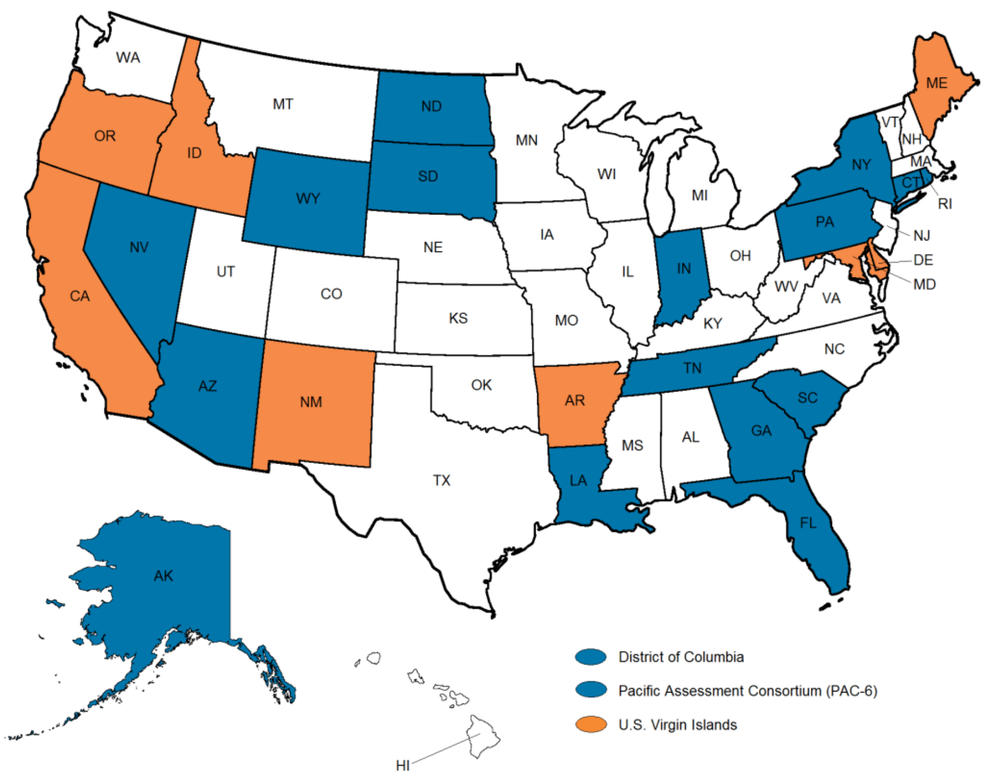
ncsc

National Center and State Collaborative

NCSC is a collaborative of 18 states and five organizations.

The states include (shown in blue on map): Alaska, Arizona, Connecticut, District of Columbia, Florida, Georgia, Indiana, Louisiana, Nevada, New York, North Dakota, Pacific Assessment Consortium (PAC-6)[1], Pennsylvania, Rhode Island, South Carolina, South Dakota, Tennessee, and Wyoming.

Tier II states are partners in curriculum, instruction, and professional development implementation but are not part of the assessment development work. They are (shown in orange on map): Arkansas, California, Delaware, Idaho, Maine, Maryland, New Mexico, Oregon, and U.S. Virgin Islands.



[1] The Pacific Assessment Consortium (including the entities of American Samoa, Commonwealth of the Northern Mariana Islands, Federated States of Micronesia, Guam, Republic of Palau, and Republic of the Marshall Islands) partner with NCSC as one state, led by the University of Guam Center for Excellence in Developmental Disabilities Education, Research, and Service (CEDDERS).

National Center and State Collaborative

The five partner organizations include: The National Center on Educational Outcomes (NCEO) at the University of Minnesota, The National Center for the Improvement of Educational Assessment (Center for Assessment), The University of North Carolina at Charlotte, The University of Kentucky, and edCount, LLC.

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National Center and State Collaborative

NCSC Math Activities with Scripted Systematic Instruction (MASSI): Elementary Equations

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January 2013

Reposted April 25, 2013

MASSI: Math Activities with Scripted Systematic Instruction

Activity: Gathering Materials for Art Activity

Grade Band: Grades 3-5

Concept: Equations

Common Core State Standard	Core Content Connectors	MASSI OBJECTIVES
3.OA.8 Solve problems involving the four operations, and identify and explain patterns in arithmetic.	3rd 3.NO.2e1 Solve and check one or two-step word problems requiring addition, subtraction, or multiplication with answers up to 100.	Selecting an expression that matches a word problem
3.OA.1 Represent and solve problems involving multiplication and division.	4th 4.NO.2d8: Match an accurate addition and multiplication equation to a representation	Matching an expression to a representation
6.EE.4 Reason about and solve one-variable equations and inequalities.	5th 5.SE.1b Evaluate whether both sides of an equation are equal.	Indicate whether an equation is true

Be sure to provide specific practice to students on the skills that correspond to their grade level.

Teaching materials: Graphic Organizer (GO): Addition; Graphic Organizer (GO): Subtraction; Addition and Subtraction Templates (cut out and laminated); Symbol Cards (cut out and laminated); Key Word Cards (laminated); Three Teacher Demonstration Cards (e.g., Lukas, Tameka, and Luther word problems and answer choices); Equality Visual; Teacher should create word problems and answer choices for steps #24, 25, 29, 30, and 31; and answer choices for #43, 45.

Other materials: Art supplies to count like markers, erasers, crayons, glue sticks, etc. (or use counting chips if art supplies are too cumbersome or distracting)

Worksheets: There are student worksheets to review each component of the lesson.

Assessments: Progress Monitoring for taking data during the lesson; Skills Test

TEACHING OVERVIEW: The first section of the MASSI provides remedial practice on addition and subtraction of sets. For students with few to no literacy skills, placing objects on the graphic organizers to create the set may be the starting point. Be sure to write equations for each addition/ subtraction problem you introduce. 3rd graders will learn to select the equation for the problem (an expression has no "=" sign, see Curriculum Resource Guide for further explanation). This is a good review for your 4th and 5th graders. The 4th graders will be selecting equations that match an array of objects. This is useful for your 3rd and 5th graders to gain fluency with equations. The 5th graders have to solve each of two sides of an equation. While the 5th graders work on equivalency, you can let the 3rd and 4th graders practice solving one side of the equation to review their addition/ subtraction skills.

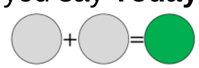
SCRIPT FOR LESSON

BUILD ESSENTIAL UNDERSTANDING: CONCEPT AND SYMBOLS: Combining Sets to Add

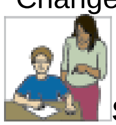
(Skip this section for students who understand these relationships and can identify these concepts).

INTRODUCE ACTIVITY: Making art can be a lot of fun. When we draw or paint we can make our pictures using different techniques with paint brushes and markers. Let's watch a video about creative ways to draw [paint]. View a video about water color or markers:
Marker ideas: http://www.youtube.com/watch?v=LGidADrMZgg&list=UUGEA1vUDxgkwsToRTEACsvA&index=104&feature=plcp
Water color ideas: http://www.youtube.com/watch?v=LMif8s15UdA&list=UUGEA1vUDxgkwsToRTEACsvA&index=102&feature=plcp
(Both videos are under three minutes. If your students have a short attention span, feel free to show them a brief clip pertaining to the art activity you plan to do that day instead of the entire video). **Before we can start an art project we have to make sure we have all the materials we need. First let's review addition.**

INTRODUCE THE PROBLEM: Write 6 + 2 = on board as you say **Today in our art project, we began with 6 markers. Then we added 2 more.**



MODEL THE PROCESS: Show student the GO: Addition  . **Let's use our graphic organizer to make and add our sets. First, I said we had 6 markers.** Put 6 in first circle. **Then I said we had 2 more.** Put 2 in second circle. **Now I add them together.** Move sets to last circle and count to add. **One. Two. Three. Four. Five. Six. Seven. Eight.** Write answer 8 on the equation 6 + 2 = 8. **We have 8 markers altogether.**
*Change numbers in equation each day the lesson is taught (sums ≤ 10).

**STUDENT PRACTICE:** Change numbers in equations above to solve. Use SYSTEM OF LEAST PROMPTS script as needed to help students with each step. Note: If students find using actual art supplies distracting or cumbersome, give them counting chips to complete problems instead.

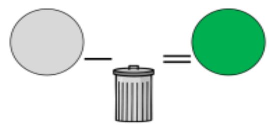
CHECK AND SCORE

Step	Teacher Says/Does	Student Response
1.	**Here is a new equation. Put the first number on your set maker.**	Student places first number under first circle.

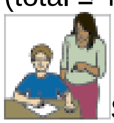
2.	**Put second number on your set maker.**	Student places second number under second circle.
3.	**Count out the number of markers you need to put in the first circle.**	Student counts out __ markers and puts them in first circle.
4.	**Count out the number of markers you need to put in the second circle.**	Student counts out __ markers and puts them in second circle.
5.	**Show me what to do to find out how many you have altogether.**	Student slides markers into 3rd circle (or indicates where to place markers with eye gaze).
6.	**Count the markers.**	Student counts the markers.
7.	**How many do you have altogether?**	Student says/indicates total number of markers.

BUILD ESSENTIAL UNDERSTANDING: CONCEPT AND SYMBOLS: Decomposing Sets to Subtract

INTRODUCE THE PROBLEM: Sometimes we use our art supplies and have to see what's left. Write 5-3. **We had 5 markers, but used up 3 of them.**



MODEL THE PROCESS: Hold up the GO: Subtraction handout  **. Let's use our graphic organizer to make and subtract our sets We had 5 pens**. Put 5 pens in first circle. **We used up 2.** Put 2 in garbage can on the handout. **Let's see how many are left.** Move the pens to the circle and count. **One. Two.** Write answer 5-3=2. **There are 2 markers left.** * Change number in equations each day the lesson is taught (total ≤ 10).

**STUDENT PRACTICE.** Change numbers in equations above to solve. Use SYSTEM OF LEAST PROMPTS script to help student with each step as needed. Note: If students find using actual art supplies distracting or cumbersome, give them counting chips to complete problems instead.

CHECK AND SCORE

Step	Teacher Says/Does	Student Response
8.	**Let's practice with the Take Away chart (GO: subtraction). Here is a new equation. Put the first number on your Take Away chart.**	Student places first number under first circle.
9.	**Put the second number on your Take Away chart.**	Student places second number under the trash can.
10.	**Count out the number of markers you need to put in the first circle.**	Student counts out __ markers and puts them in first circle.
11.	**Now take away the number of markers you need to put in the trash can.**	Student takes away __ markers from first circle and puts them in trash can.
12.	**Show me what to do to find out how many you have left over.**	Student slides markers into green circle (or indicates where to place markers with eye gaze).

13.	**Count the markers.**	Student counts the markers.
14.	**How many do you have left over?**	Student says/indicates total number of markers.

BUILD ESSENTIAL UNDERSTANDING: SYMBOL USE: +, -, =

INTRODUCE THE PROBLEM: When we solve and write problems to add and subtract, we use symbols.
MODEL THE PROCESS: Display +. **This is plus.** Display -. **This is minus.** Display =. **This is equal.**

STUDENT PRACTICE: Now it is you turn to show me the symbols. Use the CONSTANT TIME DELAY script to teach students to identify each of the key symbols. When you praise the students after a correct response, define the symbol (Examples: **You got it! Equal means "the same."/ Plus is the addition sign. It means add together./ Minus is the subtraction sign. It means take away.**). Note: Do not put +, -, or = as a distracter during trials for each of the target symbols. Only use non-targeted symbols (e.g., ?, !, x, or ÷) as distracters for each trial. Make sure to move cards around with each trial.

CHECK AND SCORE

Step	Teacher Says/Does	Student Response
15.	**Show me equal.**	Student points/eye gazes to the equal sign.
16.	**Again. Show me equal.**	Student points/eye gazes to the equal sign.
17.	**Show me plus.**	Student points/eye gazes to the plus sign.
18.	**Again. Show me plus.**	Student points/eye gazes to the plus sign.
19.	**Show me minus.**	Student points/eye gazes to the minus sign.
20.	**Again. Show me minus.**	Student points/eye gazes to the minus sign.

This may be a good stopping point. Let students use the art supplies to do an art project. There is a worksheet with this level. You can use this for additional guided practice or to send home as homework.

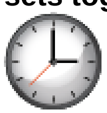
	Teacher Says/Does	Student Response	Error Correction
INDEPENDENT PRACTICE: Equations Skill Test	Give each student the *Equations Skills Test 1*. **Read directions for each problem and have student select response. Record whether response is correct or incorrect.**	Only provide praise for completing assessment (if student needs encouragement). Do not provide specific praise for correct answers while student is testing.	Once the student has completed the test, review missed problems with the student.

NOW	*NEXT*
Stop the lesson here and repeat tomorrow if student is not yet getting at least 12 independent correct responses. Score responses 1-20 on the Equation Progress Monitoring Sheet if you did not do so while teaching.	*Remember the goal is for students to be able to match equations to word problems (***3rd***), match visuals to equations (***4th***), and indicate whether an equation is true (***5th***). As soon as possible, move into the remainder of the lesson to hit the target CCCs for this grade band. Be sure to provide some practice for students at their specific grade level concept, even if they have not achieved mastery of previous concepts.*

3rd GRADE BUILD A GRADE ALIGNED COMPONENT: MATCHING EXPRESSIONS TO WORD PROBLEMS
4th and 5th GRADE BUILD ESSENTIAL UNDERSTANDING: SYMBOL USE

INTRODUCE THE PROBLEM: Our lesson today is about art supplies. Remember yesterday when we practiced adding and subtracting to find the answer to our equation? Write an equation on the board or paper (must have "=" sign, e.g., 3+2=5). **This is an equation. Sometimes we do not know what the equation is and we have to figure it out on our own.** Remove the equation you wrote. **Let's read a story about Lukas to learn about finding the equation.**

MODEL THE PROCESS ADDITION: Display Teacher Demonstration card #1 with word problem. **It says, "Lukas collected the markers. He had 3 markers in the first canister and 2 markers in the second."** Display array of 3 expressions (e.g., 3+2, 3x2, 3-2). **Which of these will show us how many Lukas has altogether?** Hold up and point to plus symbol card. **When a story problem says "in all", "total", or "altogether" we use this symbol. Plus tells us to put our sets together.** Point to 3 expressions again. **Remember our problem, "Lukas has 3 markers in the first canister and 2 markers in the second. Which of these will show us how many Lukas has altogether?** Point to "3+2". **This tells us to put our sets together. It has a plus.**



STUDENT PRACTICE: Now it's your turn to show me the symbol that tells us to put our sets together. Use the CONSTANT TIME DELAY script to teach students to identify the symbol or equation. Note: Do not put - or = as a distracter during trials for addition. Only use non-targeted symbols ?, !, x, or ÷ as distracters for each trial. Note: Each time you teach this, vary the order that you display the answer choices so that students do not memorize the answer based on placement.

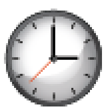
CHECK AND SCORE

Step	Teacher Says/Does	Student Response
21.	**A story problem says "in all." Show me the symbol that tells us to put our sets together.**	Student selects +.
22.	**A story problem says, "total." Show me the symbol that tells us to put our sets together.**	Student selects +.

23.	**A story problem says "altogether." Show me the symbol that tells us to put our sets together.**	Student selects +.
24.	**Here is a new word problem. It says, "Elijah passed out the paintbrushes. He had 2 paintbrushes in his left hand and 4 in his right hand. Which of these will show how many paintbrushes Elijah has <u>in all</u>?"** Emphasize "in all." **Listen again: Which of these will show how many paintbrushes Elijah has in all?** Read each expression aloud while pointing to it: "**2 + 4", "2 x 4", "2 ÷ 4"**	Student selects correct expression.
25.	**Let's try one more. Listen for the word that tells you which symbol the equation must have.** Display new word problem and equation choices. **Arlo collected the crayons in pencil cases. He had 6 crayons in one case and 3 crayons in the other. How many crayons did he have <u>total</u>?** Emphasize "total." **Listen again. Which of these show how many crayons he had total?** Read each expression aloud while pointing to it: "**6 – 3", "6 + 3", "6 x 3".**	Student selects correct expression.

MODEL THE PROCESS:

SUBTRACTION: Display Teacher Demonstration card #2. **Here is another word problem about our art supplies. It says, "Tameka bought 9 pencils. She gave 4 pencils to her students. Which of these will show how many pencils she has <u>left</u>?"** Display array of 3 expressions (e.g., 9+4, 9x4, 9-4). **Before we can decide which of these equations will show how many pencils Tameka has left, we need to practice finding the symbol that we use to take away.** Hold up and point to subtraction symbol card. **When a story problem says "left" or "remain" we use the symbol that tells us to take away. We can also find the symbol in an expression.** Point to 3 expressions again. **Remember our problem, "Tameka bought 9 pencils. She gave 4 pencils to her students. Which of these will show how many pencils she has <u>left</u>?"** Point to "9-4". **This tells us to take away. It has a minus.**



STUDENT PRACTICE: Now it's your turn to show me the symbol that tells us to take away. Use the CONSTANT TIME DELAY script to teach students to identify the symbol or equation. Note: Do not put + or = as distracters during trials for subtraction. Only use non-targeted symbols ?, !, x, or ÷ as distracters for each trial. Note: Each time you teach this, vary the order that you display the answer choices so that students do not memorize the answer based on placement.

CHECK AND SCORE

Step	Teacher Says/Does	Student Response
26.	**A story problem says "left." Show me the symbol that tells us to take away.**	Student selects -.

27.	**A story problem says, "remain." Show me the symbol that tells us to take away.**	Student selects -.
28.	**A story problem says, "difference." Show me the symbol that tells us to take away.**	Student selects -.
29.	**Here is a new word problem. It says, "Zatrel had 10 markers. He passed out 4. How many markers did he have <u>left</u>?** Emphasize "left." **Listen again: Which of these will show how many markers Zatrel has left?** Read each expression aloud while pointing to it: "**10 – 4", "10 + 4", "10 X 4"**	Student selects correct expression.
30.	**Nehemiah collected 6 crayons. He threw 2 broken ones away. How many crayons <u>remain</u>?** Emphasize "remain." **Listen again: Which of these shows how many crayons remain?** Read each expression aloud while pointing to it: "**6 ÷ 2", "6 X 2", "6 – 2"**	Student selects correct expression.
31.	**Hannah has 8 apples. Karen has 4. What is the <u>difference</u> between the number of apples Hannah has and the number Karen has?** Emphasize "difference." **Listen again: Which of these will show the difference between the number of apples Hannah has and the number of apples Karen has?** Read each expression aloud while pointing to it: **"8+4," "8X4," "8-4."**	Student selects correct expression.

MODEL THE PROCESS:
VARYING NUMBERS IN EXPRESSION: You have done a great job listening for the symbol. Now listen very carefully. We are going to practice choosing the correct expression for our problems by listening for the symbol AND the numbers. Watch me first. Display Teacher Demonstration Card #3 and answer choices. **"Luther had 5 markers. He found 3 more. How many did he have in all?"** Point to the numbers in the word problem as you say, **First I will find the numbers. They are 5 and 3.** Point to answer choices "5 +3" and "5x3". **Look, there are two answer choices with 5 and 3. I can remove the choice that does not have 5 and 3** (remove answer choice so two choices are left). **Now, watch very carefully! I have to make sure I choose the one with the right symbol.** Point to highlighted "in all" in word problem. **I have to find the words that tell me which symbol to use. This says "in all." Remember, "in all" tells me to put the sets together with the plus sign**. **Now, I have to find the answer choice with a plus symbol.** Point to the "x" symbol between "5 x 3". **This does <u>not</u> have a plus.** Point to correct choice "5 + 3". **Does this have a plus symbol?** Pause 2 seconds then say, **Yes, this says plus. The answer is 5 plus 3.**

STUDENT PRACTICE: Now it's your turn to listen for the symbol AND the numbers. Use the CONSTANT TIME DELAY script to teach students to identify the symbol or equation.

CHECK AND SCORE

Step	Teacher Says/Does	Student Response
32.	**This says, "Cora has 4 crayons. She gave 2 to Mike. How many does she have left?"** Read each answer choice aloud. "**4 – 2", "3 + 7", "4 X 2". Point to an answer choice that has the same numbers.**	Student points to an answer choice with numbers 4 and 2 (doesn't have to be correct answer yet).
33.	**Now, point to the word that tells you what symbol to use.**	Student points to "left."
34.	Point to the answer choice student selected. **Now look here. Is this the symbol that tells us how many are left?**	Student indicates yes or no.
35.	*****If yes: **Good job. You found the expression 4 minus 2.** Mark this step correct on assessment. *If no: Read each answer choice aloud. **Try again. "4 – 2", "3 + 7", "4 X 2". Which one tells us how many she has left?**	Student selects "4 – 2".

BUILD ESSENTIAL UNDERSTANDING: CONCEPT: EQUALITY ("=")

INTRODUCE THE PROBLEM: Sometime we need equal supplies, like making sure we have equal paper and pencils for each student. Let's see if I have equal supplies before we do more.

MODEL THE PROCESS: Point to equal sign. **Remember, this is the equal sign.** Display equality visual with equal sign present. **The equal sign means the total amount is same on both sides.** Show the students a number (e.g., 9) and the equal sign (e.g., 9 = ____). **If the number 9 is on this side, then we put the number 9 on this side too, to make it the same. When both sides are equal it means that the equation is true.**

**STUDENT PRACTICE:** Display equality sign erased and place markers on each side. **Now you practice making the equations true.** Use SYSTEM OF LEAST PROMPTS script as needed to help students with each step. Note: Each time you complete this section change the number of markers (≤ 10 each side) and vary whether the amounts are equal.

CHECK AND SCORE

Step	Teacher Says/Does	Student Response
34.	Write a number and = sign (e.g., 5 = ___). Read **'five equals ___.' Which number goes on this side?** Point to right side.	Student says or selects the same number (e.g., 5).
35.	Write a different number and = sign (e.g., 3 = ___). Read **'three equals ___.' Which number goes on this side?** Point to right side.	Student says or selects the same number (e.g., 3).
36.	Write a different number and = sign (e.g.,8 = ___). Read **'eight equals ___.' Which number goes on this side?** Point to right side.	Student says or selects the same number (e.g., 8).

