



Arizona Mathematics Standards

Precalculus

ARIZONA DEPARTMENT OF EDUCATION
HIGH ACADEMIC STANDARDS FOR STUDENTS
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Precalculus: Overview

Precalculus strengthens students' conceptual understanding of problems, mathematical reasoning, and mathematical modeling in solving problems. This course combines concepts of trigonometry, geometry, and functions to prepare students for studies in STEM related courses and college and career entry mathematics courses.

Precalculus courses have traditionally served to prepare students for entrance into Calculus. These standards have been written with the intent for any fourth-year math student to experience extended mathematical concepts regardless of their future pathway. The standards are designed to create coherence between and within strands and to promote making sense of mathematical ideas and problem-solving skills. The unifying themes across all strands encourage students to reason mathematically and model real-world contexts.

Reasoning Strands

Because the standards that comprise this course are extensions of the plus standards as well as concepts needed to prepare students for the study of Calculus, students who enroll in Precalculus should have securely held knowledge of real numbers from the first three high school mathematics courses. The study of the topics, concepts, and procedures of Precalculus deepens students' understanding of the use of numbers and quantities and extends students' ability to apply them at a higher conceptual level, as a tool, in the study of other topics. Therefore, Number and Quantity have become embedded concepts in the standards for Precalculus rather than a separate entity.

These Precalculus standards were constructed around four major strands: Reasoning with Functions and Relations, Reasoning with Trigonometry, Reasoning with Vectors, and Reasoning with Matrices. An intentional attempt was made to build coherence both **within** the four strands and **across** the four strands. For example, ideas involving trigonometry are seen throughout the standards. In the Reasoning with Functions and Relations strand, students will have opportunities to analyze all kinds of functions including trigonometric functions. This leads to the Reasoning with Trigonometry strand where students have opportunities to solve problems using triangle trigonometry (right and non-right triangles). In the Reasoning with Vectors strand, students continue to apply their understanding of trigonometry to make sense of vector quantities and polar coordinates. Other major topics can be seen through the standards as they naturally emerge in a coherent manner.

Mathematical Modeling

Another unifying theme throughout these standards is the idea of using mathematical modeling for the purpose of solving real-world problem situations.

Mathematical models are representations of the behavior of quantities or the relationships among quantities. Mathematical models are representations of reality; it is important for students to be able to identify realistic models, generate models, validate them, use them to solve problems, and describe the limitations of their model. However, models are exactly that, representations, and are themselves real only as models, and are never to be confused with the reality we are trying to model. Thus, if the behavior predicted by our models does not reflect what we see or measure in quantities we are trying to model, it is the models (or the model's scope) that need to be fixed—and not the quantities. The motivation for creating a model to represent quantities and their relationships is to help us understand the quantities/relationships better and to solve problems about the quantities/relationships.

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Mathematical modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Although “Model with Mathematics” is one of Arizona’s Standards for Mathematical Practice, problems that are solved through authentic use of modeling naturally promote students’ engagement with the other mathematical practices. For example, students must make sense of the problem to propose assumptions and limitations of their proposed model as well as persevere to find a satisfactory solution that makes sense for the quantities/relationships they are trying to model. While sharing their models, or judging the merit of others’ models, students construct viable arguments and critique the reasoning of others. Modeling provides opportunities for students to engage in meaningful discourse and to question one another about their decisions throughout the modeling process. Further, modeling allows students to consider the appropriateness of mathematics concepts they already know, and make decisions about representations that highlight the aspects of the model of interest. This course is designed to develop students’ modeling skills not as a separate standard but in the context of investigating various families of functions.

Use of Technology

“It is essential that teachers and students have regular access to technologies that support and advance mathematical sense making, reasoning, problem solving, and communication. Effective teachers optimize the potential of technology to develop students’ understanding, stimulate their interest, and increase their proficiency in mathematics. When teachers use technology strategically, they can provide greater access to mathematics for all students” (NCTM, 2011). These standards were created with the intention of providing many opportunities for students to explore mathematical concepts and real-world problems with the aid of technology and other manipulatives. Technology is one tool that is essential for exploration at this level of mathematics. The use of technology is expected to go beyond the use of handheld calculators and basic computation. Technology may include, but is not limited to, spreadsheets, electronic data sources, and dynamic software for the purposes of exploration, communication, and analysis.

Common Definitions

Explore--to develop new concepts and representations by investigating the formulation of questions then testing and refining their conjectures about the given situation or concept.

Fluency

Wherever the word *fluently* appears, the word includes *efficiently, accurately, flexibly, and appropriately*. Being fluent means that students are able to choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and are able to explain their approaches, and they are able to produce accurate answers efficiently.^[1]

- **Efficiency**—carries out easily, keeps track of sub-problems, and makes use of intermediate results to solve the problem.
- **Accuracy**—reliably produces the correct answer.
- **Flexibility**—knows more than one approach, chooses a viable strategy, and uses one method to solve and another method to doublecheck.
- **Appropriately**—knows when to apply a particular procedure.

