Arizona Mathematics Standards

Quantitative Reasoning

ARIZONA DEPARTMENT OF EDUCATION
HIGH ACADEMIC STANDARDS FOR STUDENTS
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Quantitative Reasoning: Overview

Quantitative reasoning is knowledge of and confidence with basic mathematical/analytical concepts and operations required for problem solving, decision making, economic productivity, and real-world applications. Quantitative reasoning will prepare students for an increasingly information-based society in which the ability to use and critically evaluate information, especially numerical information, is central to becoming an informed citizen. Students who successfully complete a course in Quantitative Reasoning should acquire the skills necessary to make rational decisions based on real data. Students will be able to report their conclusions in a precise and accurate manner using the language, tools, and symbolism of mathematics.

Quantitative Reasoning (QR) is a habit of mind and a QR course focused on developing this habit of mind may look different from previous math courses. While previous mathematics courses may have mostly focused on specific mathematical and/or statistical methods and procedures, this course should focus on understanding and communication. This approach is based on the understanding that there are real-world quantities that productive citizens need to resolve and address. The vision of this course is to develop and practice the Quantitative Reasoning Habits of Mind:

- Interpreting the circumstances of the situation, making estimates as necessary to decide what investigation or study is merited, and gleaning critical information.
- Representing the information in an appropriate mathematical form.
- Performing calculations and manipulations on the mathematical form.
- Analyzing or synthesizing the quantitative information produced.
- Evaluating assumptions that have been made.
- Communicating the results by reflecting back into the original circumstance.

These habits of mind often require careful reading of both continuous prose and discontinuous prose (such as graphical representations), using mathematics or statistics, and then interpreting or critiquing the original prose in light of mathematical results. Critical reasoning is required throughout. Problems may have multiple reasonable responses. Consequently, conclusions require explanations of reasoning that led to the conclusions. Students should always read critically, understand a situation, and draw conclusions based on evidence. Communicating the results, including the evidence that underlies the results is the critical end product. So, reading and writing - communication - is a substantial portion of quantitative reasoning.

Assessing QR involves judging written analyses and reflections and the quality of evidence given in support of arguments or conclusions. Frequent reminders may be necessary for students to supply the following:

- Evidence supporting reasoning or assertions.
- Calculations that produce numerical results.
- Correct units on quantities.
- Complete and correct sentences stating evidence and conclusions.
- Precision of language in stating questions and results.

In order to have sufficient opportunity to develop QR Habits of Mind, it is recommended that QR courses have students read and interpret authentic texts such as advertisements, consumer information, government forms, and newspaper articles containing quantitative information, including graphical displays of quantitative information. These texts may be as long as standard magazine articles and include expectations to compare, analyze, and/or synthesize multiple forms of quantitative information.

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Quantitative Reasoning Habits of Mind
Interpretation - Ability to explain information presented in mathematical form (e.g., equations, graphs, diagrams, tables, words). The student provides accurate explanations of information presented in mathematical forms and makes appropriate inferences based on that information. For example, accurately explains the trend data shown in a graph and makes reasonable predictions regarding what the data suggest about future events.
Representation - Ability to convert relevant information into various mathematical forms (e.g., equations, graphs, diagrams, tables, words). The student skillfully converts relevant information into an insightful mathematical portrayal in a way that contributes to a further or deeper understanding.
Calculation - Ability to perform arithmetic and mathematical calculations. Calculations attempted by the student are essentially all successful and sufficiently comprehensive to solve the problem. Calculations are also presented elegantly (clearly, concisely, etc.).
Analysis/Synthesis - Ability to make judgements and draw appropriate conclusions based on the quantitative analysis of data, while recognizing the limits of this analysis. The student uses the quantitative analysis of data as the basis for deep and thoughtful judgments, drawing insightful, carefully qualified conclusions from this work.
Assumptions - Ability to make and evaluate important assumptions in estimation, modeling, and data analysis. The student explicitly describes assumptions and provides compelling rationale for why each assumption is appropriate. The student shows awareness that confidence in final conclusions is limited by the accuracy of the assumptions.
Communication - Expressing quantitative evidence in support of the argument or purpose of the work (in terms of what evidence is used and how it is formatted, presented, and contextualized). The student's use of quantitative, logical and statistical information enhances the argument or purpose of the work. The student presents the quantitative evidence in an effective format.