MODEL THE PROCESS: Display Equality Visual with "=" written between the two ovals. **Here's a rule for using "=" in equations.** Point to the left side of equals and say, **We must end with the same number on this side and** (point to right side) **the other side of the equal sign.** Place equal amounts of markers on each side of the Equality Visual. **Let's see if we have equal amounts of markers on both sides of the equals sign.** Point to left side. **Count the markers with me on this side as I touch them.** Count aloud pointing to each marker as you count. **There are ____ markers on this side, so we must have ____ on the other side.** Point to right side. **Count the markers with me on this side as I touch them.** Count aloud pointing to each marker as you count. **We ended with ____ markers on both sides, so the sides are equal.**

Let's try another problem. Display Equality Visual with equal sign erased. **Remember the equal rule. We must have the same number on this side** (point to one side) **and the other side** (point to other side)**.** Place unequal amounts of markers on each side of the Equality Visual. **Let's see if we have the same number of markers on both sides.** Point to left side and say, **Count the markers with me on this side as I touch them.** Count aloud pointing to each marker as you count. Write number below after counting. **There are ____ markers on this side, so there must be ____ markers on the other side.** Point to right side. **Count the markers with me on this side as I touch them.** Count aloud pointing to each marker as you count. Write number below after counting. **There are ____ markers on this side. The numbers are not the same, so the sides are not equal. I do NOT write an equal sign.**


STUDENT PRACTICE: Display Equality Visual with no "=" sign and place markers on each side. You can vary items used each day to build generalization. **Now you practice.** Use SYSTEM OF LEAST PROMPTS script as needed to help students with each step. Note: Each time you complete this section change the number of markers (≤ 10 each side) and vary whether the amounts are equal or not equal.

CHECK AND SCORE

Step	Teacher Says/Does	Student Response
37.	Point to left side. **How many on this side?**	Student counts items.
38.	Point to right side. **How many on this side?**	Student counts items.
39.	**Are the numbers on each side the same? If they are, write the equal sign.**	Student indicates yes or no and writes the equal sign between two amounts if amounts are the same (students who cannot write may eye gaze to or Velcro the equal sign).

This may be a good stopping point. Let students use the art supplies to do an art project. There is a worksheet with this level. You can use this for additional guided practice or to send home as homework.

	Teacher Says/Does	Student Response	Error Correction
INDEPENDENT PRACTICE: Equations Skill Test	Give each student the *Equations Skills Test 2*. **Read directions for each problem and have student select response. Record whether response is correct or incorrect.**	Only provide praise for completing assessment (if student needs encouragement). Do not provide specific praise for correct answers while student is testing.	Once the student has completed the test, review missed problems with the student.
NOW *Stop the lesson here and repeat tomorrow if student is not yet getting at least 9 independent correct responses. Score responses 21-36 on the Equation Progress Monitoring Sheet if you did not do so while teaching.*		***NEXT*** *Remember the goal is for students to be able to match equations to word problems (***3rd***), match visuals to equations (***4th***), and indicate whether an equation is true (***5th***. As soon as possible, move into the remainder of the lesson to hit the target CCCs for this grade band. Be sure to provide some practice for students at their specific grade level concept, even if they have not achieved mastery of previous concepts.*	

4th BUILD A GRADE ALIGNED COMPONENT: MATCHING EQUATIONS TO REPRESENTATIONS

INTRODUCE ACTIVITY AND PROBLEM: Use actual classroom supplies (e.g., paperclips, erasers, etc.) or create pictures with arrays of classroom items. **We have been working with equations to help us organize our art supplies. Today, let's see if we can match an equation to the supplies (or pictures). We can line things up into rows with the same amount in each row to help us keep things organized and make them easier to count.**

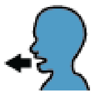
MODEL THE PROBLEM: Watch me line up some paper clips so that every row has an equal amount of paperclips. Create 3 or 4 rows of objects, with the same number of objects in each row (see Worksheet 3 for an example). **I can write this as an equation. First, let's count how many paperclips are in each row.** Count aloud pointing to each paperclip in the row as you count. **There are ___ in each row. Now, let's count how many rows there are.** Count aloud pointing to each row as you count. **There are** ___ **rows. The number of rows tells us how many times we need to add the amount in each row.** Write and say the equation. For example: 3+3+3+3=12. "There are 3 in each row and there are 4 rows, so we add 3 four times: <u>3</u> +<u>3</u>+<u>3</u> +<u>3</u>=12.

STUDENT PRACTICE: Let's try some more examples. I'll line up the supplies, and then you help me write the equation. If students cannot write, provide answer choices. Do 3-4 examples. If students do not get correct response, use MODEL-LEAD-TEST script for error correction. Note: If student finds use of paper clips distracting use chips or draw dots on paper. Change the number of rows and items per row each time you teach this section.

CHECK AND SCORE

Step	Teacher Says/Does	Student Response
40.	**Show me the rows.**	Student runs fingers (or eye gazes) across each row.
41.	**How many in each row?**	Student counts each item in row.
42.	**Count the rows.**	Student counts the rows.
43.	**Find the equation that matches.**	Student selects correct addition equation.

MODEL THE PROBLEM: There is another way we can think about this equation to check our work. We can count the total number in all. Count the paperclips with me. One, two, three... Point to the equations. **Can you find an equation that equals (<u>total</u>)?** Pause three seconds to see if student(s) can locate equation independently, then point to correct equation. **This equation equals (<u>total</u>), so this is the correct answer.**



STUDENT PRACTICE: Display new array of paper clips (or erasers). **Now it's your turn.** If students cannot write, give them options on a paper to select. Do 3-4 examples. If students do not get correct response, use MODEL-LEAD-TEST script for error correction. Note: If student finds use of paper clips distracting use chips or draw dots on paper. Change the number of rows and items per row each time you teach this step.

CHECK AND SCORE

Step	Teacher Says/Does	Student Response
44.	**How many paper clips are there in all?**	Student counts the paperclips.
45.	**Point to the equation that equals ___.**	Student selects correct equation.

This may be a good stopping point. Let students use the art supplies to do an art project. There is a worksheet with this level. You can use this for additional guided practice or to send home as homework.

	Teacher Says/Does	Student Response	Error Correction
INDEPENDENT PRACTICE: Equations Skill Test	Give each student the *Equations Skills Test 3*. **Read directions for each problem and have student select response. Record whether response is correct or incorrect.**	Only provide praise for completing assessment (if student needs encouragement). Do not provide specific praise for correct answers while student is testing.	Once the student has completed the test, review missed problems with the student.
NOW *Stop the lesson here and repeat tomorrow if student is not yet getting at least 4 independent correct responses. Score responses 37-42 on the Equation Progress Monitoring Sheet if you did not do so while teaching.*		***NEXT*** *Remember the goal is for students to be able to match equations to word problems (**3rd**), match visuals to equations (**4th**), and indicate whether an equation is true (**5th**) .As soon as possible, move into the remainder of the lesson to hit the target CCCs for this grade band. Be sure to provide some practice for students at their specific grade level concept even if they have not achieved mastery of previous concepts.*	

5th BUILD A GRADE ALIGNED COMPONENT: INDICATE WHETHER AN EQUATION IS TRUE

INTRODUCE ACTIVITY AND PROBLEM: We have been using equations to help us organize and think about our art supplies. Today we want to decide if we have the same number of supplies left after some activities. Here is our problem. Write the problem, 3 + 2 □ 9 **–** 4, as you say: **We had 3 red pens and got 2 new ones. We had 9 papers, but used 4. Do we have the same number of pens as paper? We have to decide if these are equal. Find the equal sign.** If students are unclear on equal sign, use the time delay script to find the equal sign.

MODEL THE PROBLEM: Get out GO: Addition and GO: Subtraction. **One side of our equation says to add. The other says to subtract. Show me which of these we use to add.** If student is incorrect, point out the + sign. **Now I add 3+2.** Demonstrate on GO: Addition using chips that 3+2=5. **Next I subtract 9-4**. Demonstrate with GO: subtraction that 9-4=5. **Look, these are both 5. The same. They are equal**. Put = sign between the two. **We can say this equation is true because the values on both sides of the equals sign are the same**. Note: On alternate days, demonstrate equations that are NOT equal, and thus, not true.

STUDENT PRACTICE: Write equations that are/are not true on paper using an expression on both sides of the equal sign (e.g., 4+2=5+1). Let students solve expressions on each side of equation and decide if should use the = sign. Use SYSTEM OF LEAST PROMPTS script as needed to help students with each step. Write different equations each day. Note: If student has difficulty solving the equation review steps 1-14 in lesson.

CHECK AND SCORE

Step	Teacher Says/Does	Student Response
46.	**Show me which chart you will use to solve this side.**	Student indicates correct chart.
47.	**Show me which chart you will use to solve the other side.**	Student indicates correct chart.
48.	**You are going to tell me if this equation is true. An equation is true if the expressions on both sides of the equation are equal. Here is an equation. Before you decide if it is true you have to solve each side.** Point to left side. **Solve this side and write your answer below.**	Student solves left side of equation and writes/indicates answer.
49.	Point to right side of equation. **Now solve this side and write your answer below.**	Student solves right side of equation and writes/indicates answer.
50.	**Are both sides equal?**	Student answers yes (or no).
51.	**Is the equation true?**	Student answers yes (or no).

This is the end of the lesson. Let students use the art supplies to do an art project. There is a worksheet with this level. You can use this for additional guided practice or to send home as homework.

	Teacher Says/Does	Student Response	Error Correction
INDEPENDENT PRACTICE: Equations Skills Test	Give each student the *Equations Skills Test 4.* **Read directions for each problem and have student select response. Record whether response is correct or incorrect.**	Only provide praise for completing assessment (if student needs encouragement). Do not provide specific praise for correct answers while student is testing.	Once the student has completed the test, review missed problems with the student.

Troubleshooting and Data-based Decision making for Equations Skills Test
If student is unable to complete any items on the equations test independently and correctly go back and teach one problem step-by-step.

MASSI CULMINATING ACTIVITY: Have students complete an art activity related to the season, an upcoming holiday, or a science or social studies theme. Be sure to include a variety of art materials and have student practice creating sets, matching equations to sets, and determining if they have equal amount of supplies using skills they learned during MASSI lesson.

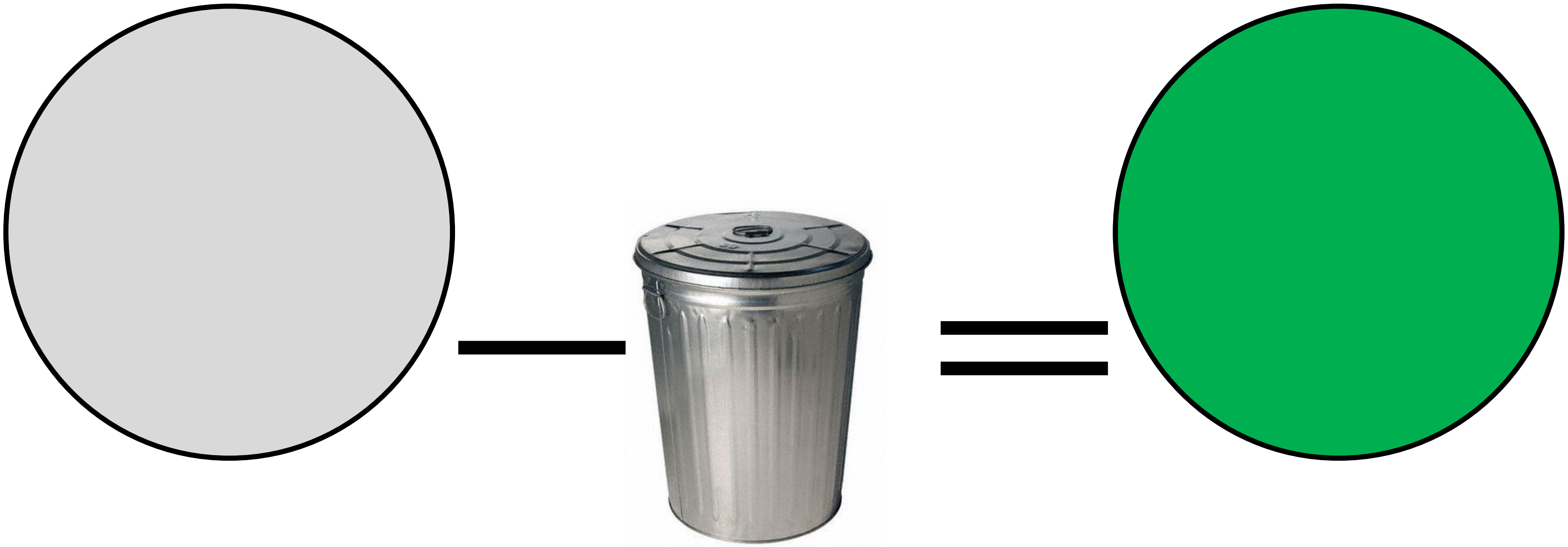
BUILD TOWARDS FULL GRADE LEVEL COMPETENCE

Here are ideas to build competence towards the full grade level competence using this same activity. See the unit plan and talk with the general education teacher for more ideas.

Component	Activity	What Student Does	Generalization/ Fluency
Matching multiplication equation to representation. Students should build an understanding of the relationship between repeated addition and multiplication.	Student is presented with a set of art materials.	Write equation demonstrating number of rows times number in each row.	Vary amounts and items daily.
Solving multiplication and division problems.	For multiplication: Student is given number of students and supplies needed for each student and asked to find the total using repeated addition and multiplication. For division: Student is given large amount of art items and is asked to determine how many each student can have.	Multiplies number of people by supplies per person to get total amount. Divides total number of art items by number of students in class.	Vary amounts and items daily.

Graphic Organizer: Subtraction

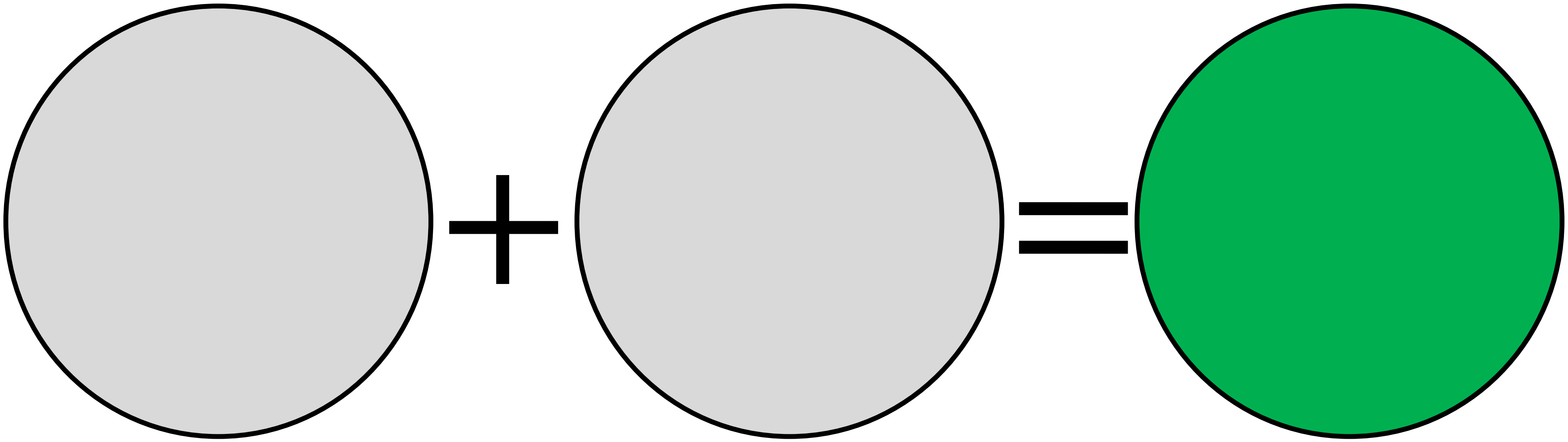
1 for teacher and 1 for each student (Laminate) – May want to print as 12 x 15 if using real art supplies



______ − ______ = ______

Graphic Organizer: Addition

1 for teacher and 1 for each student (Laminate) – May want to print as 12 x 15 if using real art supplies



________ + ________ = ________

Subtraction Equation Template: 1 for teacher (Cut out and laminate)

$$\underline{\hspace{3cm}} - \underline{\hspace{3cm}} = \underline{\hspace{3cm}}$$

Addition Equation Template: 1 for teacher (Cut out and laminate)

$$\underline{\hspace{3cm}} + \underline{\hspace{3cm}} = \underline{\hspace{3cm}}$$

Symbol Cards 1 set for teacher (Cut out and laminate)

=	−	÷	+
×	!	?	

Addition Key Words: 1 set for teacher (laminate)

altogether	in all
total	

Subtraction Key Words: 1 set for teacher (laminate)

left	remain
difference	

Teacher DEMONSTRATION card #1 (laminate)

*Velcro answer choices on back to keep organized

Lukas collected the markers. He had 3 markers in the first canister and 2 markers in the second. Which of these will show how many markers Lukas has altogether?

Cut out answer choices and laminate

3 + 2	3 x 2	3 – 2

Teacher DEMONSTRATION card #2 (laminate)

*Velcro answer choices on back to keep organized

Tameka bought 9 pencils. She gave 4 pencils to her students. Which of these will show many pencils she has left?

Cut out answer choices and laminate

9 + 4	9 x 4	9 – 4

Teacher DEMONSTRATION card #3 (laminate)

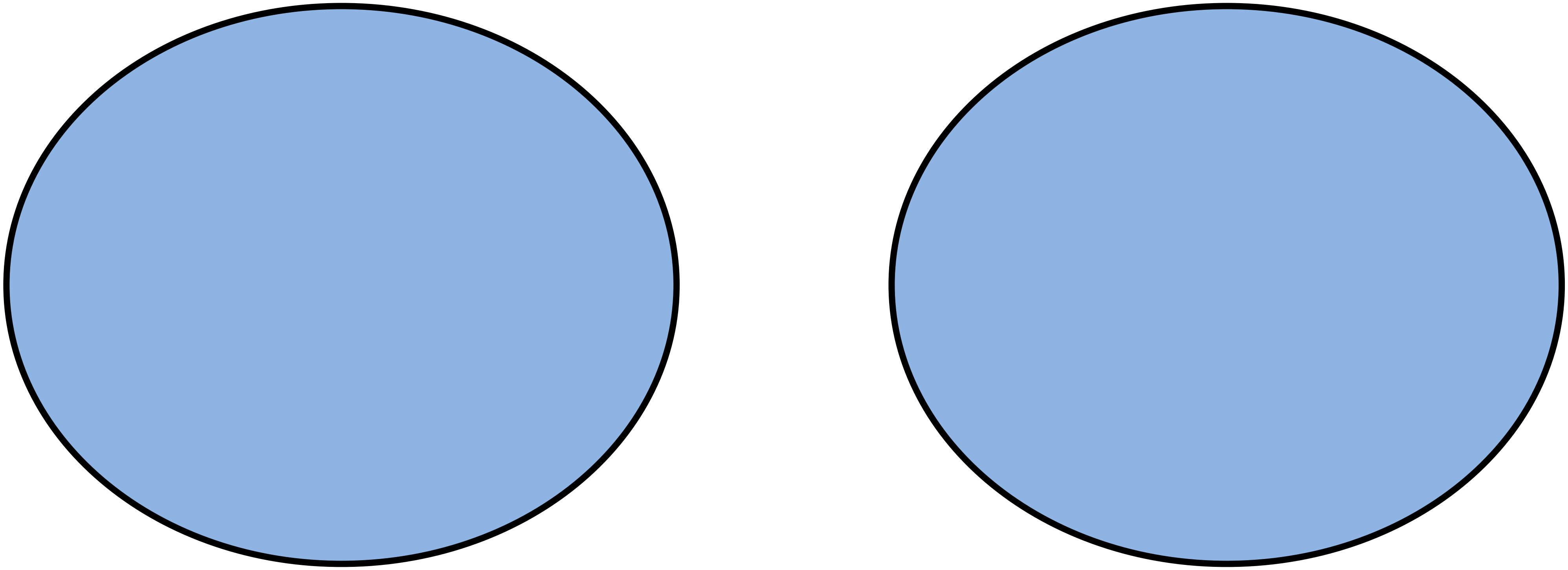
*Velcro answer choices on back to keep organized

Luther had 5 markers. He found 3 more. How many did he have in all?

Cut out answer choices and laminate

5 + 3	5 x 3	9 – 4

Equality Visual (Laminate)



Level 1 Worksheet: Building the Foundation using graphic organizer to complete simple addition and subtraction; identifying symbols

Name: ____________________

Use your setmaker and chips to find out how many all together in this equation.

$$3 + 2 = \_\_\_$$

Use your take away chart and chips to find out how many all together in this equation.

$$5 - 1 = \_\_\_$$

Circle minus.

? x –

Circle equal.

= ÷ !

Circle plus.

! + ?

Level 3 Worksheet: Matches equation to representation

Name: ____________________

__ Point to symbol for story problem that says, "in all."

x ? +

__ Point to symbol for story problem that says, "total."

+ x ÷

__ Point to symbol for story problem that says, "altogether."

+ ! ?

__Troy had 2 crayons. He bought 4 more. Which of these will show how many Troy has altogether?

2 – 4	2 + 4	2 x 4

__ Point to symbol for story problem that says, "left."

= – ?

