8^{th} Grade – Summary of Revisions and Planning Guidance - *Arizona Mathematics Standards - Adopted in 2016*

Additions

8.NS.A.3 – Understand that given any two distinct **rational** numbers, $a < b$, there exist a rational number c and an irrational number d such that $a < c < b$ and $a < d < b$. Given any two distinct **irrational** numbers, $a < b$, there exist a rational number c and an irrational number d such that $a < c < b$ and $a < d < b$.
8.SP.B – NEW CLUSTER - Investigate chance processes and develop, use, and evaluate probability models.
8.SP.B.5 - Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
a. Understand that the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using organized lists, tables, tree diagrams and other methods. Identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events.

Parameter Changes/Clarifications

8.NS.A	Irrational number terminology is used in the Cluster.
8.NS.A.1	The focus on understanding rational numbers in the standard has changed to a focus on irrational numbers.
8.NS.A.3	**NEW STANDARD**
8.EE.A.1	Instead of knowing properties, students need to understand, raising cognitive demand
8.EE.A.2	Limits of perfect squares are set at 225 and limits of perfect cubes at 1000.
8.EE.A.4	Removed the interpretation of scientific notation generated by technology
8.EE.C.7	Fluently solving linear equations and inequalities in one variable is now stated.
8.EE.C.8	Part b specifically states the need to graph equations with no solutions and infinite solutions.
8.F.B.4	A statement of tracking the values of how the two quantities change together is included in the standard.
8.G.A.1	Properties are now stated in the standard.
8.G.B.6	Rather than explaining a proof, students must **understand** the Pythagorean Theorem and its converse, raising the cognitive demand of this standard.
8.G.C.9	Instead of knowing the formulas, students need to **understand and use** formulas, raising the cognitive demand.
8.SP.B	**NEW CLUSTER**
8.SP.B.5	**NEW STANDARD**

Defining Standards, Curriculum and Instruction

Standards – What a student needs to know, understand, and be able to do by the end of each grade. Standards build across grade levels in a progression of increasing understanding and through a range of cognitive demand levels. Standards are adopted at the state level by the State Board of Education.
Curriculum – The resources used for teaching and learning the standards. Curricula are adopted at a local level by districts and schools.
Instruction – The methods used by teachers to teach their students. Instructional techniques are employed by individual teachers in response to the needs of the students in their classes to help them progress through the curriculum in order to master the standards.

Fluency Expectations

7^{th}	**7.NS.A.1.d -** Apply properties of operations as strategies to add and subtract rational numbers. **7.NS.A.2.c -** Apply properties of operations as strategies to multiply and divide rational numbers. **7.EE.B.4.a -** Fluently solve one-variable equations of the form $px + q = r$ and $p(x + q) = r$
8^{th}	**8.EE.C.7 -** Fluently solve linear equations and inequalities in one variable.
Alg.1	**A1.F-IF.C.7 -** Graph functions expressed symbolically and show key features of the graph. **A1.A-SSE.A.2 -** Use structure to identify ways to rewrite numerical and polynomial expressions.

Fluency Definition

Fluency standard instruction should begin at the beginning of the year and continue throughout the school year. Wherever the word *fluently* appears in a content standard, the word includes ***efficiently, accurately, flexibly,*** *and* ***appropriately***. Being fluent means that students are able to choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and are able to explain their approaches, and they are able to produce accurate answers efficiently.

- **Efficiency**—carries out easily, keeps track of sub-problems, and makes use of intermediate results to solve the problem.
- **Accuracy**—reliably produces the correct answer.
- **Flexibility**—knows more than one approach, chooses a viable strategy, and uses one method to solve and another method to double-check.
- **Appropriately**-knows when to apply a particular procedure.

8th Grade Content Emphasis

The Number System (NS)	
●	Understand that there are irrational numbers, and approximate them using rational numbers.
Expressions and Equations (EE)	
●	Work with radicals and integer exponents.
●	Understand the connections between proportional relationships, lines, and linear equations.
●	Analyze and solve linear equations and pairs of simultaneous linear equations.
Functions (F)	
●	Define, evaluate, and compare functions.
●	Use functions to model relationships between quantities.
Geometry (G)	
▲	Understand congruence and similarity.
▲	Understand and apply the Pythagorean Theorem.
▲	Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.
Statistics and Probability (SP)	
▲	Investigate patterns of association in bivariate data.
▲	Investigate chance processes and develop, use, and evaluate probability models.