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Strand Overview

Arizona Quantitative Reasoning courses should give students opportunity to develop the Habits of Mind, discussed above, while deepening their ability to reason numerically, covariationally, statistically, probabilistically, graphically (discrete), and financially. These content reasoning strands are introduced below:

1. Numerical Reasoning involves generating new mathematical understandings through problems involving numerical data that arise in everyday life, society, and the workplace. The students will draw conclusions and/or make decisions based on analysis and critique of quantitative information. The students will also effectively justify and communicate conclusions in ways appropriate to the audience. The student is expected to:

- Use precision and accuracy in real-life situations related to measurement and significant figures.
- Solve problems involving quantities that are not easily measured using proportionality.
- Solve real-life problems requiring interpretation and comparison of various representations of ratios (i.e. fractions, decimals, rate, and percentages).

2. Covariational Reasoning calls for students to engage in cognitive activities involved in coordinating two varying quantities while attending to the ways in which the two quantities change in relation to each other (Carlson, 1998; Carlson & Larsen, 2001; Carlson, Jacobs, & Larsen, 2001). The overarching goal of this strand is to engage students in reasoning about an association (or the relationships) between two quantities or reasoning about bivariate data, which involves knowing how to judge and interpret a relationship(s) between two quantities. Such activities include but are not limited to:

- Meaningful exploration of relative and absolute change.
- Reason and communicate meaningfully about proportional relationships and relationships that are not proportional. Use these relationships to solve contextualized problems.
- Choose and create models for bivariate data sets, and use the models to solve problems and make decisions.

3. Statistical and Probabilistic Reasoning involves using the process standards in mathematics to generate new understandings of probability and statistics. The students make interpretations based on, and inferences from, data. The students analyze statistical information and evaluate risk and return to connect mathematical ideas and make informed decisions. The students use multiple representations to communicate effectively the results of real data analysis from current events, the critical analysis of published statistical studies, and may include student-generated statistical studies. Statistical problem solving is an investigative process that involves four components:

- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- Select and use appropriate statistical methods to analyze data;
- Develop and evaluate inferences and predictions that are based on data; and
- Understand and apply basic concepts of probability. (*GAISE Report, 2007*)

This set of standards was developed from the *GAISE (Guidelines for Assessment and Instruction in Statistics Education)* report and will call for students to:

- Gain insight about a solution to a statistical question by describing features of the data through the use of graphical and tabular representations and numerical summaries.
- Develop probability models to describe the long-run behavior of observations of a random variable.
- Recognize probability as an essential tool of statistics and understand the role of probability in statistical reasoning.
- Draw appropriate conclusions from data in ways that acknowledge random variation, and acknowledge potential limitations and bias.

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4. Discrete Mathematical Reasoning involves the study of vertex-edge graphs as mathematical models for analyzing the relationships among a finite number of objects shown by vertices that are connected by edges. Students should be able to understand, analyze, and apply vertex-edge graphs and adjacency matrices to model and make informed decisions related to paths, circuits, networks, and relationships in real-world settings.

5. Financial Reasoning includes contextual applications that facilitate opportunities to learn the basics of spending and saving, credit and debt, employment and income, investing, risk management and insurance, and financial decision making in order to obtain a combination of awareness, knowledge, skill, attitude and behavior necessary to make sound financial decisions and ultimately achieve individual financial wellbeing. "Financial literacy, if taught outside of a mathematics course, often doesn't address the mathematical aspects of financial literacy. To succeed in life, students need both an understanding of financial issues and the math skills to make financially sound choices." (Peters, S., Bay-Williams, J., & Martinie, S., 2016)
Students are expected to:

- Budget with awareness of income and expenses.
- Use investment strategies to plan for the future.
- Use models to communicate various mathematical financial scenarios.
- Understand the personal and societal effect of financial decisions.

Use of Technology

"It is essential that teachers and students have regular access to technologies that support and advance mathematical sense making, reasoning, problem solving, and communication. Effective teachers optimize the potential of technology to develop students' understanding, stimulate their interest, and increase their proficiency in mathematics. When teachers use technology strategically, they can provide greater access to mathematics for all students" (NCTM, 2011). These standards were created with the intention of providing many opportunities for students to explore mathematical concepts and real-world problems with the aid of technology and other manipulatives. Technology is one tool that is essential for exploration at this level of mathematics.