__ Point to symbol for story problem that says, "remain."

? ÷ –

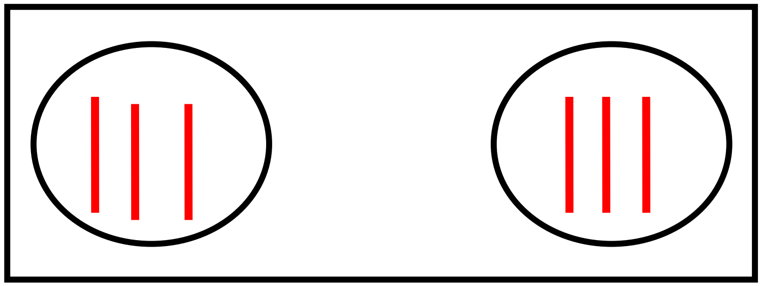
__ Jackie had 3 markers. 1 was dry so she threw it away. How many markers remain?

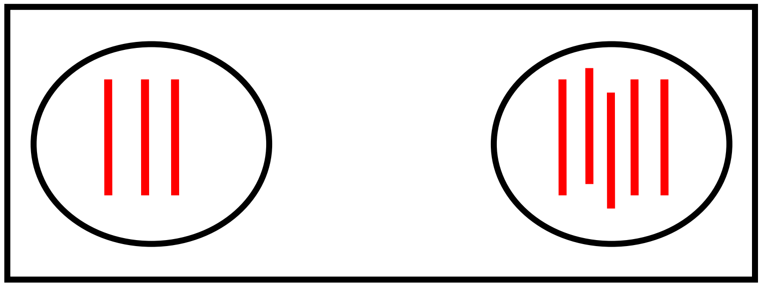
3 – 1	3 + 1	3 x 1

__ Daniel collected 4 crayons. He found 2 more. How many crayons does Daniel have in all?

8 – 6	4 + 2	4 x 2

__ Are the sets on each side the same? If they are, write the equal sign.

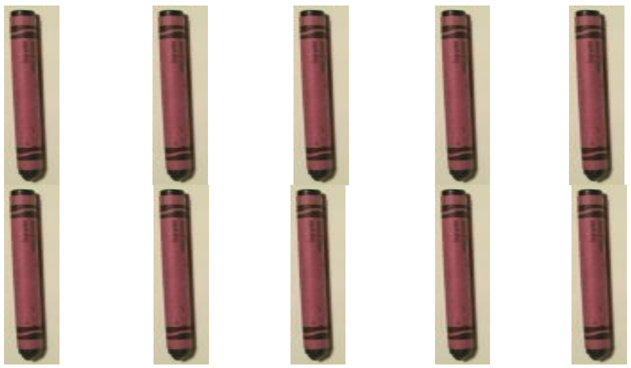




Level 4 Worksheet: Indicating whether an equation is true

Name: ____________________

__ Which equation shows what you see in this picture?



$$2 + 2 + 2 = 6$$

$$5 + 5 = 10$$

$$2 + 5 = 7$$

__ Which equation shows what you see in this picture?

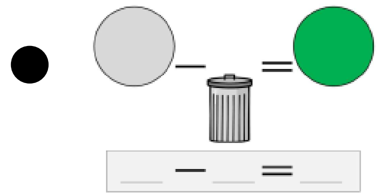
$$3 + 4 = 7$$

$$3 + 3 + 3 = 9$$

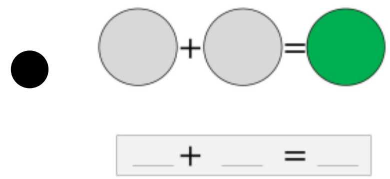
$$4 + 4 + 4 = 12$$

__ Match the equations to the chart you can use to solve it.

7 + 2 ●



4 -   1●



_ Solve the each side of the equation and write your answer below.

3 + 2 = 6 – 1

___ = ___

__ Are both sides equal?

Yes   No

__ Is the equation true?

Yes   No

National Center and State Collaborative

NCSC Math Activities with Scripted Systematic Instruction (MASSI): Elementary Equations Progress Monitoring and Skills Test

National Center and State Collaborative

The National Center and State Collaborative (NCSC) is applying the lessons learned from the past decade of research on alternate assessments based on alternate achievement standards (AA-AAS) to develop a multi-state comprehensive assessment system for students with significant cognitive disabilities. The project draws on a strong research base to develop an AA-AAS that is built from the ground up on powerful validity arguments linked to clear learning outcomes and defensible assessment results, to complement the work of the Race to the Top Common State Assessment Program (RTTA) consortia.

Our long-term goal is to ensure that students with significant cognitive disabilities achieve increasingly higher academic outcomes and leave high school ready for post-secondary options. A well-designed summative assessment alone is insufficient to achieve that goal. Thus, NCSC is developing a full system intended to support educators, which includes formative assessment tools and strategies, professional development on appropriate interim uses of data for progress monitoring, and management systems to ease the burdens of administration and documentation. All partners share a commitment to the research-to-practice focus of the project and the development of a comprehensive model of curriculum, instruction, assessment, and supportive professional development. These supports will improve the alignment of the entire system and strengthen the validity of inferences of the system of assessments.

IDEAs that Work
U.S. Office of Special Education Programs

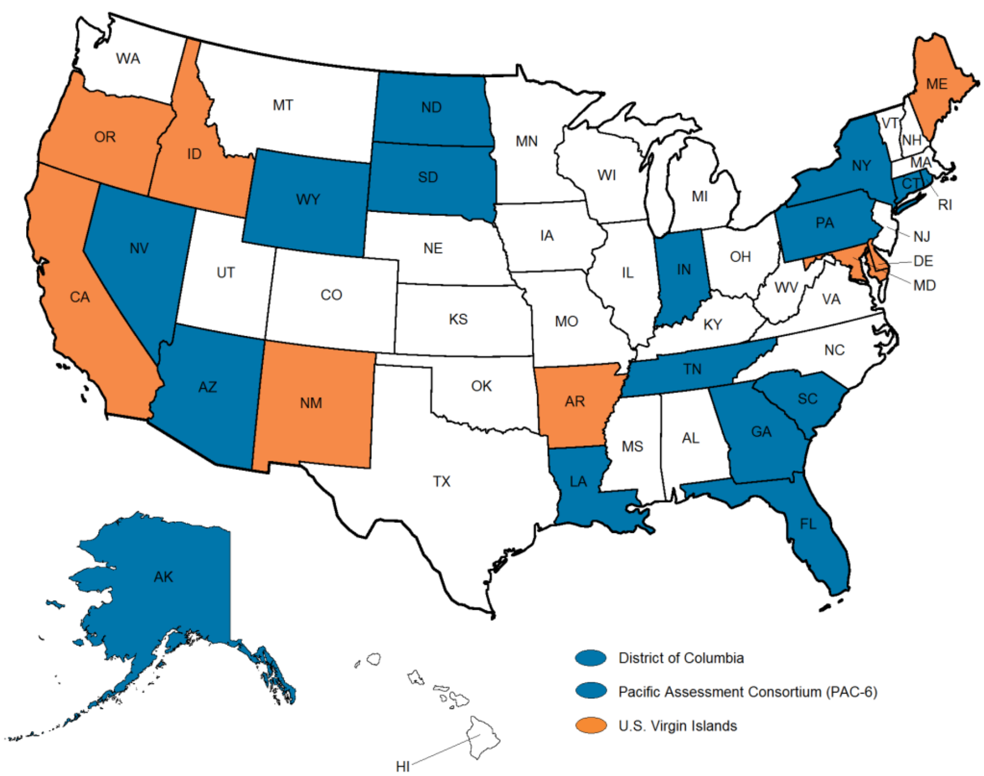
The contents of this assessment were developed as part of the National Center and State Collaborative by Keri Bethune, Julie Thompson, Alicia Saunders, and Diane Browder at University of North Carolina at Charlotte and verified by Amy Lehew, math content expert, under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the U.S. Department of Education and no assumption of endorsement by the Federal government should be made.

National Center and State Collaborative

NCSC is a collaborative of 18 states and five organizations.

The states include (shown in blue on map): Alaska, Arizona, Connecticut, District of Columbia, Florida, Georgia, Indiana, Louisiana, Nevada, New York, North Dakota, Pacific Assessment Consortium (PAC-6)[1], Pennsylvania, Rhode Island, South Carolina, South Dakota, Tennessee, and Wyoming.

Tier II states are partners in curriculum, instruction, and professional development implementation but are not part of the assessment development work. They are (shown in orange on map): Arkansas, California, Delaware, Idaho, Maine, Maryland, New Mexico, Oregon, and U.S. Virgin Islands.



[1] The Pacific Assessment Consortium (including the entities of American Samoa, Commonwealth of the Northern Mariana Islands, Federated States of Micronesia, Guam, Republic of Palau, and Republic of the Marshall Islands) partner with NCSC as one state, led by the University of Guam Center for Excellence in Developmental Disabilities Education, Research, and Service (CEDDERS).

National Center and State Collaborative

The five partner organizations include: The National Center on Educational Outcomes (NCEO) at the University of Minnesota, The National Center for the Improvement of Educational Assessment (Center for Assessment), The University of North Carolina at Charlotte, The University of Kentucky, and edCount, LLC.

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NCSC Math Activities with Scripted Systematic Instruction (MASSI): Elementary Equations Progress Monitoring and Skills Test

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Amy Lehew

January 2013

Reposted March 11, 2013

Student Name: ____

MASSI: Elementary School Equations

Options for Progress Monitoring/Formative Assessment

1. Elementary Equations Progress Monitoring (pg. 6-9): record student responses made during instruction on data sheet provided; teacher records each step correct during the lesson.

2. Elementary Equations Skills Test (pg. 10-15): a brief on demand performance assessment; could be given weekly to see if student has mastered this lesson; also helps student practice responding in a test format.

   a. NOTE: The Skill Test can be used as a baseline assessment to check for any skills the student may already have prior to beginning the MASSI.

   b. NOTE: The Skill Test can also be readministered to check for maintenance throughout the year

Student Name: ____

Elementary Equations Progress Monitoring

Directions: Score each step during instruction or as soon as the lesson is complete. Score the step as unprompted correct with a "+." Use a system to code level of prompting required for incorrect responses (e.g., V = verbal prompt, G = gesture, P = physical). Graph the number of unprompted correct responses to monitor progress.

BUILD ESSENTIAL UNDERSTANDING: CONCEPT AND SYMBOLS: Composing and Decomposing Sets
BUILD ESSENTIAL UNDERSTANDING: SYMBOL USE: +, -, =

Materials and Directions for Teacher	*Instructional Cue*	*Student Expected Response* *Date:*						
1. GO: Addition and new addition equation	**Put the first number on your Setmaker.**	Student places first number under first circle.						
2. See above.	**Put second number on your Setmaker.**	Student places second number under second circle.						
3. See above.	**Count out the number of _____ you need to put in the first circle.**	Student counts out __ [items] and puts them in first circle.						
4. See above.	**Count out the number of ____ you need to put in the second circle.**	Student counts out __ [items] and puts them in second circle.						
5. See above.	**Show me what to do to find out how many you have altogether.**	Student slides [items] into 3rd circle (or indicates where to place [items] with eye gaze).						
6. See above.	**Count the _______.**	Student counts the [items].						
7. See above.	**How many do you have altogether?**	Student says/indicates total number of [items].						
8. GO: Subtraction and a new subtraction equation.	**Now, put the first number on your Take Away chart.**	Student places first number under first circle.						
9. See above.	**Put second number on your Take Away chart.**	Student places second number under the trash can.						
10. See above.	**Count out the number of ____ you need to put in the first circle.**	Student counts out __ [items] and puts them in first circle.						
11. See above.	**Now take away the number of ____ you need to put in the trash can.**	Student takes away __ [items] from first circle and puts them in trash can.						
12. See above.	**Show me what to do to find out how many you have left over.**	Student slides [items] into green circle (or indicates where to place items with eye gaze).						
13. See above.	**Count the _______.**	Student counts the [items].						
14. See above.	**How many do you have left over?**	Student says/indicates total number of [items].						
15. Symbol flash cards (+, -, =, ?, !, x, ÷)	**Show me equal.**	Student points/eye gazes to the equal sign.						

Student Name: ______________________________________

16. Move symbol flash cards around.	**Again. Show me equal.**	Student points/eye gazes to the equal sign.						
17. Move symbol flash cards around.	**Show me plus.**	Student points/eye gazes to the plus sign.						
18. Move symbol flash cards around.	**Again. Show me plus.**	Student points/eye gazes to the plus sign.						
19. Move symbol flash cards around.	**Show me minus.**	Student points/eye gazes to the minus sign.						
20. Move symbol flash cards around.	**Again. Show me minus.**	Student points/eye gazes to the minus sign.						
		NUMBER CORRECT:						

3rd GRADE BUILD A GRADE ALIGNED COMPONENT: MATCHING EXPRESSIONS TO WORD PROBLEMS
4th and 5th GRADE BUILD ESSENTIAL UNDERSTANDING: SYMBOL USE
BUILD ESSENTIAL UNDERSTANDING: CONCEPT: EQUALITY (“=”)

21. Student has word problem. Display + and 2 distracters AND “in all” card.	**A story problem says “in all.” Show me the symbol that tells us to put our sets together.**	Student selects +.						
22. Display + and 2 distracters in different order AND “total” card. Point to “total” card.	**A story problem says, “total.” Show me the symbol that tells us to put our sets together.**	Student selects +.						
23. Display + and 2 distracters in different order AND “altogether” card.	**A story problem says “altogether.” Show me the symbol that tells us to put our sets together.**	Student selects +.						
24. Display new word problem and expression choices (2 + 4, 2 x 4, 2 ÷ 4).	**“Elijah passed out the paintbrushes. He had 2 paintbrushes in his left hand and 4 in his right hand. Which of these will show how many paintbrushes Elijah has in all?”** Read each expression aloud while pointing to it: **2 + 4, 2 x 4, 2 ÷ 4**	Student selects correct expression.						
25. Display new word problem and expression choices (6 – 3, 6 + 3, 6 x 3).	**Arlo collected the crayons in pencil cases. He had 6 crayons in one case and 3 crayons in the other. Which of these show how many crayons he had total?** Read each expression aloud while pointing to it: **6 – 3, 6 + 3, 6 x 3**	Student selects correct expression.						
26. Display - and 2 distracters AND “left” card.	**A story problem says “left.” Show me the symbol that tells us to take away.**	Student selects -.						
27. Display - and 2 distracters in different order AND “remain” card.	**A story problem says, “remain.” Show me the symbol that tells us to take away.**	Student selects -.						

28. Display – and 2 distracters in different order AND “difference” card.	**A story problem says, “difference.” Show me the symbol that tells us to take away.**	Student selects -.						
29. Display new word problem and expression choices (10 – 4, 10 + 4, 10 X 4).	**Zatrel had 10 markers. He passed out 4. Which of these will show how many markers Zatrel has left?** Read each expression aloud while pointing to it: **10 – 4, 10 + 4, 10 X 4**	Student selects correct expression.						
30. Display new word problem and expression choices.	**Nehemiah collected 6 crayons. He threw 2 broken ones away. Which of these show how many crayons remain?** Read each expression aloud while pointing to it. **6 ÷ 2, 6 X 2, 6 – 2.**	Student selects correct expression.						
31. Display new word problem and expression choices.	**Hannah has 8 apples. Karen has 4 apples. Which of these shows the difference between the number of apples Hannah has and the number Karen has?** Read each expression aloud while pointing to it: **8+4, 8-4, 8X4.**	Student selects correct expression.						
32. Display subtraction word problem and expression choices.	**This says Cora has 4 crayons. She gave 2 to Mike. How many does she have left?** Read each expression aloud. **4 – 2, 3 + 7, 4 X 2. Point to an answer choice that has the same numbers.**	Student points to an answer choice with numbers 4 and 2. (doesn’t have to be correct answer yet)						
33. See above.	**Now, point to the word that tells you what symbol to use.**	Student points to “left.”						
34. See above.	Point to the answer choice student selected. **Now look here. Is this the symbol that tells us how many are left?**	Student indicates yes or no.						
35. See above.	*****If yes: **Good job. You found the expression 4 minus 2.** Mark this step correct on assessment. *If no: Read each answer choice aloud. **Try again. 4 – 2, 3 + 7, 4 X 2. Which one tells us how many she has left?**	Student selects 4 – 2.						
36. Write a number and = sign (e.g., 5 = ___).	Read **‘five equals ___.’ Which number goes on this side?** Point to right side.	Student says or selects the same number (e.g., 5).						
37. Write a different number and = sign (e.g., 3 = ___).	Read **‘three equals ___.’ Which number goes on this side?** Point to right side.	Student says or selects the same number (e.g., 3).						
38. Write a different number and = sign (e.g., 8 = ___).	Read **‘eight equals ___.’ Which number goes on this side?** Point to right side.	Student says or selects the same number (e.g., 8).						
39. Display Equality Visual with sign erased and place markers on each side.	Point to left side. **How many on this side?**	Student counts items.						
40. See above.	Point to right side. **How many on this side?**	Student counts items.						

Student Name: ______________________________________

41. See above.	**Are the numbers on each side the same? If they are, write the equal sign.**	Student indicates yes or no and/or writes the equal sign if same amount (students who cannot write may eye gaze to or Velcro the equal sign).						
		NUMBER CORRECT:						
4th BUILD A GRADE ALIGNED COMPONENT: MATCHING EQUATIONS TO REPRESENTATIONS								
42. Display new array of [items] with different number of rows and items per row and three equations.	**Show me the rows.**	Student runs fingers (or eye gazes) across each row.						
43. See above.	**How many in each row?**	Student counts each item in row.						
44. See above.	**Count the rows.**	Student counts the rows.						
45. See above.	**Find the equation that matches.**	Student selects correct addition equation.						
46. Display new array of [items] with different number of rows and items per row and three equations.	**How many [items] are there in all?**	Student counts the [items].						
47. See above.	**Point the equation that equals ___.**	Student selects correct equation.						
		NUMBER CORRECT:						
5th BUILD A GRADE ALIGNED COMPONENT: INDICATE WHETHER AN EQUATION IS TRUE								
48. Display GO: Addition and GO: Subtraction Give student an equation (e.g., 3 + 4).	**Show me which chart you will use to solve this equation.**	Student indicates correct chart.						
49. Present a new equation.	**Show me which chart you will use to solve this equation.**	Student indicates correct chart.						
50. Display new equation.	**You are going to tell me if this equation is true. An equation is true if the expressions on both sides of the equation are equal. Here is an equation. Before you decide if it is true you have to solve each side.** Point to left side. **Solve this side and write your answer below.**	Student solves left side of equation and writes/indicates answer.						
51. See above.	Point to right side of equation. **Now solve this side and write your answer below.**	Student solves right side of equation and writes/indicates answer.						
52. See above.	**Are both sides equal?**	Student answers yes (or no).						
53. See above.	**Is the equation true?**	Student answers yes (or no).						
		NUMBER CORRECT:						

Student Name: ______________________________

EQUATION SKILL TEST 1: CONCEPT AND SYMBOLS

Note to teachers: It may be helpful to use a cover sheet of paper. Pull the cover sheet down far enough to show the model and read the text. Then, pull the sheet of paper down to show the problem and read the directions. Record “+” for an independent correct response or “-“ for incorrect response in blank.

__ **MODEL: Watch me as I use the Setmaker to solve this addition equation.**

$$4 + 5 = \_\_\_$$

STUDENT PROBLEM: Use your Setmaker to solve this addition equation.

$$6 + 2 = \_\_\_$$

__ **MODEL: Watch me as I use the Take Away chart to solve this subtraction problem.**

$$7 – 2 = \_\_\_$$

STUDENT PROBLEM: Use your Take Away chart to solve this subtraction problem.

$$6 – 3 = \_\_\_$$

Student Name: ______________________________________

__ **MODEL: Watch me point to equal.**

$$=$$

STUDENT PROBLEM: Now you point to equal

= ? x

__ **MODEL: Watch me point to minus.**

$$-$$

STUDENT PROBLEM: Now you point to minus.

! ÷ −

__ **MODEL: Watch me point to plus.**

$$+$$

STUDENT PROBLEM: Now you point to plus.

÷ ? +

Student Name: ______________________________________

EQUATION SKILL TEST 2: Matching Expressions to Word Problems

Note to teachers: It may be helpful to use a cover sheet of paper. Pull the cover sheet down far enough to show the model and read the text. Then, pull the sheet of paper down to show the problem and read the directions. Record "+" for an independent correct response or "-" for incorrect response in blank.

__ Point to symbol for a story problem that says, "in all."

+ ? =

__ Point to symbol for a story problem that says, "total."

+ ! ÷

__ Point to symbol for a story problem that says, "altogether."

x ! +

__ Maddie has 2 erasers she picks up 2 more. How many does she have total? Which of these matches the word problem?

2 – 2	2 + 2	2 x 2

__ Point to symbol for a story problem that says, "left."

– ? !

__ Point to symbol for a story problem that says, "remain."

÷ x –

Student Name: ____

__ Point to symbol for a story problem that says, "difference."

$+$ $-$ X

__ La'Shandra has 7 pencils. She trashes 3 broken ones. How many remain?

6 + 2	7 X 3	7 – 3

__ Are the numbers on each side the same? If they are, write the equal sign.

__ Are the numbers on each side the same? If they are, write the equal sign.

Student Name: ____

EQUATION SKILL TEST 3: Matching Equations to Representation

Note to teachers: It may be helpful to use a cover sheet of paper. Pull the cover sheet down far enough to show the model and read the text. Then, pull the sheet of paper down to show the problem and read the directions. Record "+" for an independent correct response or "-" for incorrect response in blank.

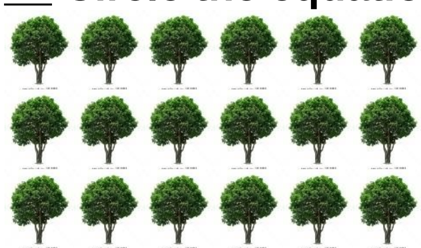
__ Circle the equation that shows what you see in this picture.

