4th Grade – Summary of Revisions and Planning Guidance - *Arizona Mathematics Standards - Adopted in 2016*

Additions	Deletions
• **4.OA.C.6**–When solving problems, assess the reasonableness of answers using mental computation and estimation strategies including rounding.	• **AZ.4.OA.A.3.1**-Solve a variety of problems based on the multiplication principle of counting. a. Represent a variety of counting problems using arrays, charts, and systematic lists, e.g., tree diagram. b. Analyze relationships among representations and make connections to the multiplication principle of counting.

Parameter Changes/Clarifications	
4.OA.A.1	A description of the interpretation of multiplication is included in the e.g.
4.OA.A.2	This standard now contains a limit of within 1000.
4.OA.A.3	The understanding of a remainder is a fraction of the divisor is included. Removed the statement to assess reasonableness as it is now included in 4.OA.C.6
4.OA.B.4	Determining the number as prime or composite was removed. (See 5.OA.B.4 for the standard on understanding prime numbers)
4.OA.C.5	Explaining the pattern informally is now part of the standard.
4.OA.C.6	**NEW STANDARD**
4.NBT.A.1	The cognitive demand has increased in this standard stating that students need to apply knowledge to understand the place value relationships.
4.NBT.B.4	Replaced "the" standard algorithm with "a" standard algorithm.
4.NBT.B.6	The strategies and representations were removed from the standard. The cognitive demand of this standard increased as students need to demonstrate understanding.
4.NF.B.4	Unit fractions are specifically mentioned in parts a and b. The numerical examples were removed and general rules are included in parts a and b.
4.NF.C.6	Tenths and hundredths are explicitly stated and the expectation for locating these on a number line was specifically added.
4.MD.A.3	Reference to Table 2 is added to this standard.

Defining Standards, Curriculum and Instruction

Standards – What a student needs to know, understand, and be able to do by the end of each grade. Standards build across grade levels in a progression of increasing understanding and through a range of cognitive demand levels. Standards are adopted at the state level by the State Board of Education.

Curriculum – The resources used for teaching and learning the standards. Curricula are adopted at a local level by districts and schools.

Instruction – The methods used by teachers to teach their students. Instructional techniques are employed by individual teachers in response to the needs of the students in their classes to help them progress through the curriculum in order to master the standards.

Fluency Expectations

3rd	**3.NBT.A.2 -** Fluently add and subtract within 1000. **3.OA.C.7 -** Fluently multiply and divide within 100. By the end of 3rd grade, know from memory all multiplication products through 10 x 10 and division quotients when both the quotient and divisor are less than or equal to 10.
4th	**4.NBT.B.4 -** Fluently add and subtract multi-digit whole numbers using a standard algorithm.
5th	**5.NBT.B.5 -** Fluently multiply multi-digit whole numbers using a standard algorithm.

Fluency Definition

Fluency standard instruction should begin at the beginning of the year and continue throughout the school year. Wherever the word *fluently* appears in a content standard, the word includes ***efficiently, accurately, flexibly,*** *and* ***appropriately***. Being fluent means that students are able to choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and are able to explain their approaches, and they are able to produce accurate answers efficiently.

- **Efficiency**—carries out easily, keeps track of sub-problems, and makes use of intermediate results to solve the problem.
- **Accuracy**—reliably produces the correct answer.
- **Flexibility**—knows more than one approach, chooses a viable strategy, and uses one method to solve and another method to double-check.
- **Appropriately**-knows when to apply a particular procedure.