● - Major Content ▲ - Supporting Content

Major Content (●) from the content emphasis section should account for approximately 70% of instructional time.

Changes in Cognitive Demand

There are times in which the standards were changed, resulting in an increase or decrease in cognitive demand expectations within the standards. This is an important aspect of the standard to examine, keeping in mind that cognitive demand refers to the complexity of thinking involved in which students interact with the content; it does **not** refer to difficulty.

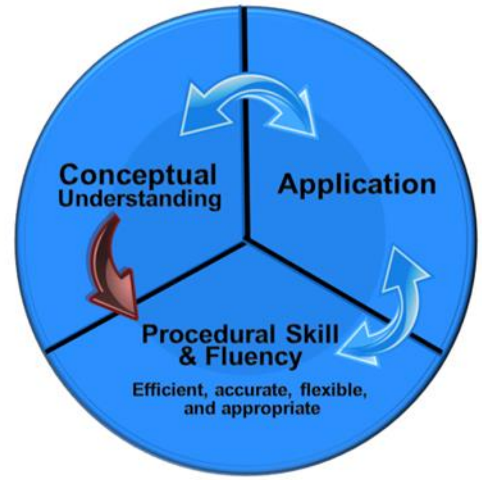
Changes in Cognitive Demand in the 8th Grade Standards		
8.G.B.6	**8.G.C.9**	**8.EE.A.1**

The Standards for Mathematical Practice

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

The Arizona Mathematics Standards now include narratives for each of the 8 Mathematical Practices.

Balance of Rigor in the Math Classroom



"Tasks that ask students to perform a memorized procedure in a routine manner lead to one type of opportunity for student thinking; tasks that require students to think conceptually and that stimulate students to make connections lead to a different set of opportunities for student thinking."

(Stein & Smith, 1998)

Comparison of Arizona Mathematics Standards – 2010 to 2016

Adopted 2010		Adopted 2016		
Number Systems (NS)		**The Number System (NS)**		
8.NS.A	**Know that there are numbers that are not rational, and approximate them by rational numbers.**			
	8.NS.A.1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.	**8.NS.A** **Understand that there are irrational numbers, and approximate them using rational numbers.**	**8.NS.A.1**	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion. Know that numbers whose decimal expansions do not terminate in zeros or in a repeating sequence of fixed digits are called irrational.
	8.NS.A.2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., p^2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*		**8.NS.A.2**	Use rational approximations of irrational numbers to compare the size of irrational numbers. Locate them approximately on a number line diagram, and estimate their values.
			8.NS.A.3	Understand that given any two distinct **rational** numbers, $a < b$, there exist a rational number c and an irrational number d such that $a < c < b$ and $a < d < b$. Given any two distinct **irrational** numbers, $a < b$, there exist a rational number c and an irrational number d such that $a < c < b$ and $a < d < b$.
Expressions and Equations (EE)		**Expressions and Equations (EE)**		
8.EE.A	**Work with radicals and integer exponents.**			
	8.EE.A.1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 x 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.	**8.EE.A** **Work with radicals and integer exponents.**	**8.EE.A.1**	Understand and apply the properties of integer exponents to generate equivalent numerical expressions.

	8.EE.A.2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where *p* is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.	**8.EE.A (cont.)**	**8.EE.A.2**	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Know that √2 is irrational. a. Evaluate square roots of perfect squares less than or equal to 225. b. Evaluate cube roots of perfect cubes less than or equal to 1000.
	8.EE.A.3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.		**8.EE.A.3**	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and express how many times larger or smaller one is than the other.
	8.EE.A.4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.		**8.EE.A.4**	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities.
8.EE.B	**Understand the connections between proportional relationships, lines, and linear equations.**			
	8.EE.B.5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*	**8.EE.B Understand the connections between proportional relationships, lines, and linear equations.**	**8.EE.B.5**	Graph proportional relationships interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

	8.EE.B.6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.		**8.EE.B.6**	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. Derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at (0, b).
8.EE.C	**Analyze and solve linear equations and pairs of simultaneous linear equations.**			
	8.EE.C.7. Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	**8.EE.C** **Analyze and solve linear equations, inequalities, and pairs of simultaneous linear equations.**	**8.EE.C.7**	Fluently solve linear equations and inequalities in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solution. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations and inequalities with rational number coefficients, including solutions that require expanding expressions using the distributive property and collecting like terms.