The use of technology is expected to go beyond the use of handheld calculators and basic computation. Technology may include, but is not limited to, spreadsheets, presentation software, electronic data sources, and dynamic software for the purposes of exploration, communication, and analysis.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Mathematical modeling is integrated throughout Quantitative Reasoning course by utilizing real-world contexts.

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Numerical Reasoning (NR)	QR.NR.1: Represent quantities, using equivalent forms when appropriate, to investigate and describe quantitative and geometric relationships and solve problems in real-world contexts.
	QR.NR.2: Reason, model, and communicate with and about percentages (change, incorrect, deceptive, relative and absolute).
	QR.NR.3: Understand and compare magnitudes of numbers utilizing real-world context. Understand the importance and impact of unit selection.
	QR.NR.4: Use and justify estimation skills, and know why, how, and when to estimate results. Assess and justify the reasonableness of estimations using the context and comparisons to other known values.
Covariational Reasoning (CR)	QR.CR.1: Analyze and compare growth and decay using absolute and relative change utilizing real-world contexts.
	QR.CR.2: Compare, reason and communicate about proportional and non-proportional models utilizing real-world contexts.
	QR.CR.3: Identify, create, and use appropriate models for bivariate data sets (i.e. linear, exponential) to estimate solutions for contextual questions, identify patterns and identify how changing parameters affect the models.

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Statistical and Probabilistic Reasoning (SPR)	QR.SPR.1: Reason and communicate about the validity of claims based on empirical, theoretical, and subjective probabilities. Draw conclusions or make decisions related to risk, pay-off, expected value, and false negatives/positives in various probabilistic contexts. *Encompasses P.S-CP.B.9, P.S-MD.A.2, P.S-MD.A.3, P.S-MD.A.4, P.S-MD.B.5, P.S-MD.B.7*
	QR.SPR.2: Analyze statistical information and identify limitations, strengths, or lack of information in studies including data collection methods (e.g. sampling, experimental, observational) and possible sources of bias. Identify errors or misuses of statistics to justify particular conclusions. *Encompasses P.S-IC.B.3*
	QR.SPR.3: Represent numerical summaries and visual displays of real-world data to make informed decisions. Reason, communicate, and describe strengths, limitations, and fallacies of various displays. *Encompasses P.S-IC.B.6*
	QR.SPR.4: Represent center, shape, and spread of two or more data sets. Reason, communicate, and compare data sets in context.
Discrete Mathematical Reasoning (DMR)	QR.DMR.1: Understand, analyze, and apply vertex-edge graphs to model and make informed decisions related to paths, circuits, networks, and relationships in real-world settings. *Encompasses P.CM-DM.A.1, P.CM-DM.A.2*
	QR.DMR.2: Devise, analyze, and apply algorithms for solving vertex-edge graph problems. *P.CM-DM.A.3*
	QR.DMR.3: Extend work with adjacency matrices for graphs, such as interpreting row sums and using the *n*th power of the adjacency matrix to count paths of length *n* in a graph. *P.CM-DM.A.4*

Financial Reasoning (FR)	QR.FR.1: Identify and research a career goal. Develop a plan and time table for achieving it including educational/training requirements, costs, and other factors (e.g. cost versus savings, income and debt).
	QR.FR.2: Understand and apply strategies to monitor income and expenses, plan for spending, implement a diversified investment strategy, and save for future goals.
	QR.FR.3: Use models to solve and communicate about contextual financial questions such as credit card debt, installment savings, amortization schedules, mortgage and other loan scenarios.
	QR.FR.4: Identify and explain personal and societal consequences of financial decisions.

Standards for Mathematical Practice (MP)	
QR.MP.1	**Make sense of problems and persevere in solving them.** Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, “Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.
QR.MP.2	**Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.
QR.MP.3	**Construct viable arguments and critique the reasoning of others.** Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming or questioning the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.
QR.MP.4	**Model with mathematics.** Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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QR.MP.5	**Use appropriate tools strategically.** Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.
QR.MP.6	**Attend to precision.** Mathematically proficient students clearly communicate to others using appropriate mathematical terminology, and craft explanations that convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.
QR.MP.7	**Look for and make use of structure.** Mathematically proficient students use structure and patterns to assist in making connections among mathematical ideas or concepts when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.
QR.MP.8	**Look for and express regularity in repeated reasoning.** Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.

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