Fluency Expectations, Precalculus

Fluency is important in high school mathematics. For example, fluency with the Unit Circle is demonstrated when students efficiently carry out calculations in both radians and degrees. Students can accurately approximate a value such as the Sin (2 radians) and explain why $\sin(1.1 \text{ radians}) > \sin(0.3 \text{ radians})$. Students are flexible in using different strategies when solving trigonometric problems. Students can appropriately apply Unit Circle concepts when working with trigonometric identities, equations, and graphs.

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Strand Overview

Arizona Precalculus courses should give students opportunities to develop the habits of mind as indicated in the Standards for Mathematical Practices as well as develop understanding and reasoning of the content in the following strands:

1. Reasoning with Functions and Relations involves students understanding functions and the reasoning of functions beyond Algebra 2. It provides students with opportunities to extend the behavior of functions and relations by using multiple representations and covariational reasoning to investigate and explore quantities, their relationships, and how these relationships change. Reasoning with functions and relations provides students with the algebraic tools necessary to analyze a variety of function types and the understanding of these functions in real-world situations.

2. Reasoning with Trigonometry involves students extending their trigonometric reasoning beyond the skills learned in Geometry and Algebra 2. Students should deepen their reasoning of relationships in right triangles from using the mnemonic *SohCahToa* to being able to interpret the meaning of the trigonometric ratios as multiplicative comparisons of the appropriate sides of a right triangle. Trigonometric reasoning will be used in situations where students are finding unknown sides and/or angles of right triangles. Students will have the opportunity to extend their reasoning with trigonometric reasoning to non-right triangles and make sense of the Law of Sines and Law of Cosines. Students will use the Law of Sines and the Law of Cosines to solve problems and to make connections to formulas for the area of a triangle. Furthermore, students will become fluent with the Unit Circle. The Unit Circle should be used to explain the symmetry and periodicity of trigonometric functions. Students should have the opportunity to model real-world situations with trigonometry including inverse trigonometric functions and periodic functions. Trigonometric expressions, like algebraic expressions, can be manipulated from one form to another using mathematical rules and properties. Students should have opportunities to identify ways to rewrite an expression in order to simplify or produce an equivalent expression to explain properties or solve trigonometric equations, including inverse trigonometric functions, utilizing real-world context. When solving trigonometric equations, students should be able to evaluate and interpret the solution in terms of context. Finally, students will extend their knowledge of trigonometry to the Polar Coordinate system. Students should have the opportunity to graph, analyze, and interpret polar equations, and solve problems in real-world context with and without technology, as appropriate.

3. Reasoning with Vectors involves recognizing that a vector consists of both magnitude and direction. Students should understand that motion in space, such as velocity, can be represented by a vector. Students should be proficient using appropriate symbols to represent vectors and their magnitudes. Furthermore, students should be able to determine a vector from its initial point and terminal point, add and subtract vectors, and multiply a vector by a scalar.

4. Reasoning with Matrices involves using matrices to represent and manipulate data and as a tool to create transformations or calculate area of geometric figures. Students should have learning opportunities with matrices that allow them to make connections to the real numbers. Students will explore the roles and properties of matrices to solve systems of linear equations.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Mathematical modeling is integrated throughout the Precalculus course by utilizing real-world context.

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Reasoning with Functions and Relations (RFR)			
		Standard	Students should have opportunities to...
RFR.AF Analyze functions <i>Standards within this strand encompass P.F-IF</i>	RFR.AF.1	Interpret parameters of a function defined by an expression in the context of the situation.	a) Define domain and range in the real-world context.
	RFR.AF.2	Sketch the graph of a function that models a relationship between two quantities, identifying key features.	b) Calculate, interpret, and use average rate of change.
	RFR.AF.3	Interpret key features of graphs and tables for a function that models a relationship between two quantities in terms of the quantities.	c) Use multiple representations of function models appropriately.
	RFR.AF.4	Use limits to describe long-range behavior, asymptotic behavior, and points of discontinuity.	d) Work with different families of functions beyond linear and quadratic functions including but not limited to exponential, logarithmic, rational, polynomial, logistic, radical, and piecewise-defined functions.
	RFR.AF.5	Sketch the graph of all six trigonometric functions, identifying key features.	e) Graph rational functions including those whose graphs contain horizontal asymptotes, vertical asymptotes, oblique asymptotes, and/or holes. f) Make sense of radian measure. g) Use the unit circle as a tool to graph the trig functions in radians and degrees. h) Develop fluency with the unit circle. Include opportunities beyond the special angles, for example, explain why $\sin(1.1) > \sin(0.3)$ in radians. i) Graph sine, cosine, tangent functions in radians and degrees and analyze/explain the characteristics of each. j) Explore the reciprocal trig functions using technology.