	8.EE.C.8. Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6. c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*		**8.EE.C.8**	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations including cases of no solution and infinite number of solutions. Solve simple cases by inspection. c. Solve mathematical problems and problems in real-world context leading to two linear equations in two variables.
Functions (F)		**Functions (F)**		
8.F.A	**Define, evaluate, and compare functions.**			
	8.F.A.1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.)	**8.F.A** **Define, evaluate, and compare functions.**	**8.F.A.1**	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.)
	8.F.A.2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*		**8.F.A.2**	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

	8.F.A.3. Interpret the equation *y* = *mx* + *b* as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function A =* s^2 *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*		**8.F.A.3**	Interpret the equation *y* = m*x* + *b* as defining a linear function whose graph is a straight line; give examples of functions that are not linear. *For example, the function A =* s^2 *giving the area of a square as a function of its side length in not linear because its graph contains the points (1,1), (2,4), and (3,9) which are not on a straight line.*
8.F.B	**Use functions to model relationships between quantities.**			
	8.F.B.4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	**8.F.B** **Use functions to model relationships between quantities.**	**8.F.B.4**	Given a description of a situation, generate a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or a graph. Track how the values of the two quantities change together. Interpret the rate of change and initial value of a linear function in terms of the situation it models, its graph, or its table of values.
	8.F.B.5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.		**8.F.B.5**	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
Geometry (G)		**Geometry (G)**		
8.G.A	**Understand congruence and similarity using physical models, transparencies, or geometry software.**			
	8.G.A.1. Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	**8.G.A** **Understand congruence and similarity.**	**8.G.A.1**	Verify experimentally the properties of rotations, reflections, and translations. Properties include: lines are taken to lines, line segments are taken to line segments of the same length, angles are taken to angles of the same measure, parallel lines are taken to parallel lines.

	8.G.A.2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.		**8.G.A.2**	Understand that a two-dimensional figure is congruent to another if one can be obtained from the other by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that demonstrates congruence.
	8.G.A.3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.		**8.G.A.3**	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
	8.G.A.4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.		**8.G.A.4**	Understand that a two-dimensional figure is similar to another if, and only if, one can be obtained from the other by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that demonstrates similarity.
	8.G.A.5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*		**8.G.A.5**	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
8.G.B	**Understand and apply the Pythagorean Theorem.**			
	8.G.B.6. Explain a proof of the Pythagorean Theorem and its converse.	**8.G.B** **Understand and apply the Pythagorean Theorem.**	**8.G.B.6**	Understand the Pythagorean Theorem and its converse.
	8.G.B.7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.		**8.G.B.7**	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world context and mathematical problems in two and three dimensions.

	8.G.B.8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.		**8.G.B.8**	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
8.G.C	**Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**			
	8.G.C.9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	**8.G.C** **Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**	**8.G.C.9**	Understand and use formulas for volumes of cones, cylinders and spheres and use them to solve real-world context and mathematical problems.
Statistics and Probability (SP)		**Statistics and Probability (SP)**		
8.SP.A	**Investigate patterns of association in bivariate data.**			
	8.SP.A.1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	**8.SP.A** **Investigate patterns of association in bivariate data.**	**8.SP.A.1**	Construct and interpret scatter plots for bivariate measurement data to investigate and describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
	8.SP.A.2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	**8.SP.A (cont.)**	**8.SP.A.2**	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

	8.SP.A.3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*		**8.SP.A.3**	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.
	8.SP.A.4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*		**8.SP.A.4**	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.
		8.SP.B **Investigate chance processes and develop, use, and evaluate probability models.**	**8.SP.B.5**	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using organized lists, tables, tree diagrams and other methods. Identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events.

Standards for Mathematical Practice

8.MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, "Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.
8.MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.
8.MP.3 Construct viable arguments, and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming or questioning the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.
8.MP.4 Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
8.MP.5 Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.
8.MP.6 Attend to precision. Mathematically proficient students clearly communicate to others using appropriate mathematical terminology, and craft explanations that convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their

representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.
8.MP.7 Look for and make use of structure. Mathematically proficient students use structure and patterns to assist in making connections among mathematical ideas or concepts when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.
8.MP.8 Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.