2 + 2 + 2 + 2 = 8

2 + 4 = 6

4 + 4 + 4 = 12

__ Circle the equation that shows what you see in this picture.



3 + 3+ 3 = 9

3 + 6 = 9

6 + 6 + 6 = 18

Student Name: ______________________________________

__ **Circle the equation that shows what you see in this picture.**

5 + 3 = 8

5 + 5 + 5 = 15

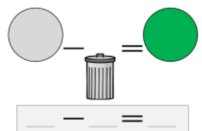
3 + 3 + 3 = 9

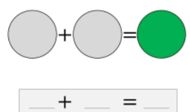
Student Name: ________________________________

EQUATION SKILL TEST 4: Indicating Whether an Equation is True

Note to teachers: It may be helpful to use a cover sheet of paper. Pull the cover sheet down far enough to show the model and read the text. Then, pull the sheet of paper down to show the problem and read the directions. Record “+” for an independent correct response or “-“ for incorrect response in blank.

__ Match the equations to the chart you can use to solve it.

8 – 1 ●   ● 

2 + 9 ●   ● 

__ Solve the each side of the equation and write your answer below.

$$10 - 1 = 8 + 2$$

___ = ___

__ Are both sides equal?

Yes   No

__ Is the equation true?

Yes   No

__ Solve the each side of the equation and write your answer below.

$$4 + 2 = 8 - 2$$

___ = ___

__ Are both sides equal?

Yes   No

__ Is the equation true?

Yes   No

National Center and State Collaborative

NCSC Math Activities with Scripted Systematic Instruction (MASSI): Middle School Equations

Reposted for NCSC state use on March 11, 2013. All materials in this version have been approved for public distribution with all necessary permissions. Selected excerpts are accompanied by annotated links to related media freely available online at the time of the publication of this document.

National Center and State Collaborative

The National Center and State Collaborative (NCSC) is applying the lessons learned from the past decade of research on alternate assessments based on alternate achievement standards (AA-AAS) to develop a multi-state comprehensive assessment system for students with significant cognitive disabilities. The project draws on a strong research base to develop an AA-AAS that is built from the ground up on powerful validity arguments linked to clear learning outcomes and defensible assessment results, to complement the work of the Race to the Top Common State Assessment Program (RTTA) consortia.

Our long-term goal is to ensure that students with significant cognitive disabilities achieve increasingly higher academic outcomes and leave high school ready for post-secondary options. A well-designed summative assessment alone is insufficient to achieve that goal. Thus, NCSC is developing a full system intended to support educators, which includes formative assessment tools and strategies, professional development on appropriate interim uses of data for progress monitoring, and management systems to ease the burdens of administration and documentation. All partners share a commitment to the research-to-practice focus of the project and the development of a comprehensive model of curriculum, instruction, assessment, and supportive professional development. These supports will improve the alignment of the entire system and strengthen the validity of inferences of the system of assessments.

IDEAs that Work
U.S. Office of Special Education Programs

The contents of this lesson were developed as part of the National Center and State Collaborative by Keri Bethune, Julie Thompson, Alicia Saunders, and Diane Browder at University of North Carolina at Charlotte and verified by Amy Lehew, math content expert, under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the U.S. Department of Education and no assumption of endorsement by the Federal government should be made.

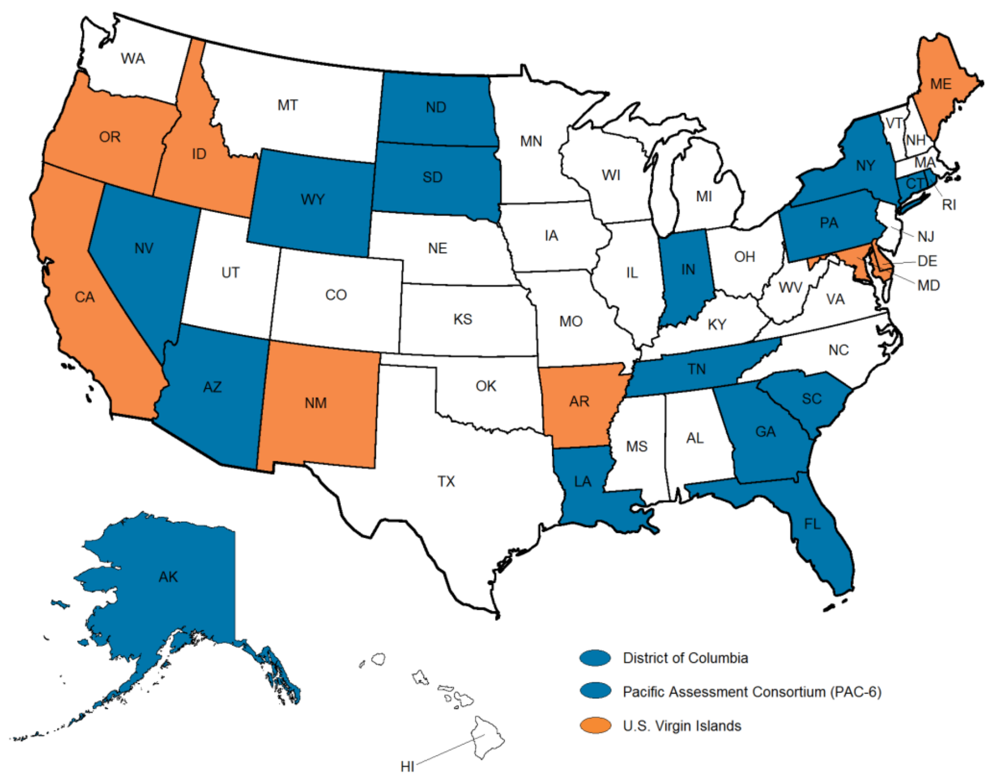
NCSC

National Center and State Collaborative

NCSC is a collaborative of 18 states and five organizations.

The states include (shown in blue on map): Alaska, Arizona, Connecticut, District of Columbia, Florida, Georgia, Indiana, Louisiana, Nevada, New York, North Dakota, Pacific Assessment Consortium (PAC-6)[1], Pennsylvania, Rhode Island, South Carolina, South Dakota, Tennessee, and Wyoming.

Tier II states are partners in curriculum, instruction, and professional development implementation but are not part of the assessment development work. They are (shown in orange on map): Arkansas, California, Delaware, Idaho, Maine, Maryland, New Mexico, Oregon, and U.S. Virgin Islands.



[1] The Pacific Assessment Consortium (including the entities of American Samoa, Commonwealth of the Northern Mariana Islands, Federated States of Micronesia, Guam, Republic of Palau, and Republic of the Marshall Islands) partner with NCSC as one state, led by the University of Guam Center for Excellence in Developmental Disabilities Education, Research, and Service (CEDDERS).

The five partner organizations include: The National Center on Educational Outcomes (NCEO) at the University of Minnesota, The National Center for the Improvement of Educational Assessment (Center for Assessment), The University of North Carolina at Charlotte, The University of Kentucky, and edCount, LLC.

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National Center and State Collaborative

NCSC Math Activities with Scripted Systematic Instruction (MASSI): Middle School Equations

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January 2013

Reposted March 11, 2013

MASSI: Math Activities with Scripted Systematic Instruction

Activity: Going Bowling

Grade Band: Grades 6-8

Concept: Equations

Common Core State Standard	Core Content Connectors	MASSI OBJECTIVES
6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.	**6th** 6. PRF.1d1: Solve real world single step linear equations	Write an equation given story problem and solve equation.
7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.	**7th** 7.PRF.1g2: Use variables to represent quantities in real-world or mathematical problems, and construct simple equations and inequalities to solve problems by reasoning about the quantities.	Write an equation given story problem and solve equation.
8.EE.7 Solve linear equations in one variable.	**8th** 8.PRF.1g3 Solve linear equations with 1 variable	Given multiplication equation solve for variable.

Be sure to provide specific practice to students on the skills that correspond to their grade level.

Teaching Materials: story problems (cut apart and laminated), equation template (one for teacher and each student, laminated), number line (one for teacher and each student, laminated), bowling photos, isolating and solving for variable template (one for teacher and each student, laminated). Note: Teachers may rewrite story problems using names of the students in the class.

Other Materials: calculators for each student; snack and other items students could practice buying and selling at a bowling alley concession stand; realistic looking pretend money or real money

Worksheets: worksheet for each section of lesson

Assessments: progress monitoring and skills tests for each section of lesson

TEACHING OVERVIEW: The first section of the MASSI provides remedial practice on identifying numbers in story problems. For students with no number recognition skills, rewrite story with numerals and add objects or pictures above numbers to help them identify. 6th and 7th graders will learn to create an addition equation, using a variable for an unknown number, from a story problem and solve for the variable. This is a good review for your 8th graders. 8th graders will be doing the same thing but with multiplication story problems. This is useful for your 6th and 7th graders to gain fluency with equations and practice calculator skills, or you can provide 6th and 7th graders with more addition problems with one variable while 8th graders work on multiplication.

SCRIPT FOR LESSON

BUILD ESSENTIAL UNDERSTANDINGS – CONCEPT AND SYMBOLS: Identifying the Number from a Sentence *(Skip this section for students who understand these relationships and can identify these concepts)*

INTRODUCE ACTIVITY: Today we are going to learn how to keep track of our money when we go bowling. Here are some pictures of a bowling alley. Have any of you been bowling before? Give students opportunity to answer yes/no. **We have to pay to go bowling. It costs money for shoes, games, and snacks. Today we are going to practice learning how to keep track of what we might spend at a bowling alley by using variables in our equations.**

INTRODUCE THE PROBLEM: Display story problem. **Here is a story about going bowling. "Lucia had $15 for bowling. Lucia spent $8 on shoes and games. She spent the rest on snacks. How much did Lucia spend on snacks?"**

MODEL THE PROCESS: The first thing I need to do to answer the story problem is to write an equation, but before I can write an equation I need to find the numbers in my story problem. Do you see any numbers? Give students opportunity to answer yes/no. **Point to a number.** Wait 4 seconds for students to respond. **Here is a number.** Circle 15. **It is number 15. Do you see another number?** Give students opportunity to answer yes/no. **Point to a number.** Wait 4 seconds for students to respond. **Good. That is number 8.** Circle 8.

STUDENT PRACTICE: Now it is your turn to practice. Display story problem. Use CONSTANT TIME DELAY script as need to help students with each step. **Note: in this exercise, change numbers daily to prevent rote memorization.*

CHECK AND SCORE

STEP	Teacher Says/Does	Student Response
1.	**Let's try another problem. "Shawn has 5 dollars in his hand. He has more in his pocket. He has 12 dollars in total. How many are in his pocket?" Circle the numbers.**	Student circles/eye gazes to numbers in story problem.

6th 7th BUILD A GRADE ALIGNED COMPONENT: Creating an Equation
8th BUILD ESSENTIAL UNDERSTANDING: Symbol Concept

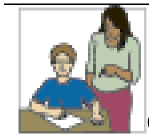
INTRODUCE PROBLEM: Display “Shawn” story problem. **We are going to solve these problems. In both of the problems there is a number we don’t know yet. We will solve the problem using a variable. A variable is a letter that can be used in place of a number you don’t know yet.**

MODEL THE PROCESS: Model using Equation Template. **Let’s review our story problem about Shawn. I am going to read the three sentences. One doesn’t have a number. We’ll use a letter when we don’t know the number. Let’s put them in our equation as we go. “Shawn has 5 dollars in his hand.” How many dollars are in his hand?** Provide wait time for students to respond. **Yes, 5.** Write five on line. **He has more in his pocket?** Provide wait time for students to respond. **We don’t know do we? There isn’t a number. Let’s use a “p” for pocket.** Write + p. **I am writing “+ p” because we know that Shawn had 5 dollars in his hand AND some dollars in his pocket. I want to add the 5 and the p.** Point to last line in problem. **He has 12 dollars in total. How many in total?** Provide wait time for students to respond. **Yes, 12. So my equation is 5 + p = 12.**

STUDENT PRACTICE: Display “Slade” story problem. Provide student with the Equation Template. Use SYSTEM OF LEAST PROMPTS script as needed to help students with each step.

CHECK AND SCORE

Step	Teacher Says/Does	Student Response
2.	**Let’s read the problem together. “Slade bought 2 candy bars. Later he bought more candy. He ate 4 pieces total.” Write the equation that represents this problem. Use the letter c to represent the unknown amount of candy. Write the first number.** ** Have the students write the numbers/digits on the template, but do not score writing ability. If students are unable to write the number, they can use number/letter stamps or direct the teacher to write it for them.	Student writes/stamps/uses Velcro numbers/points to/eye gazes to write 2. **The full correct answer is 2 + c = 4 (This is broken down into task analyzed steps)
3.	Wait for students to independently write plus or say **“What do you write next?”**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to the plus sign.
4.	Wait for students to independently write c or say **“What do you write next?”**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to write c.
5.	Wait for students to independently write equal or say **“What do you write next?”**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to the equal sign.
6.	Wait for students to independently write 4 or say **“What do you write next?”**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to write 4.

**Great work writing the equation 2 + c = 4! Let's practice one more before we read about going bowling.** Display "Jacob" story problem. Provide student with Equation Template. Use SYSTEM OF LEAST PROMPTS script as needed to help students with each step.

CHECK AND SCORE

Step	Teacher Says/Does	Student Response
7.	**Let's read the problem together. "Jacob paid 4 dollars for shoes. Then he paid for the game. He used all of the $10 that he brought." Write the equation that represents this problem. Use the letter g to represent the unknown cost of the game. Write the first number.**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to write 4. **The full correct answer is 4 + g = 10 (this is broken down into task analyzed steps)
8.	Wait for students to independently write plus or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to the plus sign.
9.	Wait for students to independently write g or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to write g.
10.	Wait for students to independently write equal or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to the equal sign.
11.	Wait for students to independently write 10 or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to write 10.

Great work writing the equation 4 + g = 10!

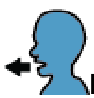
This may be a good stopping point. Let students practice working to buy a snack at the bowling alley. They can make a selection and purchase with response options provided if needed. You can use the worksheet for this level as additional guided practice or to send home as homework.

	Teacher Says/Does	Student Response	Error Correction
INDEPENDENT PRACTICE: **Equations Skills Test**	Give each student the *Equations Skills Test 1*. **Read directions for each problem and have student select response. Record whether response is correct or incorrect.**	Only provide praise for completing assessment (if student needs encouragement). Do not provide specific praise for correct answers while student is testing.	Once the student has completed the test, review missed problems with the student.
	***NOW*** *Stop the lesson here and repeat tomorrow if student is not yet getting at least 6 independent correct responses. Score responses 1-11 on the Equation Progress Monitoring Sheet if you did not do so while teaching.*	***NEXT*** *Remember the goal is for students to be able to write equations given story problem and solve the addition equation* **6th 7th** *to find the variable, and given multiplication solve to find variable* **8th** *. As soon as possible, move into the remainder of the lesson to hit the target CCCs for this grade band. Be sure to provide some practice for students at their specific grade level concept even if they have not achieved mastery of previous concepts.*	

6th 7th BUILD A GRADE ALIGNED COMPONENT: Solving Equation for Variable

8th BUILD ESSENTIAL UNDERSTANDING: Continue Building Symbol Concept

INTRODUCE THE PROBLEM: Now we have learned to write equations when we have a story problem. Let's learn how to solve the equations. Yesterday we wrote an equation for the story problem about Shawn. Let's read the story problem again: "Shawn has 5 dollars in his hand. He has more in his pocket. He has 12 dollars in total. How many are in his pocket?" Remember, the equation was 5 + p = 12. Let's learn how to solve for p.

**MODEL THE PROCESS:** Display Equation Template with number line. **We can use a number line to solve this problem**. **I will put my finger on 5 and count how many jumps it takes to get to 12.** Place your finger on five and count. **One. Two. Three. Four. Five. Six. Seven. I will write p equals seven. He has $7 in his pocket.**

STUDENT PRACTICE: Now it's your turn to solve an equation for a variable. Display story problem, write equation on paper with c=___, and give students number line. *Note: you can adapt this by making it larger for students to eye gaze towards, or make tactile by using puffy paint on each point on the number line, etc. Use MODEL-LEAD-TEST script to teach counting on number line if needed.

CHECK AND SCORE

Step	Teacher Says/Does	Student Response
12.	**Look at the story problem. "Slade bought 2 candy bars. Later he bought more candy. He ate 4 pieces total." The equation is 2 + c = 4. Use the number line to solve the problem. What number do you start at?**	Student points to, says, or otherwise indicates 2. Note: If needed circle numbers, or have student circle numbers, so they remember where to start and stop.
13.	**Good. Now what number will you count to?**	Student points to, says, or otherwise indicates 4.
14.	**Start at 2 and count until you get to 4.**	Student places finger on (or begins eye gaze) 2 and counts jumps…One. Two (stopping at number 4).
15.	**What does c equal? Write it.**	Student points to, says, or otherwise identifies 2 and writes 2.

Great work today! You solved the equation for c. C equaled 2 candy bars. Let's do another. Display "Jacob" story problem. Use MODEL-LEAD-TEST script to teach counting on number line if needed.

Step	Teacher Says/Does	Student Response
16.	**Let's do the other problem from before. Let's read the problem together. Jacob paid 4 dollars for shoes. Then he paid for the game. He used all of the $10 that he brought." We wrote that equation as: 4 + g = 10. Now, you need to solve for g.** Do not provide additional verbal instructions outside of prompting strategies.	Student points to, says, or otherwise indicates 4.
17.	Wait for students to begin counting up or say "**What's next?"**	Student places finger on (or begins eye gaze) 4 and counts jumps…One. Two. Three. Four. Five. Six (stopping at number 10).
18.	Wait for students to write 6 or say "**What's next?"**	Student points to, says, or otherwise identifies 6 and writes 6.

Good job! You solved 4 + g = 10, and g = $6. You are doing nice work today.

This may be a good stopping point. Let students practice working to buy a snack at the bowling alley. They can make a selection and purchase with response options provided if needed. Model writing down the equation before giving the order. There is a worksheet with this level, which can be used as additional guided practice or to send home as homework.

	Teacher Says/Does	Student Response	Error Correction
INDEPENDENT PRACTICE: Equations Skill Test	Give student the *Equations Skills Test 2.* **Read directions for each problem and have student select response. Record whether response is correct or incorrect.**	Only provide praise for completing assessment (if student needs encouragement). Do not provide specific praise for correct answers while student is testing.	Once the student has completed the test, review missed problems with the student.
NOW *Stop the lesson here and repeat tomorrow if student is not yet getting at least 4 independent correct responses. Score responses 12 - 18 on the Equation Progress Monitoring Sheet if you did not do so while teaching.*		***NEXT*** *Remember the goal is for students to be able to write equations given story problem and solve the addition equation* **6th 7th** *to find the variable, and given multiplication solve to find variable* **8th***. As soon as possible, move into the remainder of the lesson to hit the target CCCs for this grade band. Be sure to provide some practice for students at their specific grade level concept even if they have not achieved mastery of previous concepts.*	

8th BUILD A GRADE ALIGNED COMPONENT: Solving Multiplication Equation for Variable

INTRODUCE THE PROBLEM: Sometimes we might need to solve for a variable in a multiplication equation. Listen to this word problem. "Marion paid for 4 bowling games. It cost 20 dollars total. How much did each game cost?" The key word "each" tells us this is a multiplication problem. We want to know the price of *each* game, so we will use the variable "g" for games.

MODEL THE PROCESS: Display the equation and template for multiplication. See next page for the "process for isolating a variable." For more on isolating variables, see the Curriculum Resource Guide for Equations. Note: Students may not know that if the number (known as coefficient) and letter are next to each other that it means to multiply, even if there is no multiplication sign. They also may not know to divide when numbers are written over and under a line. Please explain this to your students if they do not have these pre-requisite skills.
The equation for this problem is "4 times g equals 20". Point to g. **The first step is to isolate the variable. That means that g needs to be alone on this side of the equation** (point to left side of equation)**. Remember, both sides of the equation must be equal! So, whenever we change one side of the equation, we must change the other side in the same way. In order to isolate the variable, we use the inverse operation to undo the side of the equation with the variable. Can you show me the side with the variable?** Wait for student(s) to point. **Inverse operation means opposite operation, so the inverse operation of multiplication is division. What is the inverse operation of multiplication?** Wait for students to respond. **Yes, division is the inverse operation, so we must divide both sides by 4 to get g alone. Let's write that on the equation template. Watch what I write.** Underline each side of the equation and write 4 below the line. **The fours on top and bottom cancel each other out. So we can draw an X over them.** Draw an X on both 4s on the left side. Rewrite the equation so that it says g = $\frac{20}{4}$. Point to this equation and say, **This is what the equation looks like after we cancel out the 4s. Now we have isolated the g. We just need to divide 20 by 4 to solve for g.** Display calculator. **I will use my calculator to solve for g. First I will type 20, then push the division sign, then push 4, and lastly push equal {or ENTER** depending on calculator type}**. The answer is 5. G = five. That means each game cost five dollars.**

Process for Isolating a Variable

$$4 \quad g = 20$$

$$\frac{\cancel{4}g}{\cancel{4}} = \frac{20}{4}$$

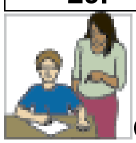
$$g = \frac{20}{4}$$

$$g = 5$$

STUDENT PRACTICE: Display story problem that begins with “3 groups of friends came to the bowling alley.” Give students blank “isolating and solving for variable" template. Use SYSTEM OF LEAST PROMPTS script as needed to help students with each step.

CHECK AND SCORE

Step:	Teacher Says/Does	Student Response
19.	**Here is another multiplication word problem. Let's read the problem together. "3 groups of friends came to the bowling alley. Each group was the same size. There were 12 friends altogether. How many were in each group?" Below is the equation for this problem. It says 3 times g = 12. g stands for group. Write the equation on your paper. Write the first number in the yellow box.** ** Have the students write the answer on the template, but do not score writing ability. If students are unable to write the number, they can use number stamps or direct the teacher to write it for them.	Student writes 3 in yellow box.
20.	**Now write the variable in the blue box.**	Student writes g in blue box.
21.	**Write the total in the purple box.**	Student writes 12 in purple box.
22.	**Remember we need to isolate the variable. Write the equation again and divide both sides by three.**	Student writes equation on next line and divides each side by three.
23.	Point to the left side of their equation. **The threes cancel each other out on this side. Cross out the threes.**	Student crosses out the threes on the left side.
24.	**Write the variable and the left over numbers below.**	Student writes the remaining variable and numbers below.
25.	**Good you isolated the variable. Now use your calculator to divide 12 by 3. Type 12.**	Student types 12 into calculator.
26.	**Push divided by.**	Student pushes division symbol.
27.	**Push 3.**	Student pushes 3.
28.	**Push enter.**	Student pushes enter.
29.	**Write your answer.**	Student writes answer.