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Reasoning with Functions and Relations (RFR)			
		Standard	Students should have opportunities to...
RFR.BF Building Functions <i>Standards within this strand encompass P.F-BF, P.F-TF</i>	RFR.BF.1	Model relationships between quantities that require adding, subtracting, multiplying, and/or dividing functions	a) Model real-world situations with the sum, difference, product, or quotient of other function models. b) Describe relationships of quantities in functions and within a composition of those functions. c) Find an inverse algebraically, for example given $y = f(x)$ algebraically find $x = f^{-1}(y)$. Work with exponential and logarithmic functions, and quadratic and square root functions at a minimum d) Understand that restricting a trigonometric function to a domain on which it is monotonic allows its inverse to be constructed. e) Describe the meaning of $f^{-1}(20)$ given a function f that takes hours as an input and gives miles as an output. f) Make sense of the covarying quantities when modeling with inverse functions.
	RFR.BF.2	Model relationships through composition and attend to the restrictions of the domain.	
	RFR.BF.3	Rewrite a function as a composition of functions.	
	RFR.BF.4	Determine if a function has an inverse. If so, find the inverse. If not, define a restriction on the domain that meets the requirement for invertibility and find the inverse on the restricted domain.	
	RFR.BF.5	Interpret the meanings of quantities involving functions and their inverses.	
	RFR.BF.6	Verify by analytical methods that one function is the inverse of another.	

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Reasoning with Functions and Relations (RFR)			
		Standard	Students should have opportunities to...
RFR.IC Interpreting Conics <i>Standards within this strand encompass P.G-GPE</i>	RFR.IC.1	Model real-world situations which involve conic sections.	a) Explore conics as loci of points satisfying stipulated conditions. b) Explore conic sections with technology and manipulatives. c) Connect the geometric and algebraic relationships of conics. d) Use the method of completing the square to put the equation of the conic section into standard form.
	RFR.IC.2	Identify key features of conic sections (foci, directrix, radii, axes, asymptotes, center) graphically and algebraically.	
	RFR.IC.3	Sketch a graph of a conic section using its key features.	
	RFR.IC.4	Use the key features of a conic section to write its equation.	
	RFR.IC.5	Given a quadratic equation of the form $ax^2+by^2 + cx + dy + e = 0$, determine if the equation is a circle, ellipse, parabola, or hyperbola.	
Reasoning with Functions and Relations (RFR)			
		Standard	Students should have opportunities to...
RFR.ISS Interpreting Sequences and Series <i>Standards within this strand encompass P.F-BF.A.2, P.A-SSE.B.4</i>	RFR.ISS.1	Model real-world situations involving sequences or series using recursive and/or explicit definitions.	a) Become fluent in working with arithmetic and geometric sequences and series. b) Explore several types of sequences, including but not limited to Fibonacci, telescoping, harmonic, alternating. c) Work with tasks without using formal limit notation, such as, “as n approaches infinity, what happens to s_n ?”
	RFR.ISS.2	Use covariational reasoning to describe sequences and series.	
	RFR.ISS.3	Represent finite or infinite series using sigma notation.	
	RFR.ISS.4	Find the sums of finite or infinite series, if they exist.	

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Reasoning with Trigonometry			
		Standard	Students should have opportunities to...
RT.ETT Extending Triangle Trigonometry <i>Standards within this strand encompass P.G-SRT, P.F-TF.B</i>	RFR.ETT.1	Model real-world situations involving trigonometry.	<p>a) Go beyond using the mnemonic <i>SohCahToa</i> by interpreting the meaning of the trigonometric ratios as multiplicative comparisons of the appropriate sides of a right triangle. Situations should involve unknown sides and/or angles, as well as periodic functions and their inverses.</p> <p>b) Make sense of the Law of Sines and the Law of Cosines.</p> <p>c) Derive formulas to find the area of a triangle.</p>
	RFR.ETT.2	Apply the Law of Sines and Law of Cosines to solve problems.	
	RFR.ETT.3	Use trigonometry to find the area of triangles.	
	RFR.ETT.4	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x , where x is any real number.	
	RFR.ETT.5	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	
	RFR.ETT.6	Use inverse functions to solve trigonometric equations utilizing real world context; evaluate the solution and interpret them in terms of context.	