4th Grade Content Emphasis	
Operations and Algebraic Thinking (OA)	
●	Use the four operations with whole numbers to solve problems.
▲	Gain familiarity with factors and multiples.
▲	Generate and analyze patterns.
Number and Operations in Base Ten (NBT)	
●	Generalize place value understanding for multi-digit whole numbers.
●	Use place value understanding and properties of operations to perform multi-digit arithmetic.
Number and Operations - Fractions (NF)	
●	Extend understanding of fraction equivalence and ordering.
●	Apply and extend previous understanding of multiplication to multiply a whole number by a fraction.
●	Understand decimal notation for fractions, and compare decimal fractions.
Measurement and Data (MD)	
▲	Solve problems involving measurement and conversion of measurements from a larger unit to a small unit.
▲	Represent and interpret data.
▲	Geometric measurement: understand concepts of angle and measure angles.
Geometry (G)	
▲	Draw and identify lines and angles, and classify shapes by properties of their lines and angles.
● - Major Content ▲ - Supporting Content	

Major Content (●) from the content emphasis section should account for approximately 70% of instructional time.

Changes in Cognitive Demand

There are times in which the standards were changed, resulting in an increase or decrease in cognitive demand expectations within the standards. This is an important aspect of the standard to examine, keeping in mind that cognitive demand refers to the complexity of thinking involved in which students interact with the content; it does **not** refer to difficulty.

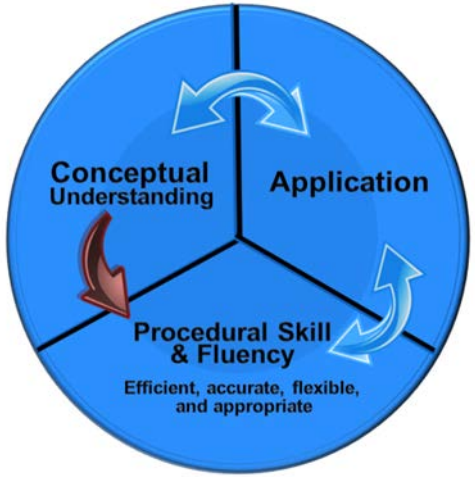
Changes in Cognitive Demand in the 4th Grade Standards	
4.NBT.A.1	**4.NBT.B.6**

Standards that reference Table 2 in 4th Grade	
4.OA.A.2	**4.MD.A.3**

The Standards for Mathematical Practice

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.
The Arizona Mathematics Standards now include narratives for each of the 8 Mathematical Practices.

Balance of Rigor in the Math Classroom



"Tasks that ask students to perform a memorized procedure in a routine manner lead to one type of opportunity for student thinking; tasks that require students to think conceptually and that stimulate students to make connections lead to a different set of opportunities for student thinking."
(Stein & Smith, 1998)

Comparison of Arizona Mathematics Standards – 2010 to 2016

Adopted 2010		Adopted 2016		
Operations and Algebraic Thinking (OA)		**Operations and Algebraic Thinking (OA)**		
4.OA.A	**Use the four operations with whole numbers to solve problems.**			
	4.OA.A.1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	**4.OA.A** **Use the four operations with whole numbers to solve problems.**	**4.OA.A.1**	Represent verbal statements of multiplicative comparisons as multiplication equations. Interpret a multiplication equation as a comparison (e.g., 35 is the number of objects in 5 groups, each containing 7 objects, and is also the number of objects in 7 groups, each containing 5 objects).
	4.OA.A.2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (See Table 2.)		**4.OA.A.2**	Multiply or divide within 1000 to solve word problems involving multiplicative comparison (e.g. , by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison). *See Table 2.*
	4.OA.A.3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.		**4.OA.A.3**	Solve multistep word problems using the four operations, including problems in which remainders must be interpreted. Understand how the remainder is a fraction of the divisor. Represent these problems using equations with a letter standing for the unknown quantity.
	AZ.4.OA.A.3.1 Solve a variety of problems based on the multiplication principle of counting. a. Represent a variety of counting problems using arrays, charts, and systematic lists, e.g., tree diagram. b. Analyze relationships among representations and make connections to the multiplication principle of counting.			