Great job solving the multiplication equation. Now let's do another. Display problem 4x=20. Give students blank "isolating and solving for variable" template. Use SYSTEM OF LEAST PROMPTS script as needed to help students with each step.

CHECK AND SCORE

Step:	Teacher Says/Does	Student Response
30.	**Here is another multiplication equation. It says 4 times x = 20. Solve for x.** Do not provide additional verbal instructions outside of prompting strategies.	Student writes equation on his/her paper.
31.	Wait for students to rewrite equation below or say "**What's next?"**	Student rewrites equation on next line.
32.	Wait for students to divide both sides by 4 or say "**What's next?"**	Student divides both sides by 4.
33.	Wait for students to cross out fours on left side or say "**What's next?"**	Student crosses out fours on left side.

34.	Wait for students to rewrite remaining numbers below or say “**What’s next?”**	Student rewrites remaining numbers below.
35.	Wait for students to use calculator to solve for x or say “**What’s next?”**	Student uses calculator to solve for x.
36.	Wait for students to write answer or say “**What’s next?”**	Student writes answer below.

Good job! You solved the equation 4x = 20 and found that x=5. You are doing nice work today.

This is the end of the lesson. Let students practice working to buy a snack at the bowling alley. They can make a selection and purchase with response options provided if needed. Model writing down the equation before giving the order. There is a worksheet with this level. You can use this for additional guided practice or to send home as homework.

	Teacher Says/Does	**Student Response**	**Error Correction**
INDEPENDENT PRACTICE: Equations Skills Test	Give student the *Equations Skills Test 2.* **Read directions for each problem and have student select response. Record whether response is correct or incorrect.**	Only provide praise for completing assessment (if student needs encouragement). Do not provide specific praise for correct answers while student is testing.	Once the student has completed the test, review missed problems with the student.

Troubleshooting and Data-Based Decision Making for Equations Test:
If student is unable to complete any items on the equations test independently and correctly, go back and teach one problem step-by-step.

MASSI CULMINATING ACTIVITY: Go bowling! Have students use the equations to figure out how much money they need.

BUILD TOWARDS FULL GRADE LEVEL COMPETENCE

Here are ideas to build competence towards the full grade level competence using this same activity. See the unit plan for more ideas and talk with the general education teacher for more ideas.

Component	Activity	What Student Does	Generalization/ Fluency
Write subtraction, multiplication, and division equations with one variable from story problem.	Students are presented with variety of story problems related to bowling.	Students writes equation based on story problem.	Different amounts and story problems daily.
Solve subtraction, multiplication, and division equations with one variable.	Present students with variety of story problems. Teach them the rule: “What you do to one side, you must do the same to the other,” as well as inverse operations.	Students solve the equations to find the value of the variable. Aim to have students do it independently without using the Task Analysis or equation template.	Different numbers, types of equations, and story problems daily.

Teaching Materials (Story Problem):

Lucia had $15 for bowling. Lucia spent $8 on shoes and games. She spent the rest on snacks. How much did Lucia spend on snacks?

Shawn has 5 dollars in his hand. He has more in his pocket. He has 12 dollars in total. How many are in his pocket?

Slade bought 2 candy bars. Later he bought more candy. He ate 4 bars total. How many bars did he buy later?

Jacob paid 4 dollars for shoes. Then he paid for the game. He used all of the $10 that he brought. How much did the game cost?

---

3 groups of friends came to the bowling alley. Each group was the same size. There were 12 friends altogether. How many were in each group?

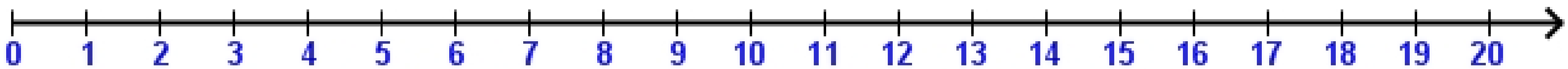
Equations:

$3g=12$

$4x=20$

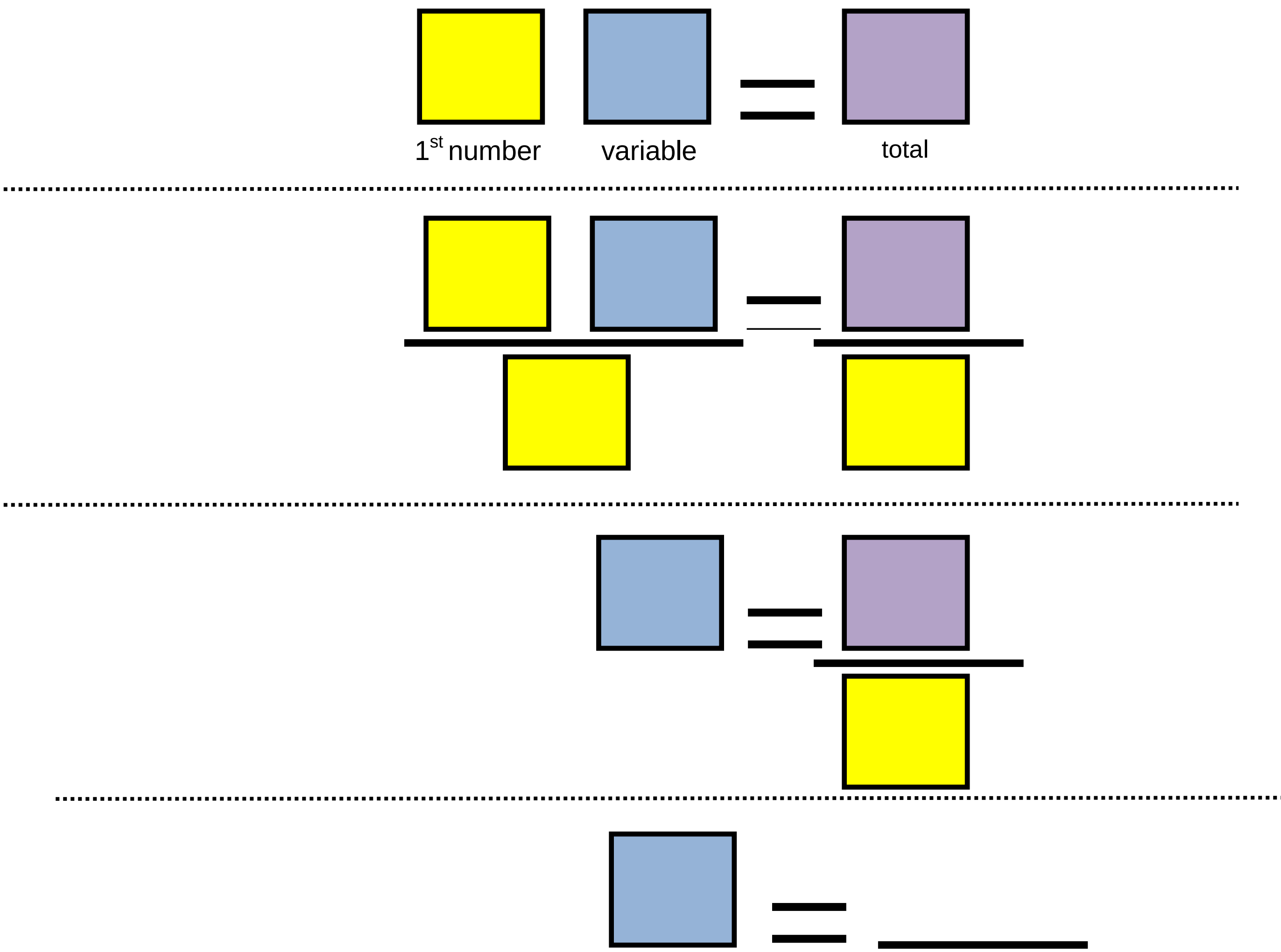
Equation Template for 6th and 7th grade skills

Equation: ______________________



Multiplication Equation: Isolating and solving for variable template (Laminate for practice during lesson)

Equation: ________



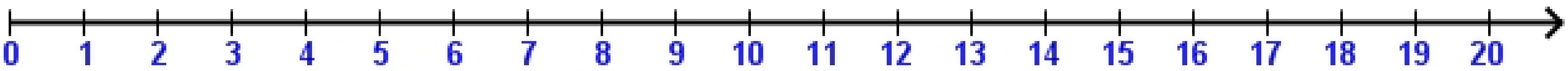
Circle the numbers in the story problem.

Mei collected 3 shells. Her father gave her a few. Now she has 9 shells.

Write an equation from the word problem.

Nasir earned $12 allowance for mowing the lawn. He earned more money washing the car. Now he had a total of $18. Write an equation. Use c as the variable for the unknown amount earned.

Equation: ______________________



1) $5 + x = 9$. Solve for x.

$x =$ ______

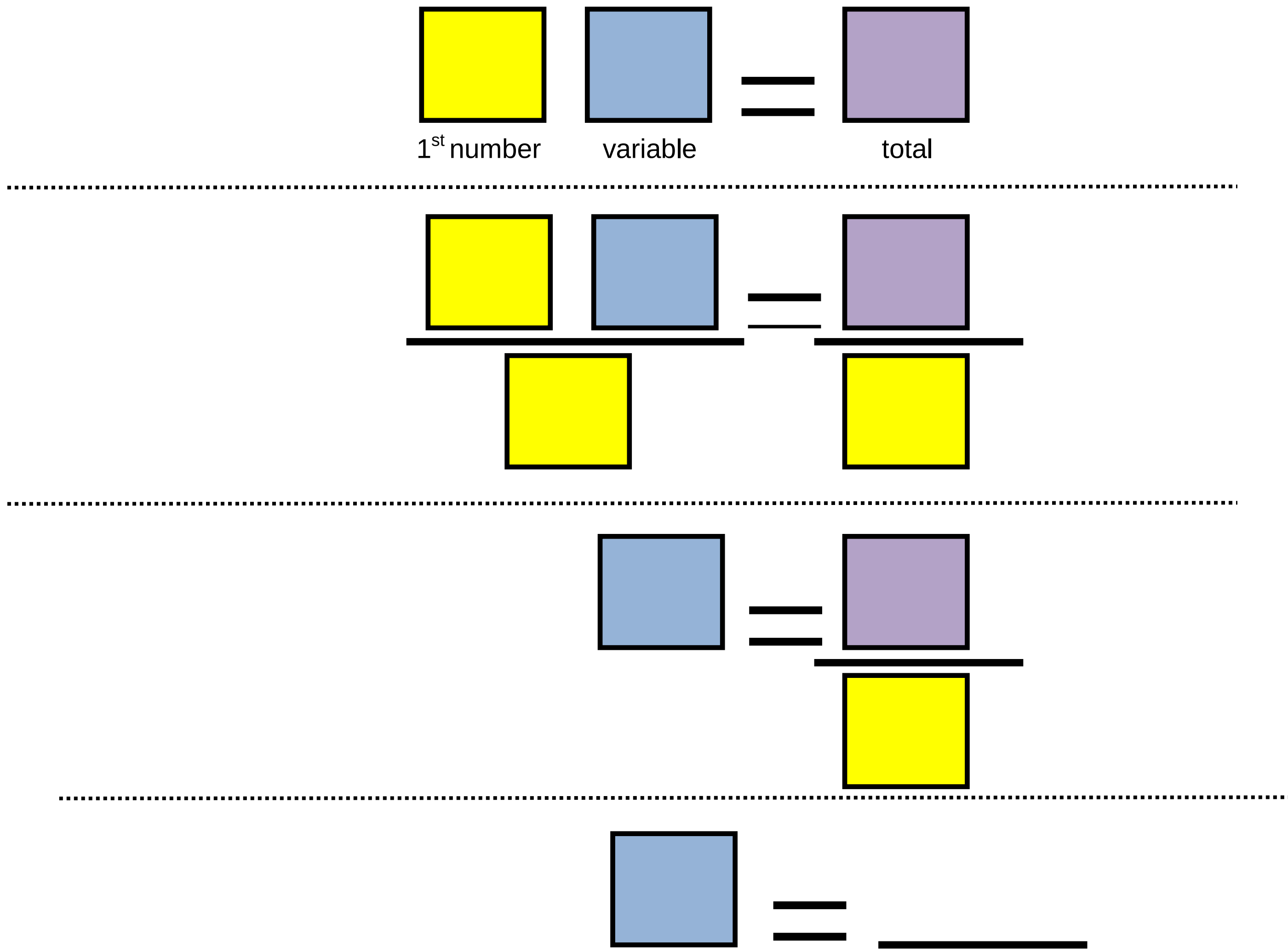
2) $8 + m = 10$. Solve for m.

$m =$ ______

3) $2 + r = 7$. Solve for r.

$r =$ ______

Equation: __________



National Center and State Collaborative

NCSC Math Activities with Scripted Systematic Instruction (MASSI): Middle School Equations Progress Monitoring and Skills Test

National Center and State Collaborative

The National Center and State Collaborative (NCSC) is applying the lessons learned from the past decade of research on alternate assessments based on alternate achievement standards (AA-AAS) to develop a multi-state comprehensive assessment system for students with significant cognitive disabilities. The project draws on a strong research base to develop an AA-AAS that is built from the ground up on powerful validity arguments linked to clear learning outcomes and defensible assessment results, to complement the work of the Race to the Top Common State Assessment Program (RTTA) consortia.

Our long-term goal is to ensure that students with significant cognitive disabilities achieve increasingly higher academic outcomes and leave high school ready for post-secondary options. A well-designed summative assessment alone is insufficient to achieve that goal. Thus, NCSC is developing a full system intended to support educators, which includes formative assessment tools and strategies, professional development on appropriate interim uses of data for progress monitoring, and management systems to ease the burdens of administration and documentation. All partners share a commitment to the research-to-practice focus of the project and the development of a comprehensive model of curriculum, instruction, assessment, and supportive professional development. These supports will improve the alignment of the entire system and strengthen the validity of inferences of the system of assessments.

IDEAs that Work
U.S. Office of Special Education Programs

The contents of this assessment were developed as part of the National Center and State Collaborative by Keri Bethune, Julie Thompson, Alicia Saunders, and Diane Browder at University of North Carolina at Charlotte and verified by Amy Lehew, math content expert, under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the U.S. Department of Education and no assumption of endorsement by the Federal government should be made.

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National Center and State Collaborative

NCSC is a collaborative of 18 states and five organizations.

The states include (shown in blue on map): Alaska, Arizona, Connecticut, District of Columbia, Florida, Georgia, Indiana, Louisiana, Nevada, New York, North Dakota, Pacific Assessment Consortium (PAC-6)[1], Pennsylvania, Rhode Island, South Carolina, South Dakota, Tennessee, and Wyoming.

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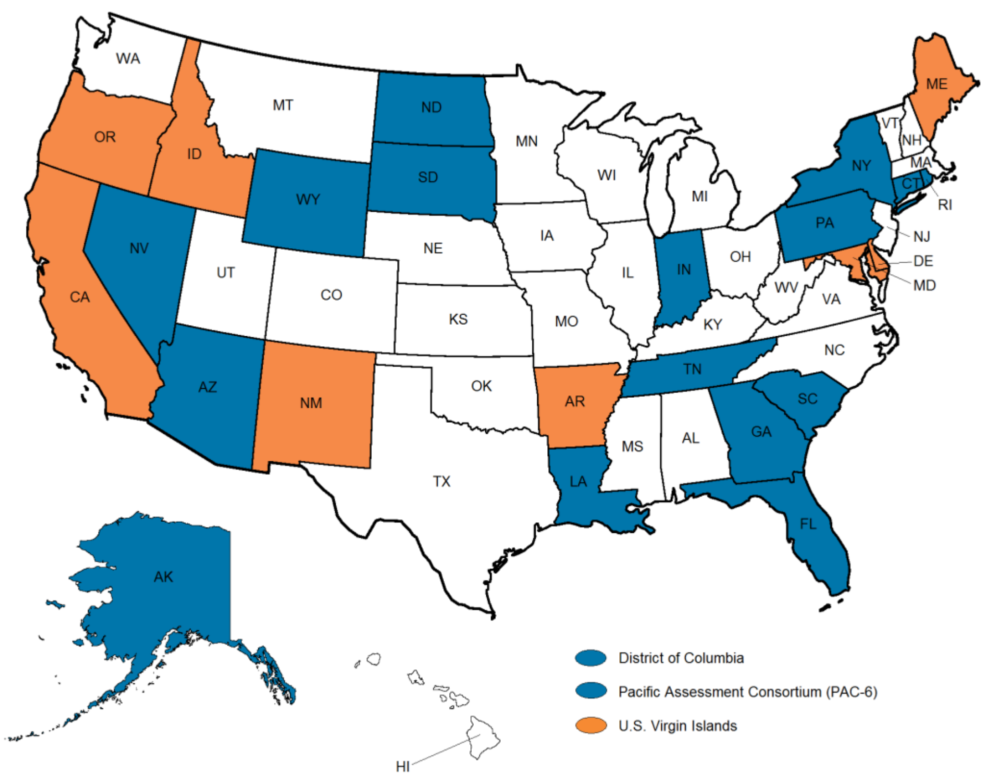

[1] The Pacific Assessment Consortium (including the entities of American Samoa, Commonwealth of the Northern Mariana Islands, Federated States of Micronesia, Guam, Republic of Palau, and Republic of the Marshall Islands) partner with NCSC as one state, led by the University of Guam Center for Excellence in Developmental Disabilities Education, Research, and Service (CEDDERS).

ncsc

National Center and State Collaborative

The five partner organizations include: The National Center on Educational Outcomes (NCEO) at the University of Minnesota, The National Center for the Improvement of Educational Assessment (Center for Assessment), The University of North Carolina at Charlotte, The University of Kentucky, and edCount, LLC.

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Center for Assessment

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National Center and State Collaborative

NCSC Math Activities with Scripted Systematic Instruction (MASSI): Middle School Equations Progress Monitoring and Skills Test

Keri Bethune
Julie Thompson
Alicia Saunders
Diane Browder
Amy Lehew

January 2013

Reposted March 11, 2013

Student Name: ____________________________________

MASSI: Middle School Equations

Options for Progress Monitoring/Formative Assessment

1. Middle School Equations Progress Monitoring (pg. 7-9): record student responses made during instruction on data sheet provided; teacher records each step correct during the lesson.

2. Middle School Equations Skills Test (pg. 10-13): a brief on demand performance assessment; could be given weekly to see if student has mastered this lesson; also helps student practice responding in a test format.

    a. NOTE: The Skill Test can be used as a baseline assessment to check for any skills the student may already have prior to beginning the MASSI.

    b. NOTE: The Skill Test can also be readministered to check for maintenance throughout the year.

Student Name: ______________________________________

Middle School Equations Progress Monitoring

Directions: Score each step during instruction or as soon as the lesson is complete. Score the step as unprompted correct with a "+" or prompted correct with a "p." Graph the number of unprompted correct responses to monitor progress.

BUILD ESSENTIAL UNDERSTANDINGS – CONCEPT AND SYMBOLS: Identifying the Number from a Sentence

6th 7th BUILD A GRADE ALIGNED COMPONENT: Creating an Equation

8th BUILD ESSENTIAL UNDERSTANDING: Symbol Concept

Materials and Directions for Teacher	*Instructional Cue*	*Student Expected Response* *Date:*						
1. Student has "Shawn" story problem.	**Circle the numbers.**	Student circles/eye gazes to numbers in story problem.						
2. "Slade" story problem and equation template	**Write the first number.**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to "2".						
3. See above.	Wait for students to independently write plus or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to "+".						
4. See above.	Wait for students to independently write c or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to "c".						
5. See above.	Wait for students to independently write equal or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to "=".						
6. See above.	Wait for students to independently write 4 or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to "4".						
7. "Jacob" story problem and equation template	**Write the equation that represents this problem. Use the letter g to represent the unknown cost of the game. Write the first number.**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to "4".						
8. See above.	Wait for students to independently write plus or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to "+".						
9. See above.	Wait for students to independently write g or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to "g".						
10. See above.	Wait for students to independently write equal or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to "=".						
11. See above.	Wait for students to independently write 10 or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to "10".						
		NUMBER CORRECT:						

Student Name: ____

6th 7th BUILD A GRADE ALIGNED COMPONENT: Solving Equation for Variable 8th BUILD ESSENTIAL UNDERSTANDING: Continue Building Symbol Concept								
12. "Slade" story problem, paper with equation written on it, and number line.	**What number do you start at?**	Student points to, says, or otherwise indicates "2".						
13. See above.	**Now what number will you count to?**	Student points to, says, or otherwise indicates "4".						
14. See above.	**Start at 2 and count until you get to 4.**	Student places finger on (or begins eye gaze) 2 and counts jumps…One. Two (stopping at number 4).						
15. See above.	**What does c equal? Write it.**	Student points to, says, or otherwise identifies 2 and writes "2".						
16. "Jacob" story problem, paper with equation written on it, and number line.	**Now, you need to solve for g.**	Student points to, says, or otherwise indicates "4".						
17. See above.	Wait for students to begin counting up or say "**What's next?"**	Student places finger on (or begins eye gaze) 4 and counts jumps…One. Two. Three. Four. Five. Six (stopping at number 10).						
18. See above.	Wait for students to write 6 or say "**What's next?"**	Student points to, says, or otherwise identifies 6 and writes "6".						
		NUMBER CORRECT:						

Student Name: ______________________________________

8^{th} **BUILD A GRADE ALIGNED COMPONENT: Solving Multiplication Equation for Variable**								
19. Story problem, blank "isolating and solving for variable" template, and calculator	**Write the first number in the yellow box.**	Student writes 3 in yellow box.						
20. See above.	**Now write the variable in the blue box.**	Student writes g in blue box.						
21. See above.	**Write the total in the purple box.**	Student writes 12 in purple box.						
22. See above.	**Remember we need to isolate the variable. Write the equation again and divide both sides by three.**	Student writes equation on next line and divides each side by three.						
23. See above.	Point to the left side of their equation. **The threes cancel each other out on this side. Cross out the threes.**	Student crosses out the threes on the left side.						
24. See above.	**Write the variable and the left over numbers below.**	Student writes the remaining variable and numbers below.						
25. See above.	**Good you isolated the variable. Now use your calculator to divide 12 by 3. Type 12.**	Student types 12 into calculator.						
26. See above.	**Push divided by.**	Student pushes division symbol.						
27. See above.	**Push 3.**	Student pushes 3.						
28. See above.	**Push enter.**	Student pushes enter.						
29. See above.	**Write your answer.**	Student writes answer.						
30. Problem 4x=20, "isolating and solving for variable" template, and calculator.	**Solve for x.**	Student writes equation on his/her worksheet.						
31. See above.	Wait for students to rewrite equation below or say "**What's next?"**	Student rewrites equation on next line.						
32. See above..	Wait for students to divide both sides by 4 or say "**What's next?"**	Student divides both sides by 4.						
33. See above.	Wait for students to cross out fours on left side or say "**What's next?"**	Student crosses out fours on left side.						
34. See above.	Wait for students to rewrite remaining numbers below or say "**What's next?"**	Student rewrites remaining numbers below.						
35. See above.	Wait for students to use calculator to solve for x or say "**What's next?"**	Student uses calculator to solve for x.						
36. See above.	Wait for students to write answer or say "**What's next?"**	Student writes answer below.						
		NUMBER CORRECT:						

Student Name: ______________________________________

EQUATION SKILL TEST 1: CONCEPT AND SYMBOLS

Note to teachers: It may be helpful to use a cover sheet of paper. Pull the cover sheet down far enough to show the model and read the text. Then, pull the sheet of paper down to show the problem and read the directions. Record “+” for an independent correct response or “-“ for incorrect response in blank.