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Reasoning with Trigonometry			
		Standard	Students should have opportunities to...
RT.RTS Reasoning with Trigonometric Structure <i>Standards within this strand encompass P.F-TF.C</i>	RT.RTS.1	Use the structure of a trigonometric expression to identify ways to rewrite it.	a) Recognize that a single identity can be manipulated into another identity. For example, $(\sin x)^2 + (\cos x)^2 = 1$ can be transformed into $1 + (\cot x)^2 = (\csc x)^2$ by dividing by $(\cos x)^2$. b) Apply the Pythagorean, sum, difference, double angle, and half angle formulas for sine and cosine to reveal and explain properties. c) Solve trigonometric equations algebraically using simple identity substitutions and factoring, extending solutions to all real numbers.
	RT.RTS.2	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.	
	RT.RTS.3	Solve trigonometric equations.	
Reasoning with Trigonometry			
		Standard	Students should have opportunities to...
RT.EPE Exploring Polar Equations	RT.EPE.1	Graph polar equations.	a) Convert points between polar and rectangular forms. b) Determine equivalent polar representations for a given point. c) Analyze graphs of polar equations and look for patterns in their behavior including but not limited to the number of petals in the rose, symmetry, intersection points for two or more polar graphs, and the required domain to complete the graph. d) Reconcile preconceived notions of graphs in rectangular form with graphs in polar form. For example: the vertical line test, the meaning of constant rate of change/linearity.
	RT.EPE.2	Analyze and interpret the graphs of polar equations.	
	RT.EPE.3	Use polar equations to solve problems.	

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Reasoning with Vectors

		Standard	Students should have opportunities to...
RV.MP Modeling with Parametrics	RV.MP.1	Model real-world contexts with parametric equations.	a) Convert between parametric and rectangular representations. b) Understand parametric equations are used to express two dependent variables in terms of an independent variable, the parameter. c) Realize that two or more graphs of parametric equations may intersect at a location but not for the same parameter value. For example, the paths of two cars may cross but not at the same time. d) Understand parametric graphs can be represented using multiple parametric equations as defined by the domain of the parameter.
	RV.MP.2	Use parametric equations to solve problems.	
	RV.MP.3	Graph parametric equations and identify orientation.	
	RV.MP.4	Analyze and interpret the graphs of parametric equations.	

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Reasoning with Vectors

		Standard	Students should have opportunities to...
RV.EV Exploring Vectors <i>Standards within this strand encompass P.N-VM.A, P.N-VM.B</i>	RV.EV.1	Recognize vector quantities as having both magnitude and direction.	a) Distinguish vector quantities from scalar quantities. For example, distinguish the difference between velocity and speed. b) Make sense of operations with vectors.
	RV.EV.2	Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes.	
	RV.EV.3	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.	
	RV.EV.4	Solve problems involving velocity and other quantities that can be represented by vectors.	
	RV.EV.5	Add and subtract vectors, and multiply a vector by a scalar.	

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Reasoning with Matrices			
		Standard	Students should have opportunities to...
RM.UM Using Matrices <i>Standards within this strand encompass P.N-VM.C</i>	RM.UM.1	Use matrices to represent and manipulate data.	a) Explore the properties of matrices and their operations. For example, the commutative property of multiplication does not apply with all matrices.
	RM.UM.2	Use matrix operations to solve problems. Add, subtract, and multiply matrices of appropriate dimensions. Multiply matrices by scalars to produce new matrices.	b) Explain that the determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
	RM.UM.3	Find the inverse and determinant of a matrix.	c) Explore the roles of the zero matrix, identity matrix, inverse matrix, and the determinant of a matrix.
	RM.UM.4	Use matrices to solve systems of linear equations.	d) Work with 2 x 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. e) Use the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. f) Use matrices as a tool. Including but not limited to: producing new vectors from old vectors, creating transformations in the plane, calculating the area of geometric figures. g) Explore matrices and solve problems with and without technology, as appropriate.

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Standards for Mathematical Practice

PC.MP.1	<p>Make sense of problems and persevere in solving them.</p> <p>Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, “Does this make sense?” to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.</p>
PC.MP.2	<p>Reason abstractly and quantitatively.</p> <p>Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.</p>
PC.MP.3	<p>Construct viable arguments and critique the reasoning of others.</p> <p>Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming or questioning the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.</p>
PC.MP.4	<p>Model with mathematics.</p> <p>Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.</p>

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PC.MP.5	<p>Use appropriate tools strategically.</p> <p>Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.</p>
PC.MP.6	<p>Attend to precision.</p> <p>Mathematically proficient students clearly communicate to others using appropriate mathematical terminology, and craft explanations that convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.</p>
PC.MP.7	<p>Look for and make use of structure.</p> <p>Mathematically proficient students use structure and patterns to assist in making connections among mathematical ideas or concepts when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.</p>
PC.MP.8	<p>Look for and express regularity in repeated reasoning.</p> <p>Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.</p>

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