4.OA.B	**Gain familiarity with factors and multiples.**			
	4.OA.B.4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	**4.OA.B Gain familiarity with factors and multiples.**	**4.OA.B.4**	Find all factor pairs for a whole number in the range 1 to 100 and understand that a whole number is a multiple of each of its factors.
4.OA.C	**Generate and analyze patterns.**			
	4.OA.C.5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*	**4.OA.C Generate and analyze patterns.**	**4.OA.C.5**	Generate a number pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself and explain the pattern informally (e.g., given the rule "add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers).
			4.OA.C.6	When solving problems, assess the reasonableness of answers using mental computation and estimation strategies including rounding.
Number and Operations in Base Ten (NBT) (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000)		**Number and Operations in Base Ten (NBT)** *Note: Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.*		
4.NBT.A	**Generalize place value understanding for multi-digit whole numbers.**			
	4.NBT.A.1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*	**4.NBT.A Generalize place value understanding for multi-digit whole numbers.**	**4.NBT.A.1**	Apply concepts of place value, multiplication, and division to understand that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.
	4.NBT.A.2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.		**4.NBT.A.2**	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
	4.NBT.A.3. Use place value understanding to round multi-digit whole numbers to any place.		**4.NBT.A.3**	Use place value understanding to round multi-digit whole numbers to any place.

4.NBT.B	**Use place value understanding and properties of operations to perform multi-digit arithmetic.**			
	4.NBT.B.4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.	**4.NBT.B** **Use place value understanding and properties of operations to perform multi-digit arithmetic.**	**4.NBT.B.4**	Fluently add and subtract multi-digit whole numbers using a standard algorithm.
	4.NBT.B.5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.		**4.NBT.B.5**	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
	4.NBT.B.6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.		**4.NBT.B.6**	Demonstrate understanding of division by finding whole-number quotients and remainders with up to four-digit dividends and one-digit divisors.
Number and Operations—Fractions (NF) (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100)		**Number and Operations—Fractions (NF)** *Note: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.*		
4.NF.A	**Extend understanding of fraction equivalence and ordering.**			
	4.NF.A.1. Explain why a fraction *a*/*b* is equivalent to a fraction (n x a)/(n x b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	**4.NF.A** **Extend understanding of fraction equivalence and ordering.**	**4.NF.A.1**	Explain why a fraction *a/b* is equivalent to a fraction (*n x a*)/(*n x b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to understand and generate equivalent fractions.
	4.NF.A.2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.		**4.NF.A.2**	Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators and by comparing to a benchmark fraction). a. Understand that comparisons are valid only when the two fractions refer to the same size whole. b. Record the results of comparisons with symbols >, =, or <, and justify the conclusions.

4.NF.B	**Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers.**			
	4.NF.B.3. Understand a fraction a/b with a > 1 as a sum of fractions 1/b. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8=1/8+1/8+1/8 ; 3/8=1/8+2/8; 2 1/8=1 + 1+1/8=8/8+8/8 +1/8. c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	**4.NF.B** **Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers.**	**4.NF.B.3**	Understand a fraction *a/b* with a > 1 as a sum of unit fractions (1/*b*). a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way (e.g., 3/8 = 1/8 + 1/8+1/8; 3/8 = 2/8 + 1/8; 2 1/8 = 1 + 1 + 1/8 + or 2 1/8 = 8/8 + 8/8 + 1/8). c. Add and subtract mixed numbers with like denominators (e.g., by using properties of operations and the relationship between addition and subtraction and/or by replacing each mixed number with an equivalent fraction). d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.