__ MODEL: Watch me as I circle the numbers in the word problem.

Adolfo had 4 pencils. He found more. Now he has 6 pencils.

STUDENT PROBLEM: Your turn. Circle the numbers in the word problem.

Puja collected 3 watercolor sets. Her teacher handed her some more. She has 7 total.

Student Name: ______________________________________

__ **MODEL: Watch me as I write an equation from the word problem.**

Izzy painted 3 flowers. Later he painted some more. He painted 7 flowers in all.

Equation: ______________________

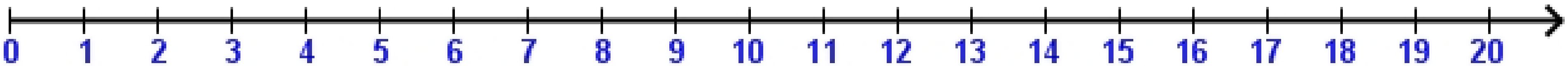
STUDENT PROBLEM: Write an equation from the word problem.

Jesse bought 1 postcard. Then she bought some more. Now she had 4 postcards.

Equation: ______________________

Student Name: ______________________________________

EQUATION SKILL TEST 2: Solving equation for variable.



1) $2 + k = 9$. Solve for k.

$k =$ ______

2) $9 + w = 15$. Solve for w.

$w =$ ______

3) $1 + b = 11$. Solve for b.

$b =$ ______

Student Name: ______________________________________

EQUATION SKILL TEST 3: Solving multiplication equation for variable.

____ 1. Solve for d.

$$6d = 18$$

d = ______

____ 2. Solve for f.

$$4f = 24$$

f = ______

National Center and State Collaborative

NCSC Math Activities with Scripted Systematic Instruction (MASSI): High School Equations

Reposted for NCSC state use on March 11, 2013. All materials in this version have been approved for public distribution with all necessary permissions. Selected excerpts are accompanied by annotated links to related media freely available online at the time of the publication of this document.

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U.S. Office of Special Education Programs

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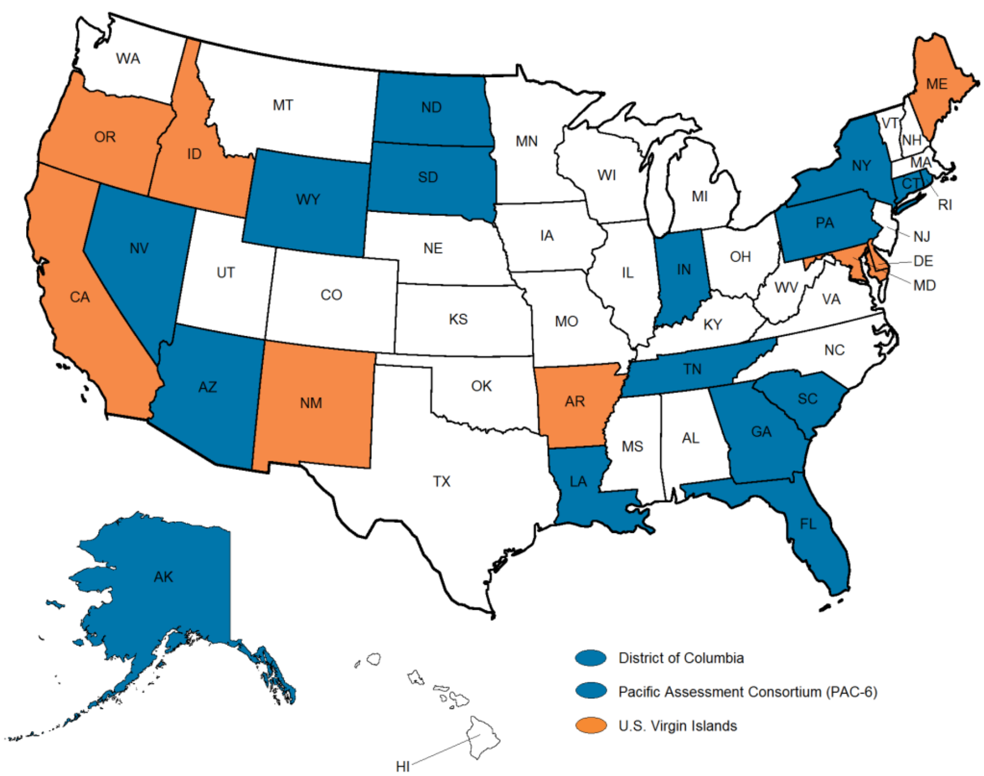

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NCSC Math Activities with Scripted Systematic Instruction (MASSI): High School Equations

Keri Bethune
Julie Thompson
Alicia Saunders
Diane Browder
Amy Lehew

January 2013

Reposted March 11, 2013

MASSI: Math Activities with Scripted Systematic Instruction

Activity: Working at a Hardware Store

Grade Band: High School

Concept: Equations

Common Core State Standard	Core Content Connectors	MASSI OBJECTIVES
*This lesson is linked to standards of mathematical practice and not specific CCSS.	HS H.NO.3a2 Rewrite mathematical statements (e.g., an expression) in multiple forms.	• Making Sets • Using a variable to represent an unknown number • Writing expressions with unknown numbers • Simplifying expressions

Be sure to provide specific practice to students on the skills that correspond to their grade level.

Combined materials provided: bolts (at least 100 total), washers (at least 100 total), small brackets (at least 100 total), zip top bags

Teacher materials: Pictures of hardware stores, dry erase marker

Student materials (need one set for each student)**:** bolts, washers, small brackets, zip top bags, pencil

Worksheets: There are student worksheets to review each component of the lesson.

Assessments: Progress Monitoring for taking data during the lesson; Skills Test

TEACHING OVERVIEW: The first section of the MASSI provides remedial practice on making sets and using a letter to represent an unknown number. The lesson then moves to learning the symbol use and how to write an expression to represent unknown numbers. The lesson moves on to introduce the CCC target component where students learn to write the expression multiple ways by simplifying it. The CCC targets used for the MASSIs will be aligned with the Assessment Tasks but use different examples.

SCRIPT FOR LESSON

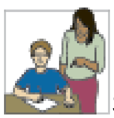
BUILD ESSENTIAL UNDERSTANDING: CONCEPT AND SYMBOLS: Creating Sets

(Skip this section for students who understand these relationships and can identify these concepts).

INTRODUCE ACTIVITY: Today we are going to learn skills you will need when you have a job. One type of job that you might have is working at a hardware store. Here are some pictures of a hardware store. Have any of you ever been to a hardware store? Give students the opportunity to answer yes/no. **What are some hardware stores you've been to?** Give students a moment to respond with local hardware store names, you may provide choices for students that need support. **Hardware stores sell a lot of different types of items. Some items they sell are bolts, washers, and brackets. Here are some bolts, washers, and brackets.** Show students the bolts, washers, and brackets.

INTRODUCE THE PROBLEM: Today we are going to practice learning how to find formulas to help us gather items like bolts, washers, and brackets for a customer at a hardware store.

MODEL THE PROCESS: Show students the bin of bolts. **Bolts get sold in bags of 5. In order to get these bolts ready to sell, we need to each make five sets of five. Watch me make two sets of five.** Count aloud as you take each bolt and place it in one pile **(1, 2, 3, 4, 5),** then count aloud again as you place each bolt in a second pile **(1, 2, 3, 4, 5). Then we put each set into a zip top bag, like this.** Put each set of bolts into its own bag.



STUDENT PRACTICE: Give each student 21-24 bolts, so student has more bolts than needed to ensure that s/he stops at the proper stopping point. Use LEAST INTRUSIVE PROMPTS script as needed to help students with each step.

***Note: Change numbers daily to prevent rote memorization.*
***Note: If students are unable to physically move the bolts, the teacher may move them while the student counts aloud. If the student is nonverbal, counting may be done using an assistive technology device or with a response board (containing the numbers 1-10 or higher when appropriate). The student should "count" by pointing to the bolt, then pointing to the corresponding number. For example, the student points to the first bolt and points to the number one, then points to the second bolt and points to the number two, etc. Student may also tap as teacher counts or move hands/AT scanner from number to number as teacher counts. Look for an action that the student can perform independently and encourage this action as the tiles are counted. Some examples are: a tap, a head nod, blink, leg movement, finger movement.*

CHECK AND SCORE

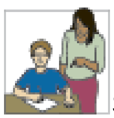
Step	Teacher Says/Does	Student Response
1.	**I need some help. Make five sets of five bolts.**	Student moves five bolts into the first set while counting 1, 2, 3, 4, 5.

2.	Wait after student finishes the first set for them to independently initiate making the second set of five or say **"What's next?"**	Student moves five bolts into the second set while counting 1, 2, 3, 4, 5.
3.	Wait after student finishes the second set for them to independently initiate making the third set of five or say **"What's next?"**	Student moves five bolts into the third set while counting 1, 2, 3, 4, 5.
4.	Wait after student finishes the third set for them to independently initiate making the fourth set of five or say **"What's next?"**	Student moves five bolts into the fourth set while counting 1, 2, 3, 4, 5.
5.	Wait after student finishes the fourth set for them to independently initiate making the fifth set of five or say **"What's next?"**	Student moves five bolts into the fifth set while counting 1, 2, 3, 4, 5.

Good work making sets. Now let's put each set into a zip top bag. Have students put each set into its own zip top bag. **I'll take any leftovers you have.**

INTRODUCE THE PROBLEM: Show students the bin of washers. **Washers get sold in bags of 8. In order to get these washers ready to sell, we need to each make four sets of eight.**

MODEL THE PROCESS: Watch me make two sets of eight. Count aloud as you take each washer and place it in one pile **(1, 2, 3, 4, 5, 6, 7, 8)**, then count aloud again as you place each washer in a second pile **(1, 2, 3, 4, 5, 6, 7, 8)**. **Then we put each set into a zip top bag, like this.** Put each set of washers into its own bag.

**STUDENT PRACTICE.** Give each student 33-36 washers, so student has more washers than needed to ensure that s/he stops at the proper stopping point. Use LEAST INTRUSIVE PROMPTS script to help student with each step as needed.

CHECK AND SCORE

Step	Teacher Says/Does	Student Response
6.	**I need some help. Make four sets of eight washers.**	Student moves eight washers into the first set while counting 1, 2, 3, 4, 5, 6, 7, 8.
7.	Wait after student finishes the first set for them to independently initiate making the second set of eight or say **"What's next?"**	Student moves eight washers into the second set while counting 1, 2, 3, 4, 5, 6, 7, 8.
8.	Wait after student finishes the second set for them to independently initiate making the third set of eight or say **"What's next?"**	Student moves eight washers into the third set while counting 1, 2, 3, 4, 5, 6, 7, 8.
9.	Wait after student finishes the third set for them to independently initiate making the fourth set of eight say **"What's next?"**	Student moves eight washers into the fourth set while counting 1, 2, 3, 4, 5, 6, 7, 8.

Good work making sets. Now let's put each set into a zip top bag. Have students put each set into its own zip top bag. **I'll take any leftovers you have.**

INTRODUCE THE PROBLEM: Show students the bin of brackets. **Brackets get sold in bags of 3. In order to get these brackets ready to sell, we need to each make six sets of three.**

MODEL THE PROCESS: Watch me make two sets of three. Count aloud as you take each bracket and place it in one pile **(1, 2, 3)**, then count aloud again as you place each bracket in a second pile **(1, 2, 3)**. **Then we put each set into a zip top bag, like this.** Put each set of brackets into its own bag.

**STUDENT PRACTICE.** Give each student 20-25 brackets, so student has more brackets than needed to ensure that s/he stops at the proper stopping point. Use LEAST INTRUSIVE PROMPTS script to help student with each step as needed.

CHECK AND SCORE

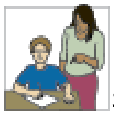
Step	Teacher Says/Does	Student Response
10.	**I need some help. Make six sets of three brackets.**	Student moves three brackets into the first set while counting 1, 2, 3.
11.	Wait after student finishes the first set for them to independently initiate making the second set of three or say **"What's next?"**	Student moves three brackets into the second set while counting 1, 2, 3.
12.	Wait after student finishes the second set for them to independently initiate making the third set of three or say **"What's next?"**	Student moves three brackets into the third set while counting 1, 2, 3.
13.	Wait after student finishes the third set for them to independently initiate making the fourth set of three or say **"What's next?"**	Student moves three brackets into the fourth set while counting 1, 2, 3.
14.	Wait after student finishes the fourth set for them to independently initiate making the fifth set of three or say **"What's next?"**	Student moves three brackets into the fifth set while counting 1, 2, 3.
15.	Wait after student finishes the fifth set for them to independently initiate making the sixth set of three or say **"What's next?"**	Student moves three brackets into the sixth set while counting 1, 2, 3.

Good work making sets. Now let's put each set into a zip top bag. Have students put each set into its own zip top bag. **I'll take any leftovers you have.**

BUILD ESSENTIAL UNDERSTANDING: CONCEPT AND SYMBOLS: Using a Variable to Represent an Unknown Number *(Skip this section for students who understand these relationships and can identify these concepts).*

INTRODUCE THE PROBLEM: Part of the job at the hardware store requires keeping track of inventory, or how many items are in stock. We may know how many bags are in stock, but what if we don't know how many bolts, washers, or brackets are in each bag? We can use a variable, or a letter, to represent the unknown number of bolts, washers, and brackets. Variables can be any letter. Sometimes people use an "x" as a variable, and sometimes people take the first letter of the item to help them remember what the variable represents. For example, they may use the letter "b" to represent "brackets" since the word "brackets" starts with the letter "b". Since we have two words that start with "b"- "bolts" and "brackets", we will practice with the variable "x" for bolts and "b" for brackets.

MODEL THE PROCESS: Show the students four new bags of bolts (possibly different sizes to make it clear they are different). **I don't know how many bolts are in each bag, but I know that each bag has the same number of bolts.** Count the number of bags. **1, 2, 3, 4. I have four bags of bolts. If I wanted to write this down, first I write the number of bags, 4** (write the number 4 on the board)**. Then I can write a variable to represent the unknown number, because I don't know how many bolts are in each bag. I'm going to use the letter "b" for bolts, so I would write 4b.** Write the letter "b" next to the number four. **Can you find the variable?** Wait for students to identify "b" as the variable.



STUDENT PRACTICE: Good, let's practice with a few for you. Give each student worksheet 1. Use LEAST INTRUSIVE PROMPTS script to help student with each step as needed.
***Note: Change numbers daily to prevent rote memorization.*
***Note: Have the students write the number and letter on the worksheet, but do not score writing ability. If students are unable to write the number/letter, they can use number stamps, Velcro numbers, an assistive device, or direct the teacher to write it for them.*

CHECK AND SCORE

Step	Teacher Says/Does	Student Response
16.	Give each student 5 bags of bolts. **If we didn't know how many bolts were in each bag, how would we write the total number of bolts? Use the letter "x" to represent an unknown number.**	Student counts the number of bags (either out loud or to themselves) and writes "5x".
17.	Give each student 7 bags of washers. **If we didn't know how many washers were in each bag, how would we write the total number of washers? Use the letter "w" to represent an unknown number.**	Student counts the number of bags (either out loud or to themselves) and writes "7w".
18.	Give each student 4 bags of brackets. **If we didn't know how many brackets were in each bag, how would we write the total number of brackets? Use the letter "b" to represent an unknown number.**	Student counts the number of bags (either out loud or to themselves) and writes "4b".

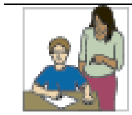
This may be a good stopping point. Let students practice working at a setup hardware store in your classroom. Demonstrate how to be the employee taking orders, then have students take turn taking orders. There is a worksheet with this level. You can use this for additional guided practice or to send home as homework.

	Teacher Says/Does	Student Response	Error Correction
Equations Skills Test	Give each student the *Equations Skills Test 1*. **Read directions for each problem and have student select response. Record whether response is correct or incorrect.**	Only provide praise for completing assessment (if student needs encouragement). Do not provide specific praise for correct answers while student is testing.	Once the student has completed the test, review missed problems with the student.
NOW *Stop the lesson here and repeat tomorrow if student is not yet getting at least 11 independent correct responses. Score responses 1-18 on the Equation Progress Monitoring Sheet if you did not do so while teaching.*		***NEXT*** *Remember the goal is for students to be able to write expressions as soon as possible, move into the second part of the lesson as soon as possible. Then to hit the target CCC for this grade level, move to the last part of the lesson (simplifying an expression). You can skip this Conceptual Foundation section to move on if students have mastered it.*	

HS BUILD A GRADE ALIGNED COMPONENT (Part 1): Creating an Expression to Represent a Story Problem

INTRODUCE THE PROBLEM: Now, you will show students how to create an expression representing an order at a hardware store. An expression has no "=" sign versus an equation that does. **When you work in a hardware store, there will be times you need to take an order for more than one customer at once. There's a way we can write down an expression to represent orders from multiple customers for multiple items. An expression is like an equation but has no equals sign.**

MODEL THE PROCESS: Use Teacher Demonstration Card. **Here is an example: "At the hardware store, Betty needs 2 bags of bolts and 5 bags of washers. Juan needs 4 bags of bolts and 1 bag of washers." Watch me as I write this on the board. We don't know how many bolts and washers are in each bag, so we will use the letter "b" to represent the number of bolts in each bag and the letter "w" to represent the number of washers in each bag. First, Betty ordered 2 bags of bolts, so I write "2b". Now I write a plus. Betty also wanted 5 bags of washers, so I write "5w". Now I write a plus. Juan wanted 4 bags of bolts, so I write "4b". Write plus. He also wanted 1 bag of washers, so I write "1w".** The expression should be 2b + 5w + 4b + 1w.

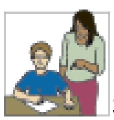
**STUDENT PRACTICE:** Each student also needs worksheet 2. **Look at your worksheet. Let's read the problem together. Nala needs 8 bags of bolts and 3 bags of washers. Wally needs 1 bag of bolts and 6 bags of washers. Use the letter "b" to represent the number of bolts in a bag and the letter "w" to represent the number of washers in a bag.** Use LEAST INTRUSIVE PROMPTS script to help student with each step as needed.

***Note: Have the students write the numbers/digits on the worksheet, but do not score writing ability. If students are unable to write the number, they can use number/letter stamps or direct the teacher to write it for them.*
**The full correct answer is 8b + 3w + 1b + 6w (this is broken down into task analyzed steps)

CHECK AND SCORE

Step	Teacher Says/Does	Student Response
19.	**Write the expression that represents this problem.**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to write 8b.
20.	Wait for students to independently write plus or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes the plus sign.
21.	Wait for students to independently write 3w or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to write 3w.
22.	Wait for students to independently write plus or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes the plus sign.
23.	Wait for students to independently write 1b or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to write 1b.
24.	Wait for students to independently write plus or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes the plus sign.
25.	Wait for students to independently write 6w or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to write 6w.

Great work writing the expression 8b + 3w + 1b + 6w! Let's practice one more before we practice working at the store.

**STUDENT PRACTICE: Look at your worksheet. Let's read the problem together. Pierre needs 1 bag of washers and 3 bags of brackets. Adara needs 4 bags of washers and 5 bags of brackets. Use the letter "w" to represent the number of washers in a bag and the letter "b" to represent the number of brackets in a bag.** Use LEAST INTRUSIVE PROMPTS script to help student with each step as needed.

**The full correct answer is 1w + 3b+ 4w + 5b (this is broken down into task analyzed steps)

CHECK AND SCORE

Step	Teacher Says/Does	Student Response
26.	**Write the expression that represents this problem.**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to write 1w.
27.	Wait for students to independently write plus or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes the plus sign.
28.	Wait for students to independently write 3b or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to write 3b.
29.	Wait for students to independently write plus or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes the plus sign.
30.	Wait for students to independently write 4w or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to write 4w.
31.	Wait for students to independently write plus or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes the plus sign.
32.	Wait for students to independently write 5b or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to write 5b.

Great work writing the expression 1w + 3b + 4w + 5b!

