Core Content Connectors

Kindergarten

ARIZONA DEPARTMENT OF EDUCATION

HIGH ACADEMIC STANDARDS FOR STUDENTS

December, 2016

Kindergarten Overview

Counting and Cardinality (CC)

- Know number names and the count sequence.
- Count to tell the number of objects.
- Compare numbers.

Operations and Algebraic Thinking (OA)

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten (NBT)

- Work with numbers 11–19 to gain foundations for place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data (MD)

- Describe and compare measurable attributes.
- Classify objects and count the number of objects in each category.

Geometry (G)

- Identify and describe shapes.
- Analyze, compare, create, and compose shapes.

Standards for Mathematical Practices (MP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Kindergarten: Critical Areas

In Kindergarten, instructional time should focus on two critical areas:

1. Develop competency with counting and cardinality.
2. Develop understanding of addition and subtraction and strategies for addition and subtraction within 10.

More learning time in Kindergarten should be devoted to quantity and number than to other topics.

(1) Students use numbers, including written numerals, to represent quantities such as counting objects in a set; counting out a given number of objects; comparing sets or numerals and recognizing the cardinalities of small sets of objects.

(2) Students use numbers including written numerals to represent and solve quantitative problems. Students choose, combine, and apply effective strategies to solve problems. They will model simple joining and separating situations with sets of objects or eventually with equations. Kindergarten students should see addition and subtraction equations; student writing of equations is encouraged but not required.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Counting and Cardinality (CC)		
K.CC.A **Know number names and the count sequence.**	**K.CC.A.1**	Count to 100 by ones and by tens. **K.NO.1a1 Rote count up to 10.** **K.NO.1a2 Rote count up to 31.** **K.NO.1a3 Rote count up to 100.**
	K.CC.A.2	Count forward from a given number other than one, within the known sequence (e.g., "Starting at the number 5, count up to 11."). **1.NO.1a7 Count forward beginning from any given number below 10.**
	K.CC.A.3	Write numbers from 0 to 20. Represent a number of objects with a written numeral 0 to 20 (with 0 representing a count of no objects). **K.NO.1d1 Identify numerals 1-10** **K.NO.1d2 Identify the numerals 1-10 when presented the name of the number.** **K.NO.1e1 Write or select the numerals 1-10.** **1. NO.1i1 Recognize zero as representing none or no objects**
K.CC.B **Count to tell the number of objects.**	**K.CC.B.4**	Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object (one to one correspondence). b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted (cardinality). c. Understand that each successive number name refers to a quantity that is one larger (hierarchical inclusion). **K.NO.1b2 Identify the set that has more.** **K.NO.1a4 Count up to10 objects in a line, rectangle, or array.** **K.NO.1b1 Match the numeral to the number of objects in a set.** **1.NO.1c1 Use a number line to count up to 31 objects by matching 1 object per number**
	K.CC.B.5	Count to answer questions about "How many?" when 20 or fewer objects are arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1 to 20, count out that many objects. **K.DPS.1a1 Select a question that is answered by collected data.**
K.CC.C **Compare numbers.**	**K.CC.C.6**	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group. (Include groups with up to ten objects.) **No CCC developed for this standard.**

	K.CC.C.7	Compare two numbers between 0 and 10 presented as written numerals. **K.NO.1f1 Identify the smaller or larger number given 2 numbers between 0-10.**
	Operations and Algebraic Thinking (OA)	
K.OA.A **Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.**	**K.0A.A.1**	Represent addition and subtraction concretely. *See Table 1.* **K.PRF.1b1 Use objects or pictures to respond appropriately to "add __" and "take away ___".** **K.PRF.1b2 Communicate answer after adding or taking away.**
	K.0A.A.2	Solve addition and subtraction word problems and add and subtract within 10. *See Table 1.* **K.PRF.1c1 Solve one step addition and subtraction word problems, and add and subtract within 10 using objects, drawings, pictures. Part b** **K.PRF.2a3 Extend a repeating numerical AB pattern** **K.NO.2a1 Count two sets to find sums up to 10.** **K.NO.2a3 Solve word problems within 10.**
	K.0A.A.3	Decompose numbers less than or equal to 10 into pairs in more than one way (e.g., using fingers, objects, symbols, tally marks, drawings, expressions). **K.NO.2a2 Decompose a set of up to ten objects into a group; count the quantity in each group.**
	K.0A.A.4	For any number from 1 to 9, find the number that makes 10 when added to the given number (e.g., using fingers, objects, symbols, tally marks, drawings, or equation). **1.NO.2a4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record or select the answer.**
	K.0A.A.5	Fluently add and subtract within 5. **No CCC developed for this standard.**

Number and Operations in Base Ten (NBT)		
K.NBT.A **Work with numbers 11 to 19 to gain foundations for place value.**	**K.NBT.A.1**	Compose and decompose numbers from 11 to 19 into ten ones and additional ones by using objects, drawings and/or equations. Understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones (e.g., 18 = 10 + 8). **1. NO.1h1 Build representations of numbers up to 19 by creating a group of 10 and some 1s (e.g., 13 = one 10 and three 1s).**
K.NBT.B **Use place value understanding and properties of operations to add and subtract.**	**K.NBT.B.2**	Demonstrate understanding of addition and subtraction within 10 using place value. *See Table 1.* **No CCC developed for this standard.**