	4.NF.B.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 x (1/4), recording the conclusion by the equation 5/4 = 5 x (1/4). b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 x (2/5) as 6 x (1/5), recognizing this product as 6/5. (In general, n x (a/b)=(n x a)/b.) c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?	**4.NF.B (cont.)**	**4.NF.B.4**	Build fractions from unit fractions. a. Understand a fraction *a/b* as a multiple of a unit fraction 1/*b*. In general, *a/b* = *a* x 1/*b* . b. Understand a multiple of *a/b* as a multiple of a unit fraction 1/*b* , and use this understanding to multiply a whole number by a fraction. In general, *n x a/b* = (*n x a*)/*b*. c. Solve word problems involving multiplication of a whole number by a fraction. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*
4.NF.C	**Understand decimal notation for fractions, and compare decimal fractions.**			
	4.NF.C.5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100. (*Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general, but addition and subtraction with unlike denominators in general is not a requirement at this grade.)	**4.NF.C Understand decimal notation for fractions, and compare decimal fractions.**	**4.NF.C.5**	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 (tenths) and 100 (hundredths). *For example, express 3/10 as 30/100, and and 3/10 + 4/100 = 34/100.* (Note: Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators, in general, is not a requirement at this grade.)
	4.NF.C.6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*		**4.NF.C.6**	Use decimal notation for fractions with denominators 10 (tenths) or 100 (hundredths), and locate these decimals on a number line.

	4.NF.C.7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.		**4.NF.C.7**	Compare two decimals to hundredths by reasoning about their size. Understand that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <.
Measurement and Data (MD)		**Measurement and Data (MD)**		
4.MD.A	**Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**			
	4.MD.A.1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs* (*1, 12*), (*2, 24*), (*3, 36*),	**4.MD.A** **Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**	**4.MD.A.1**	Know relative sizes of measurement units within one system of units which could include km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1,12), 2,24), (3,36).*
	4.MD.A.2. Solve word problems in a real-world context involving distances, intervals of time (hr, min, sec), liquid volumes, masses of objects, and money, including decimals and problems involving fractions with like denominators, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using a variety of representations that feature a measurement scale.		**4.MD.A.2**	Use the four operations to solve word problems and problems in real-world context involving distances, intervals of time (hr, min, sec), liquid volumes, masses of objects, and money, including decimals and problems involving fractions with like denominators, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using a variety of representations, including number lines that feature a measurement scale.
	4.MD.A.3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.		**4.MD.A.3**	Apply the area and perimeter formulas for rectangles in mathematical problems and problems in real-world contexts including problems with unknown side lengths. S*ee Table 2.*

4.MD.B	**Represent and interpret data.**			
	4.MD.B.4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*	**4.MD.B** **Represent and interpret data.**	**4.MD.B.4**	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots.
4.MD.C	**Geometric measurement: understand concepts of angle and measure angles.**			
	4.MD.C.5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.	**4.MD.C** **Geometric measurement: understand concepts of angle and measure angles.**	**4.MD.C.5**	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through n one-degree angles is said to have an angle measure of *n* degrees.
	4.MD.C.6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.		**4.MD.C.6**	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
	4.MD.C.7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.		**4.MD.C.7**	Understand angle measures as additive. (When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.) Solve addition and subtraction problems to find unknown angles on a diagram within mathematical problems as well as problems in real-world contexts.

Geometry (G)		Geometry (G)		
4.G.A	**Draw and identify lines and angles, and classify shapes by properties of their lines and angles**			
	4.G.A.1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	**4.G.A** **Draw and identify lines and angles, and classify shapes by properties of their lines and angles.**	**4.G.A.1**	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
	4.G.A.2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.		**4.G.A.2**	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size (e.g., understand right triangles as a category, and identify right triangles).
	4.G.A.3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.		**4.G.A.3**	Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Standards for Mathematical Practice

4.MP.1 Make sense of problems and persevere in solving them.
Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, “Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.

4.MP.2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.

4.MP.3 Construct viable arguments, and critique the reasoning of others.
Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming or questioning the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.

4.MP.4 Model with mathematics.
Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

4.MP.5 Use appropriate tools strategically.
Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.

4.MP.6 Attend to precision.
Mathematically proficient students clearly communicate to others using appropriate mathematical terminology, and craft explanations that convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.

4.MP.7 Look for and make use of structure.
Mathematically proficient students use structure and patterns to assist in making connections among mathematical ideas or concepts when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.

4.MP.8 Look for and express regularity in repeated reasoning.
Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.