This may be a good stopping point. Let students practice working at a setup hardware store in your classroom. Demonstrate how to be the employee taking orders. Create sample order scenarios with multiple orders for multiple items. Then have students take turn taking orders by creating expressions as they go. There is a worksheet with this level. You can use this for additional guided practice or to send home as homework.

	Teacher Says/Does	Student Response	Error Correction
Equations Skill Test	Give each student the *Equations Skills Test 2*. **Read directions for each problem and have student select response. Record whether response is correct or incorrect.**	Only provide praise for completing assessment (if student needs encouragement). Do not provide specific praise for correct answers while student is testing.	Once the student has completed the test, review missed problems with the student.
NOW *Stop the lesson here and repeat tomorrow if student is not yet getting at least 8 independent correct responses on this section. Score responses 19-32 on the Equation Progress Monitoring Sheet if you did not do so while teaching.*		***NEXT*** *Remember the goal is for students to hit the target CCC for this grade level, move to the last part of the lesson (simplifying an expression). You can skip this Symbol Use section to move on if students have mastered the skills.*	

HS BUILD A GRADE ALIGNED COMPONENT (Part 2): Simplifying Expressions (Writing equations in multiple forms)

INTRODUCE ACTIVITY AND PROBLEM: We've already learned how to write an expression, but we must also simplify the expression to make it easier for us to get the items the customer wants. Before, we wrote an expression for a problem about Betty and Juan's order. Now we are going to learn to simplify the expression. Today, we will use the letter "b" for bolts and "w" for washers. I showed you how to write the expression for the following problem: "Betty needs 2 bags of bolts and 5 bags of washers. Juan needs 4 bags of bolts and 1 bag of washers." I wrote: 2b + 5w + 4b + 1w. Write 2b + 5w + 4b + 1w on the board for students to look at.

MODEL THE PROBLEM: If Betty wants 2 bags of bolts and Juan wants 4 bags of bolts, we can combine them together. 2 bags plus 4 bags is 6 bags, so I write "6b". Cross off 2b and 4b in the expression above and write a new expression below that starts with 6b. **Remember, we can only add together the types that are the same, which means they have the same letter. Let's finish simplifying this expression. If Betty wants 5 bags of washers and Juan wants 1 bag of washers, how many bags of washers do we need all together? We need 6 bags of washers all together, so I write "+ 6w" here.** Cross off the 5w and 1w in the expression above and write 6w in the expression below. See below for an example of how the expression/visual should look when you write it on the board.

1. If Betty wants 2 bags of bolts and Juan wants 4 bags of bolts, we can combine them together.

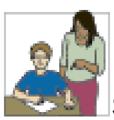
3. Remember, we can only add together the types that are the same, which means they have the same letter. Let's finish simplifying this equation. If Betty wants 5 bags of washers and Juan wants 1 bag of washers, how many bags of washers do we need all together?

$$\cancel{2b} + \cancel{5w} + \cancel{4b} + \cancel{1w}$$

$$6b + 6w$$

2. 2 bags plus 4 bags is 6 bags, so I write 6b.

4. We need 6 bags of washers, all together, so I write +6w here.



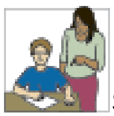
STUDENT PRACTICE: Now it's your turn. Simplify the expressions on your worksheet. Give students worksheet 3. **Look at your worksheet. It has the same problem from before. Let's read the problem together. Nala needs 8 bags of bolts and 3 bags of washers. Wally needs 1 bag of bolts and 6 bags of washers. We wrote that expression as: 8b + 3w + 1b + 6w.** Use LEAST INTRUSIVE PROMPTS script to help student with each step as needed.

***Note: Have the students write the area into the formula on the worksheet, but do not score writing ability. If students are unable to write the number, they can use number stamps or direct the teacher to write it for them.*
***Note: In the following problem, students are required to add. If students are unable to add 3+6 independently, it is ok to provide them with a calculator or other visual; however they must do the work independently. Be consistent with the type of accommodation provided here.*

CHECK AND SCORE

STEP	Teacher Says/Does	Student Response
33.	**Now, you need to simplify this expression. Start by looking at 8b, which number represents the same item (bolts)?**	Student points to, says, or otherwise identifies 1b.
34.	**Good, now add together 8b plus 1b and write your answer below.**	Student adds 8b + 1b to equal 9b and writes 9b in the expression below.
35.	**We can cross out those numbers in the first expression.**	Student crosses off the numbers 8b and 1b from the top expression.
36.	**Now look at 3w, which number represents the same item (washers)?**	Student points to, says, or otherwise identifies 6w.
37.	**Now write the plus sign below.**	Student writes/Velro's/selects/stamps the plus sign.
38.	**Good, now add together 3w plus 6w and write your answer below.**	Student adds 3w + 6w to equal 9w and writes 9w in the expression below.
39.	**You can cross those numbers off the first expression.**	Student crosses off the numbers 3w and 6w from the top expression.

Great work today! You simplified 8b + 3w + 1b + 6w to 9b + 9w. I'm very proud of you.



STUDENT PRACTICE: Let's do another by yourself. Look at your worksheet. Let's to do other problem from before. Let's read the problem together. Pierre needs 1 bag of washers and 3 bags of brackets. Adara needs 4 bags of washers and 5 bags of brackets. We wrote that expression as: 1w + 3b + 4w + 5b. Use LEAST INTRUSIVE PROMPTS script to help student with each step as needed.

CHECK AND SCORE

Steps	Teacher Says/Does	Student Response
40.	**Now, you need to simplify this expression.** Do not provide additional verbal instructions outside of prompting strategies.	Student adds 1w + 4w to equal 5w and writes 5w in the expression below.
41.	Wait for students to independently cross off the numbers or say "**What's next?**"	Student crosses off the numbers 1w and 5w from the top expression.
42.	Wait for students to independently write the plus sign next to 5b or say "**What's next?**"	Student writes/Velcro's/selects/stamps the plus sign.
43.	Wait for students to independently add 3b plus 5b or say "**What's next?**"	Student adds 3b + 5b to equal 8b and writes 8b in the expression below.
44.	Wait for students to independently cross off the numbers or say "**What's next?**"	Student crosses off the numbers 3b and 5b from the top expression.

Good job! You simplified 1w + 3b + 4w + 5b to 5w + 8b. You are doing nice work today.

This may be a good stopping point. Let students practice working at a setup hardware store in your classroom. Demonstrate how to be the employee taking orders, and then have students take turn taking orders writing down and simplify expressions as they go. There is a worksheet with this level. You can use this for additional guided practice or to send home as homework.

	Teacher Says/Does	Student Response	Error Correction
Equations Skill Test	Give each student the *Equations Skills Test 3*. **Read directions for each problem and have student select response. Record whether response is correct or incorrect.**	Only provide praise for completing assessment (if student needs encouragement). Do not provide specific praise for correct answers while student is testing.	Once the student has completed the test, review missed problems with the student.

Troubleshooting and Data-based Decision Making for Equations Test
If student is unable to complete any items on the equations test independently and correctly go back and teach one problem step-by-step.

MASSI CULMINATING ACTIVITY: Setup a mock hardware store in the classroom or other space in the school. Have students practice getting orders from customers with multiple orders for groups of items. Make sure students write down the orders and simplify the expressions appropriately.

BUILD TOWARDS FULL GRADE LEVEL COMPETENCE

Here are ideas to build competence towards the full grade level competence using this same activity. See the unit plan and talk with the general education teacher for more ideas.

Component	Activity	What Student Does	Generalization/ Fluency
Simplify linear equations with more than two types of variables	Students are presented with multiple customers with orders for bolts, washers, and brackets.	Students simplify orders for bolts, washers, and brackets.	Different amounts and items daily.
Rewrite statements in more than one form	Present students with orders for three types of items and ask them to rewrite the expressions in more than one way.	Students rewrite the expressions in more than one way.	Different numbers daily.
Substitute values for variables and simplify (e.g., 2a + 3a; where a=4 bolts)	Have students calculate the total items by substituting a value for the variable.	Students simplify the expressions by replacing the letters/variables with varying amounts.	Different numbers/amounts daily.

Teacher Demonstration Card #1

Create an expression to represent the word problem.

At the hardware store,

Betty needs 2 bags of bolts and 5 bages of washers.

Juan needs 4 bags of bolts and 1 bag of washers.

Worksheet 1: Build the Foundational Concepts

Unknown bolts = x

Unknown washers = w

Unknown brackets = b

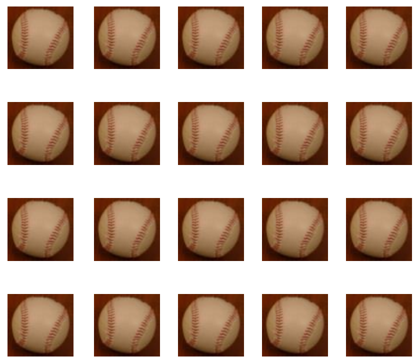
____
Number of bolts

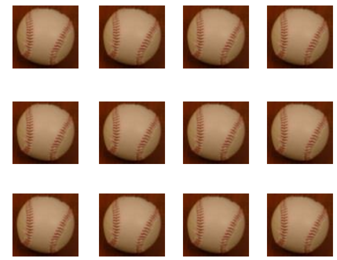
____
Number of washers

____
Number of brackets

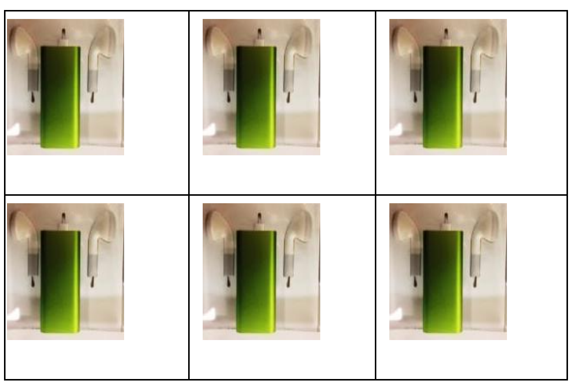
Generalization Worksheet 1 (Build the Foundational Concepts):

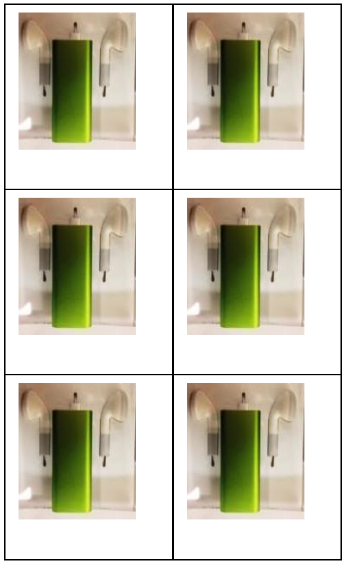
Which one shows 4 sets of 5 baseballs?

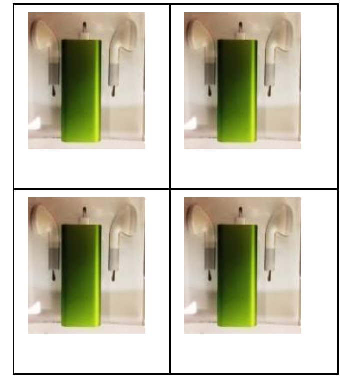




Which one shows 3 sets of 3 IPods?







Draw 8 sets of 2 dots. If students are unable to use a marker/pencil, they can use stickers or manipulatives to create the sets.

Draw 2 sets of 4 dots. If students are unable to use a marker/pencil, they can use stickers or manipulatives to create the sets.

Jeff wants to practice playing golf. He went to the driving range and bought 3 buckets of balls. He doesn't know how many golf balls are in each bucket, but he does know that each bucket has the same number of golf balls. If we use the letter "g" to represent the unknown number, how would we write the number of golf balls that Jeff bought? Students who cannot write can use Velcro numbers/letters, direct the teacher what to write, or use assistive technology to answer.



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In order to get ready to take the tests at the end of the year, Principal Jones got a cup of pencils for each of the 5 math teachers. He doesn't know how many pencils are in each cup but each cup has the same amount. If we use the letter "p" to represent an unknown number, how would you write the number of pencils there are? Students who cannot write can use Velcro numbers/letters, direct the teacher what to write, or use assistive technology to answer.





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Worksheet 2: Creating an Expression

   

Nala needs 8 bags of bolts and 3 bags of washers. Wally needs 1 bag of bolts and 6 bags of washers. Write the expression that represents this problem. Use the letter “b” to represent the number of bolts in a bag and the letter “w” to represent the number of washers in a bag.

   

Pierre needs 1 bag of washers and 3 bags of brackets. Adara needs 4 bags of washers and 5 bags of brackets. Write the expression that represents this problem. Use the letter “w” to represent the number of washers in a bag and the letter “b” to represent the number of brackets in a bag.

Worksheet 2: Generalization




Hans needs 4 bags of washers and 2 bags of brackets. Fredericka needs 2 bags of washers and 6 bags of brackets. Write the expression that represents this problem. Use the letter “w” to represent the number of washers in a bag and the letter “b” to represent the number of brackets in a bag.


Carlos works in an art supply store. The store sells charcoal pencils in packets. They also sell packets of paintbrushes. Carlos doesn’t know how many pencils or paintbrushes are in each packet, but he knows it’s the same amount in each packet. Francis comes into the store and orders 3 packets of pencils and 4 packets of paintbrushes. Then, Karen comes in and orders 1 packet of pencils and 2 packets of paintbrushes. Use the letter “c” to represent the number of charcoal pencils in each packet and the letter “p” to represent the number of paintbrushes in each packet. Write an expression to represent the customers’ orders.

Worksheet 3: Simplifying Expressions (Writing equations in multiple forms)

   

Nala needs 8 bags of bolts and 3 bags of washers. Wally needs 1 bag of bolts and 6 bags of washers. Simplify this expression.

$$8b + 3w + 1b + 6w$$

   

Pierre needs 1 bag of washers and 3 bags of brackets. Adara needs 4 bags of washers and 5 bags of brackets. Simplify this expression.

$$1w + 3b + 4w + 5b$$

Worksheet 3: Generalization

   

Hans needs 4 bags of washers and 2 bags of brackets. Fredericka needs 2 bags of washers and 6 bags of brackets. Write the expression that represents this problem. Use the letter w to represent the number of washers in a bag and the letter b to represent the number of brackets in a bag.

Now simplify the expression.

Carlos works in an art supply store. The store sells charcoal pencils in packets. They also sell packets of paintbrushes. Carlos doesn't know how many pencils or paintbrushes are in each packet, but he knows it's the same amount in each packet. Francis comes into the store and orders 3 packets of pencils and 4 packets of paintbrushes. Then, Karen comes in and orders 1 packet of pencils and 2 packets of paintbrushes. Use the letter c to represent the number of pencils in each packet and the letter d to represent the number of paintbrushes in each packet. Write an expression to represent the customers' orders.

Now simplify the expression.

NCSC Math Activities with Scripted Systematic Instruction (MASSI): High School Equations Progress Monitoring and Skills Test

National Center and State Collaborative

The National Center and State Collaborative (NCSC) is applying the lessons learned from the past decade of research on alternate assessments based on alternate achievement standards (AA-AAS) to develop a multi-state comprehensive assessment system for students with significant cognitive disabilities. The project draws on a strong research base to develop an AA-AAS that is built from the ground up on powerful validity arguments linked to clear learning outcomes and defensible assessment results, to complement the work of the Race to the Top Common State Assessment Program (RTTA) consortia.

Our long-term goal is to ensure that students with significant cognitive disabilities achieve increasingly higher academic outcomes and leave high school ready for post-secondary options. A well-designed summative assessment alone is insufficient to achieve that goal. Thus, NCSC is developing a full system intended to support educators, which includes formative assessment tools and strategies, professional development on appropriate interim uses of data for progress monitoring, and management systems to ease the burdens of administration and documentation. All partners share a commitment to the research-to-practice focus of the project and the development of a comprehensive model of curriculum, instruction, assessment, and supportive professional development. These supports will improve the alignment of the entire system and strengthen the validity of inferences of the system of assessments.

IDEAs that Work
U.S. Office of Special Education Programs

The contents of this assessment were developed as part of the National Center and State Collaborative by Keri Bethune, Julie Thompson, Alicia Saunders, and Diane Browder at University of North Carolina at Charlotte and verified by Amy Lehew, math content expert, under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the U.S. Department of Education and no assumption of endorsement by the Federal government should be made.

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These materials and documents were developed under the National Center and State Collaborative (NCSC) General Supervision Enhancement Grant and are consistent with its goals and foundations. Any changes to these materials are to be consistent with their intended purpose and use as defined by NCSC.

This document is available in alternative formats upon request.

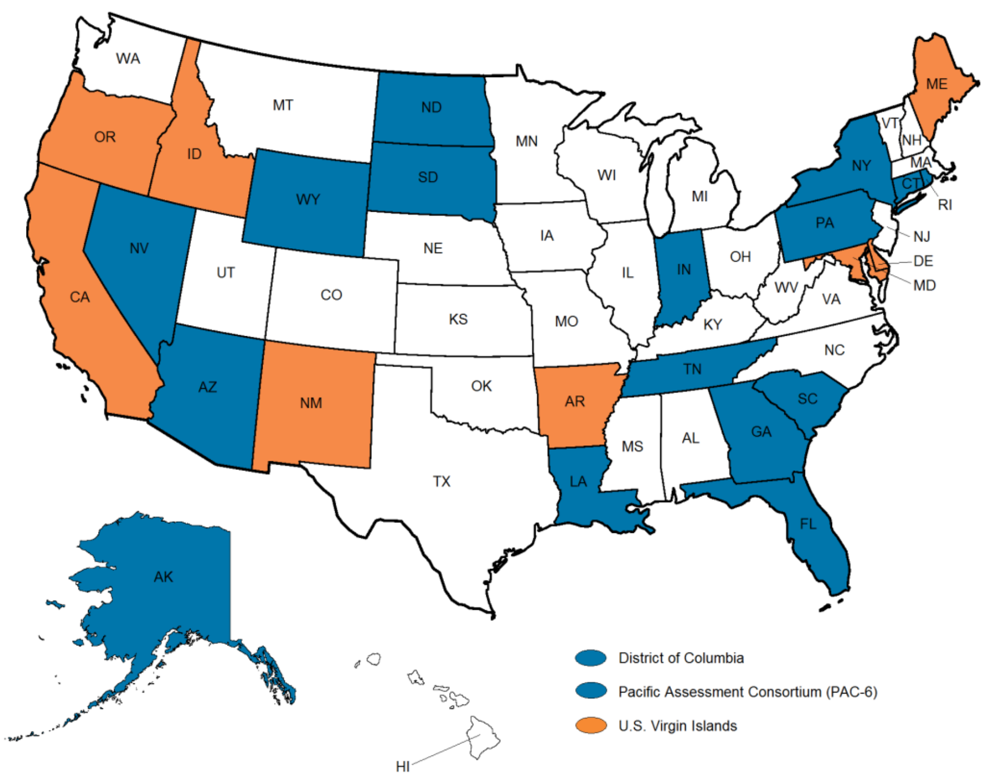
ncsc

National Center and State Collaborative

NCSC is a collaborative of 18 states and five organizations.

The states include (shown in blue on map): Alaska, Arizona, Connecticut, District of Columbia, Florida, Georgia, Indiana, Louisiana, Nevada, New York, North Dakota, Pacific Assessment Consortium (PAC-6)[1], Pennsylvania, Rhode Island, South Carolina, South Dakota, Tennessee, and Wyoming.

Tier II states are partners in curriculum, instruction, and professional development implementation but are not part of the assessment development work. They are (shown in orange on map): Arkansas, California, Delaware, Idaho, Maine, Maryland, New Mexico, Oregon, and U.S. Virgin Islands.



[1] The Pacific Assessment Consortium (including the entities of American Samoa, Commonwealth of the Northern Mariana Islands, Federated States of Micronesia, Guam, Republic of Palau, and Republic of the Marshall Islands) partner with NCSC as one state, led by the University of Guam Center for Excellence in Developmental Disabilities Education, Research, and Service (CEDDERS).

ncsc

National Center and State Collaborative

The five partner organizations include: The National Center on Educational Outcomes (NCEO) at the University of Minnesota, The National Center for the Improvement of Educational Assessment (Center for Assessment), The University of North Carolina at Charlotte, The University of Kentucky, and edCount, LLC.

NATIONAL CENTER ON EDUCATIONAL OUTCOMES

Center for Assessment

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National Center and State Collaborative

NCSC Math Activities with Scripted Systematic Instruction (MASSI): High School Equations Progress Monitoring and Skills Test

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MASSI: High School Equations

Options for Progress Monitoring/Formative Assessment

1. High School Equations Progress Monitoring (pg. 7-12): record student responses made during instruction on data sheet provided; teacher records each step correct during the lesson.

2. High School Equations Skills Test (pg. 13-21): a brief on demand performance assessment; could be given weekly to see if student has mastered this lesson; also helps student practice responding in a test format.

    a. NOTE: The Skill Test can be used as a baseline assessment to check for any skills the student may already have prior to beginning the MASSI.

    b. NOTE: The Skill Test can also be readministered to check for maintenance throughout the year.

Student Name: ____

High School Equations Progress Monitoring

Directions: Score each step during instruction or as soon as the lesson is complete. Score the step as unprompted correct with a "+." Use a system to code level of prompting required for incorrect responses (e.g., V = verbal prompt, G = gesture, P = physical). Graph the number of unprompted correct responses to monitor progress.

BUILD ESSENTIAL UNDERSTANDING: CONCEPT AND SYMBOLS: Creating Sets and Using Variables to Represent Unknown Numbers

Materials and Directions for Teacher	*Instructional Cue*	*Student Expected Response* *Date:*						
1. Give each student 21-24 bolts. Give student more bolts than needed to ensure that s/he stops at the proper stopping point.	**Make five sets of five bolts.**	Student moves five bolts into the first set while counting 1, 2, 3, 4, 5.						
2. See above.	Wait after student finishes the first set for them to independently initiate making the second set of five or say **"What's next?"**	Student moves five bolts into the second set while counting 1, 2, 3, 4, 5.						
3. See above.	Wait after student finishes the second set for them to independently initiate making the third set of five or say **"What's next?"**	Student moves five bolts into the third set while counting 1, 2, 3, 4, 5.						
4. See above.	Wait after student finishes the third set for them to independently initiate making the fourth set of five or say **"What's next?"**	Student moves five bolts into the fourth set while counting 1, 2, 3, 4, 5.						
5. See above.	Wait after student finishes the fourth set for them to independently initiate making the fifth set of five or say **"What's next?"**	Student moves five bolts into the fifth set while counting 1, 2, 3, 4, 5.						
6. Give each student 33-36 washers. Give student more washers than needed to ensure that s/he stops at the proper stopping point.	**Make four sets of eight washers.**	Student moves eight washers into the first set while counting 1, 2, 3, 4, 5, 6, 7, 8.						
7. See above.	Wait after student finishes the first set for them to independently initiate making the second set of eight or say **"What's next?"**	Student moves eight washers into the second set while counting 1, 2, 3, 4, 5, 6, 7, 8.						
8. See above.	Wait after student finishes the second set for them to independently initiate making the third set of eight or say **"What's next?"**	Student moves eight washers into the third set while counting 1, 2, 3, 4, 5, 6, 7, 8.						
9. See above.	Wait after student finishes the third set for them to independently initiate making the fourth set of eight or say **"What's next?"**	Student moves eight washers into the fourth set while counting 1, 2, 3, 4, 5, 6, 7, 8.						

Student Name: ______________________________

10. Give each student 20-25 brackets. Give student more brackets than needed to ensure that s/he stops at the proper stopping point.	**I need some help. Make six sets of three brackets.**	Student moves three brackets into the first set while counting 1, 2, 3.						
11. See above.	Wait after student finishes the first set for them to independently initiate making the second set of three or say **"What's next?"**	Student moves three brackets into the second set while counting 1, 2, 3.						
12. See above.	Wait after student finishes the second set for them to independently initiate making the third set of three or say **"What's next?"**	Student moves three brackets into the third set while counting 1, 2, 3.						
13. See above.	Wait after student finishes the third set for them to independently initiate making the fourth set of three or say **"What's next?"**	Student moves three brackets into the fourth set while counting 1, 2, 3.						
14. See above.	Wait after student finishes the fourth set for them to independently initiate making the fifth set of three say **"What's next?"**	Student moves three brackets into the fifth set while counting 1, 2, 3.						
15. See above.	Wait after student finishes the fifth set for them to independently initiate making the sixth set of three say **"What's next?"**	Student moves three brackets into the sixth set while counting 1, 2, 3.						
16. Give each student 5 bags of bolts and worksheet 1.	**If we didn't know how many bolts are in each bag, how would we write the total number of bolts. Use the letter "x" to represent an unknown number.**	Student counts the number of bags (either out loud or to themselves) and writes "5x".						
17. Give each student 7 bags of washers and worksheet 1.	**If we didn't know how many washers were in each bag, how would we write the total number of washers. Use the letter "w" to represent an unknown number.**	Student counts the number of bags (either out loud or to themselves) and writes "7w".						
18. Give each student 4 bags of brackets and worksheet 1.	**If we didn't know how many brackets were in each bag, how would we write the total number of brackets. Use the letter "b" to represent an unknown number.**	Student counts the number of bags (either out loud or to themselves) and writes "4b".						
		NUMBER CORRECT:						

Student Name: ____

HS BUILD A GRADE ALIGNED COMPONENT (Part 1): Creating an Expression to Represent a Problem								
19. Student worksheet 2.	**Look at your worksheet. Let's read the problem together. Nala needs 8 bags of bolts and 3 bags of washers. Wally needs 1 bag of bolts and 6 bags of washers. Write the equation that represents this problem. Use the letter "b" to represent the number of bolts in a bag and the letter "w" to represent the number of washers in a bag.**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to write 8b.						
20. See above.	Wait for students to independently write plus or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes the plus sign.						
21. See above.	Wait for students to independently write 3w or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to write 3w.						
22. See above.	Wait for students to independently write plus or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes the plus sign.						
23. See above.	Wait for students to independently write 1b or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to write 1b.						
24. See above.	Wait for students to independently write plus or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes the plus sign.						
25. See above.	Wait for students to independently write 6w or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to write 6w.						
26. Student worksheet 2.	**Look at your worksheet. Let's read the problem together. Pierre needs 1 bag of washers and 3 bags of brackets. Adara needs 4 bags of washers and 5 bags of brackets. Write the equation that represents this problem. Use the letter w to represent the number of washers in a bag and the letter b to represent the number of brackets in a bag.**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to write 1w.						
27. See above.	Wait for students to independently write plus or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes the plus sign.						

28. See above.	Wait for students to independently write 3b or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to write 3c.						
29. See above.	Wait for students to independently write plus or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes the plus sign.						
30. See above.	Wait for students to independently write 4w or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to write 4b.						
31. See above.	Wait for students to independently write plus or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes the plus sign.						
32. See above.	Wait for students to independently write 5b or say **"What do you write next?"**	Student writes/stamps/uses Velcro numbers/points to/eye gazes to write 5c.						
		NUMBER CORRECT:						

Student Name: ____

HS BUILD A GRADE ALIGNED COMPONENT (Part 2): Simplifying Expressions								
33. Student worksheet 3.	**Look at your worksheet. It has the same problem from before. Let's read the problem together. Nala needs 8 bags of bolts and 3 bags of washers. Wally needs 1 bag of bolts and 6 bags of washers. We wrote that expression as: 8b + 3w + 1b + 6w. Now, you need to simplify this expression. Start by looking at 8b, which number represents the same item (bolts)?**	Student points to, says, or otherwise identifies 1b.						
34. See above.	**Good, now add together 8b plus 1b and write your answer below.**	Student adds 8b + 1b to equal 9b and writes 9b in the expression below.						
35. See above.	**We can cross those numbers off the first expression.**	Student crosses off the numbers 8b and 1b from the top expression.						
36. See above.	**Now look at 3w, which number represents the same item (washers)?**	Student points to, says, or otherwise identifies 6w.						
37. See above.	**Now write the plus sign below.**	Student writes/Velcro's/selects/ stamps the plus sign.						
38. See above.	**Good, now add together 3w plus 6w and write your answer below.**	Student adds 3w + 6w to equal 9w and writes 9w in the expression below.						
39. See above.	**You can cross those numbers off the first expression.**	Student crosses off the numbers 3w and 6w from the top expression.						
40. See above.	**Look at your worksheet. Let's do other problem from before. Let's read the problem together. Pierre needs 1 bag of washers and 3 bags of brackets. Adara needs 4 bags of washers and 5 bags of brackets. We wrote that expression as: 1w + 3b + 4w + 5b. Now, you need to simplify this expression.** Do not provide additional verbal instructions outside of prompting strategies.	Student adds 1w + 4w to equal 5w and writes 5w in the expression below.						
41. See above.	Wait for students to independently cross off the numbers or say "**What's next?"**	Student crosses off the numbers 1w and 4w from the top expression.						
42. See above.	Wait for students to independently write the plus sign next to 5w or say "**What's next?"**	Student writes/Velcro's/selects/ stamps the plus sign.						

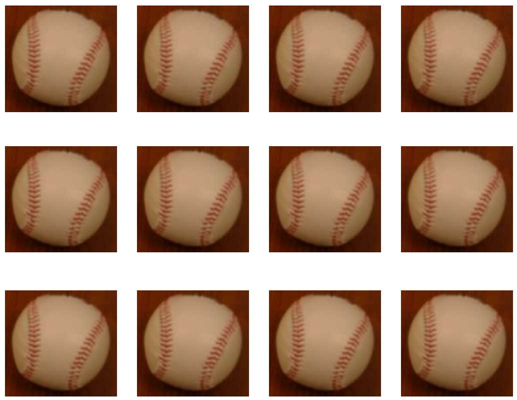
43. See above.	Wait for students to independently add 3b plus 5b or say "**What's next?**"	Student adds 3b + 5b to equal 8b and writes 8b in the expression below.						
44. Student worksheet 3.	Wait for students to independently cross off the numbers or say "**What's next?**"	Student crosses off the numbers 3b and 5b from the top expression.						
		NUMBER CORRECT:						

Student Name: ____

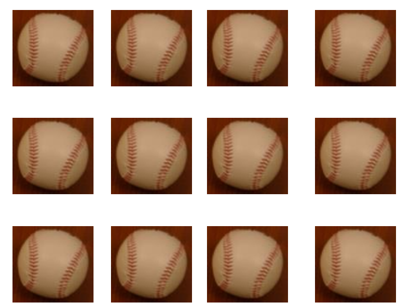
EQUATIONS SKILL TEST 1: CONCEPT AND SYMBOLS

Note to teachers: It may be helpful to use a cover sheet of paper. Pull the cover sheet down far enough to show the model and read the text. Then, pull the sheet of paper down to show the problem and read the directions. Record “+” for an independent correct response or “-“ for incorrect response besideproblem.

___ **This shows 3 sets of 4 baseballs.**

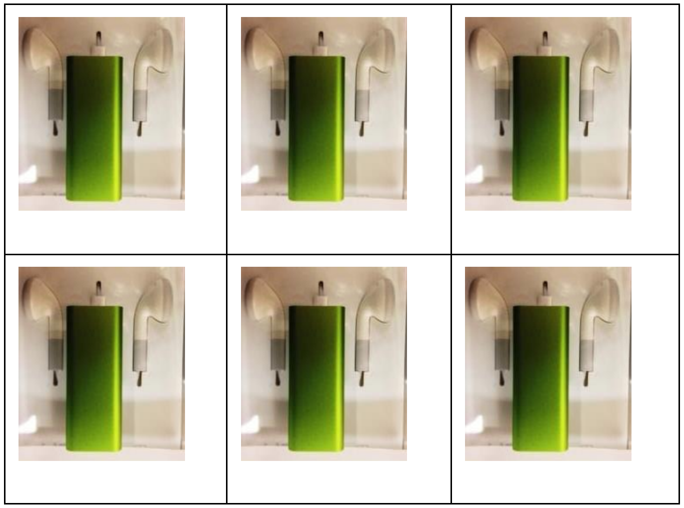


STUDENT PROBLEM: Which picture shows 4 sets of 4 baseballs?

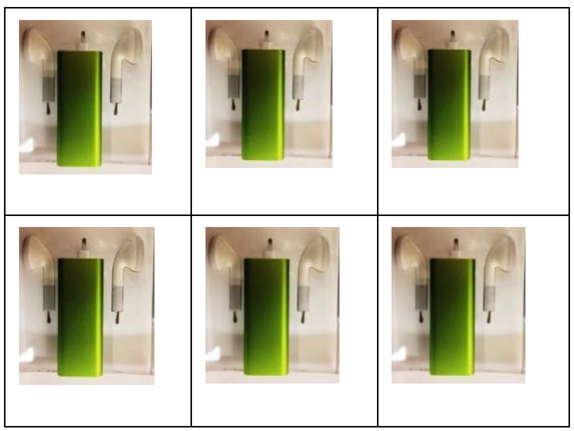


Student Name: ____

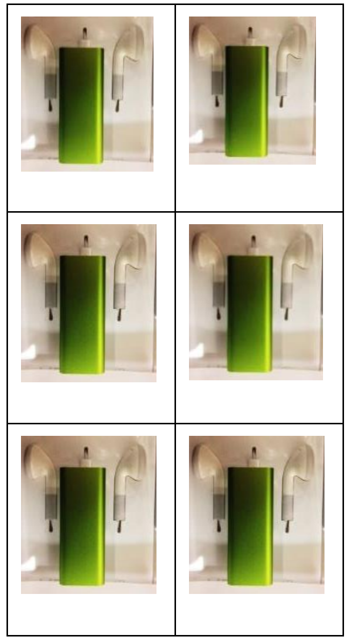
___ **This shows 2 sets of 3 IPods.**

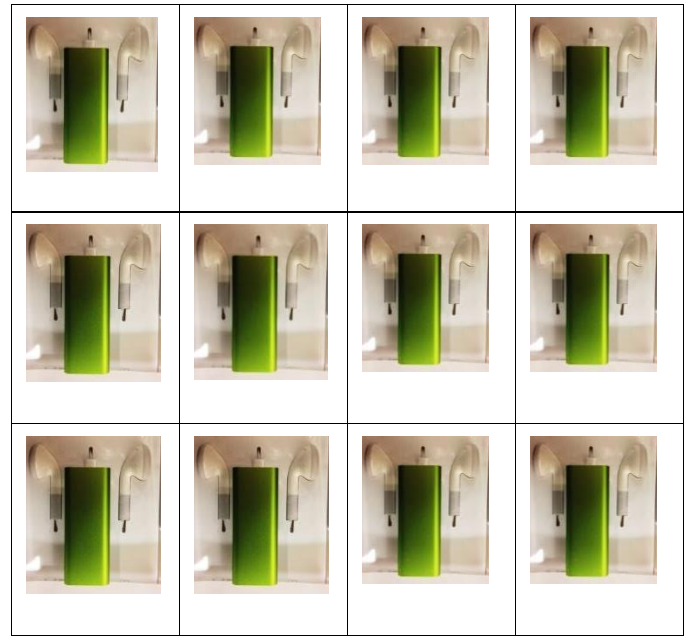


STUDENT PROBLEM: Which picture shows 3 sets of 4 IPods?



Student Name: ____





Student Name: ____

___ This picture shows 2 sets of 5 backpacks.

STUDENT PROBLEM: Which picture shows 3 sets of 6 backpacks?

Student Name: ____

___ This picture shows 2 sets of 2 calculators.

STUDENT PROBLEM: Which picture shows 2 sets of 4?

Student Name: ______________________________________

___ Sasha buys 3 bags of oranges. We don't know how many oranges are in each bag, but we know each bag has the same number of oranges. If we use the letter o to represent the unknown number, we would write the number of oranges as 3o.



3o

STUDENT PROBLEM: Ben bought 6 bags of oranges. If we use the letter o to represent the unknown number, how would we write the number of oranges that Ben bought? Students who cannot write can use Velcro numbers/letters, direct the teacher what to write, or use assistive technology to answer.






____

Student Name: ____

___ Mr. Thompson got 4 cups of pencils for his students. He doesn’t know how many pencils are in each cup. If we use the letter p to represent an unknown number, we would write the number of pencils as 4p.




4p

STUDENT PROBLEM: Mrs. Parker got 7 cups of pencils for her students. If we use the letter p to represent an unknown number, how would you write the number of pencils there are? Students who cannot write can use Velcro numbers/letters, direct the teacher what to write, or use assistive technology to answer.







____

Student Name: ____

EQUATIONS SKILLS TEST 2: Creating an Expression to Represent a Story Problem


Carlos works in an art supply store. The store sells charcoal pencils in packets. They also sell packets of paintbrushes. Carlos doesn't know how many pencils or paintbrushes are in each packet, but he knows it's the same amount in each packet. Ed comes into the store and orders 2 packets of pencils and 5 packets of paintbrushes. Then, Sue comes in and orders 6 packets of pencils and 3 packets of paintbrushes. Use the letter "p" to represent the number of pencils in each packet and the letter "b" to represent the number of paintbrushes in each packet. Write an expression to represent the customers' orders.




Stephanie needs 1 bag of bolts and 4 bags of washers. Burt needs 5 bags of bolts and 2 bags of washers. Write the expression that represents this problem. Use the letter "b" to represent the number of bolts in a bag and the letter "w" to represent the number of washers in a bag.

Student Name: ____

EQUATIONS SKILLS TEST 3: Simplifying Expressions




Boris needs 3 bags of washers and 5 bags of brackets. John needs 4 bags of washers and 7 bags of brackets. Write the expression that represents this problem. Use the letter “w” to represent the number of washers in a bag and the letter “b” to represent the number of brackets in a bag.

Now simplify that expression.

Student Name: ____

Carlos works in an art supply store. The store sells charcoal pencils in packets. They also sell packets of paintbrushes. Carlos doesn’t know how many pencils or paintbrushes are in each packet, but he knows it’s the same amount in each packet. Kenroy comes into the store and orders 4 packets of pencils and 5 packets of paintbrushes. Then, Joy comes in and orders 2 packet of pencils and 4 packets of paintbrushes. Use the letter “p” to represent the number of pencils in each packet and the letter “b” to represent the number of paintbrushes in each packet. Write an expression to represent the customers’ orders.

Now simplify that expression.

National Center and State Collaborative

January 2013

Dear Colleagues,

We are happy to provide you with a representative hard-copy set of the NCSC Instructional Resources. We have chosen one topic in mathematics (equations) to demonstrate how our resources work together to support implementation of the Common Core State Standards (CCSS) for students with significant cognitive disabilities. This set can support state leaders' understanding of the purpose, use, and content of NCSC resources as you plan for professional development opportunities for teachers in your state.

These resources will be useful to teachers as they transition to instruction based on the CCSS for students who participate in alternate assessment based on alternate achievement standards. When integrated into state and district plans for professional development, the materials and related training tools will help build the capacity of teachers to better understand the CCSS, to plan and effectively teach this challenging content, and to monitor student progress toward curricular goals.

For easy reference, the order of presentation of the instructional resources in this binder follows the NCSC *Schema for Common Core State Standards Resources*. It does not prescribe the order in which these materials should be presented in trainings. Rather, it is organized to emphasize the purpose and use of the full suite of the NCSC resources as they fit together to support teaching and learning.

We will be providing two webinars in February to orient you and other state staff of your choice to the resources and to related training tools to support their use. More information about the webinars will be provided in email communications and on Ad Hoc Calls. Additionally, you may contact your assigned NCSC trainer who can support you as you integrate these resources and training tools into your state professional development plan.

Thank you,

The NCSC Partners at NCEO, NCIEA, UNCC, UKY, and edCount

PS: Please note that the Core Content Connectors (CCCs), CCSS, and Learning Progression Frameworks (LPF) are not included in this binder. Documents related to these can be found on the NCSC SharePoint site under the Curriculum and Instruction Tab. Other topics in mathematics (in addition to this sample set on equations) are available on SharePoint. ELA will be done this summer! Please contact Laura Hart at lhart@edcount.com if you need assistance with SharePoint.

Order of Contents

1. The Schema for Common Core State Standards Resources: NCSC Instructional Resources
   a. Identifies the content (the “what”) and the instructional resources (the “how”) to plan for and implement instruction based on the Common Core State Standards (CCSS) and includes multiple assessment tools and strategies to monitor student progress.

2. The Equations Content Module
   a. Provides teachers with a deeper understanding of difficult or complex mathematical concepts and promotes teacher understanding of these concepts to support effective planning, teaching, and learning. Sample general education lesson plans and assessments are included. Additionally, the modules provide teachers with potential adaptations and modifications to consider when designing materials and instruction for students with the most significant cognitive disabilities.

3. The Graduated Understandings: Instructional Families Patterns, Relations, and Functions (includes equations)
   a. Provide visual representations of the areas of curricular emphasis and show the academic expectations (concepts and skills) within and across grades and how these expectations develop over time to promote instruction based on the CCSS.

4. The Graduated Understandings: Element Cards Patterns, Relations, and Functions (includes Equations)
   a. Provide a wide range of suggested instructional strategies, supports, and scaffolds to promote instruction of the CCCs and the broader CCSS. Include Essential Understandings that define the necessary knowledge and skills required to successfully address grade-specific academic skills and develop assessments. Provide teachers with a key resource to further support instruction on the CCSS when used as a supplement to and/or in conjunction with other instructional resources.

5. Equations Curriculum Resource Guide
   a. Offers examples of how academic content is taught in general education and ideas for teaching across content areas. Includes assessment examples, instructional ideas embedding real life use, examples of modifications and adaptations for students with specific learning needs, and ways to promote college and career readiness. Covers a range of the CCCs for grades 3 through high school. Includes examples of performance assessments of student knowledge for prioritized CCCs.

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6. Universal Design for Learning (UDL) Instructional Unit - Measurement
    a. Provides models of universally designed planning and instruction of the CCSS for an entire class of students and illustrates how to target the Core Content Connectors (CCCs) within general education lessons. Includes class-based performance assessments. Offers a model of how to engage all students in well-designed instruction and how to plan for engagement, representation, and expression. This set of materials only includes the high school and elementary levels.

7. The Equations Mathematics Activities with Scripted Systematic Instruction (MASSIs) for Elementary
    a. Offer intensive, scripted instructional lessons with increasing levels of difficulty for grades 3 – 5 that include evidence-based practices that support targeted CCCs prioritized for assessment. The first steps of the lesson are accessible to students with little to no understanding of the content. The lesson continues building understanding through a target component of the CCC. Real life, hands-on activities suggest how to teach the concept that can be easily set up in most classrooms with inexpensive materials. Includes data sheets that can be used for monitoring progress towards mastery and a skill test to provide students practice responding in a testing context.
    b. Note: To access the MASSI Webinars, go to http://coedpages.uncc.edu/ncsc and click on "all states." Username is "ncsc-secure" (make sure to include hyphen). The password is "uncc2012" ; click on Equations.

8. The Equations Mathematics Activities with Scripted Systematic Instruction (MASSIs) for Middle School
    a. Offer intensive, scripted instructional lessons with increasing levels of difficulty for grades 6 - 8 that include evidence-based practices that support targeted CCCs (see further description for Elementary Equations MASSI).

9. The Equations Mathematics Activities with Scripted Systematic Instruction (MASSIs) for High School
    a. Offer intensive, scripted instructional lessons with increasing levels of difficulty for high school that include evidence-based practices that support targeted CCCs (see further description for Elementary Equations MASSI).

10. The Instructional Resource Guide
    a. Provides guidance for teachers regarding evidence-based prompting and instructional strategies to be used to teach students with the most significant cognitive disabilities. Helps educators build knowledge of the essential systematic instructional methods and prompting strategies that are used in the MASSIs and LASSIs to teach students targeted skills. Serves as a companion document to the MASSIs (Math Activities with Scripted Systematic Instruction) and LASSIs (Language Arts Scripted Systematic Instruction).