Measurement and Data (MD)		
K.MD.A **Describe and compare measurable attributes.**	**K.MD.A.1**	Describe measurable attributes of a single object (e.g., length and weight). **K.ME.1a1 Describe objects in terms of measurable attributes (longer, shorted, heavier, lighter…).**
	K.MD.A.2	Directly compare two objects with a measurable attribute in common to see which object has "more of" or "less of" the attribute, and describe the difference (e.g., directly compare the length of 10 cubes to a pencil and describe one as longer or shorter). **K.ME.1b2 Compare 2 objects with a measurable attribute in common to see which object has more/less of the attribute (length, height, weight).**
K.MD.B **Classify objects and count the number of objects in each category.**	**K.MD.B.3**	Classify objects into given categories; count the number in each category and sort the categories by count. (Note: limit category counts to be less than or equal to 10.) **K.ME.1b1 Sort objects by characteristics (e.g., big/little, colors, shapes, etc.).**

Geometry (G)		
K.G.A **Identify and describe shapes.**	**K.G.A.1**	Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to. **K.GM.1a2 Recognize two dimensional shapes in environment regardless of orientation or size.** **K.GM.1a3 Use spatial language (e.g., above, below, etc.) to describe two-dimensional shapes.**
	K.G.A.2	Correctly name shapes regardless of their orientation or overall size (e.g., circle, triangle, square, rectangle, rhombus, trapezoid, hexagon, cube, cone, cylinder, sphere). **K.GM.1a1 Recognize two- dimensional shapes (e.g., circle, square, triangle and rectangle) regardless of orientation or size**.
	K.G.A.3	Identify shapes as two-dimensional (lying in a plane, flat) or three-dimensional (solid). **K.GM.1b1 Identify shapes as two-dimensional (lying flat) or three dimensional (solid).**
K.G.B **Analyze, compare, create, and compose shapes.**	**K.G.B.4**	Analyze and compare two-dimensional and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/corners), and other attributes (e.g., having sides of equal length). **No CCC developed for this standard.**
	K.G.B.5	Model shapes in the world by building shapes from components (e.g., use sticks and clay balls) and drawing shapes. **No CCC developed for this standard.**
	K.G.B.6	Use simple shapes to form composite shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"* **K.GM.1c1 Compose a larger shape from smaller shapes.** **1.GM.1b1 Identify shapes as two-dimensional (lying flat) or three-dimensional (solid).**

Standards for Mathematical Practice	
K.MP.1	**Make sense of problems and persevere in solving them.** Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, “Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.
K.MP.2	**Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.
K.MP.3	**Construct viable arguments and critique the reasoning of others.** Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming or questioning the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.
K.MP.4	**Model with mathematics.** Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

K.MP.5	**Use appropriate tools strategically.** Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.
K.MP.6	**Attend to precision.** Mathematically proficient students clearly communicate to others using appropriate mathematical terminology, and craft explanations that convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.
K.MP.7	**Look for and make use of structure.** Mathematically proficient students use structure and patterns to assist in making connections among mathematical ideas or concepts when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.
K.MP.8	**Look for and express regularity in repeated reasoning.** Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.

Table 1. Common Addition and Subtraction Problem Types/Situations.[1]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together / Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean ***makes*** or ***results*** in but always does mean ***is the same quantity as***.

[3]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

Core Content Connectors

First Grade

ARIZONA DEPARTMENT OF EDUCATION
HIGH ACADEMIC STANDARDS FOR STUDENTS
December, 2016

First Grade Overview

Operations and Algebraic Thinking (OA)

- Represent and solve problems involving addition and subtraction.
- Understand and apply properties of operations and the relationship between addition and subtraction.
- Add and subtract within 10.
- Work with addition and subtraction equations.

Number and Operations in Base Ten (NBT)

- Extend the counting sequence.
- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data (MD)

- Measure lengths indirectly and by iterating length units.
- Work with time and money.
- Represent and interpret data.

Geometry (G)

- Reason with shapes and their attributes.

Standards for Mathematical Practices (MP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

First Grade: Critical Areas

In first grade, instructional time should focus on three critical areas:

1. Develop understanding of addition, subtraction, and strategies for addition and subtraction within 20.
2. Develop competency of whole number relationships and place value, including grouping in tens and ones.
3. Develop understanding of linear measurement.

More learning time in 1st Grade should be devoted to working with whole numbers than to other topics.

(1) Students develop strategies for adding and subtracting whole numbers. They use a variety of models to represent add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction (*Table 1*). Students understand connections between counting and addition and subtraction. They use properties of addition to add whole numbers to solve addition and subtraction problems through 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

(2) Students work with whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense and place value, they understand the order of the counting sequence, compare whole numbers through 100, and model addition and subtraction situations. Students develop, discuss, and use efficient, accurate, and flexible strategies to add within 100 and subtract multiples of 10.

(3) Students develop an understanding of the meaning and processes of measurement, including iteration (finding the length of an object with repeated equal-sized units) and for indirect measurement (comparing the length of two objects using a third object).

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Operations and Algebraic Thinking (OA)		
1.OA.A **Represent and solve problems involving addition and subtraction.**	**1.OA.A.1**	Use addition and subtraction within 20 to solve word problems with unknowns in all positions (e.g., by using objects, drawings, and/or equations with a symbol for the unknown number to represent the problem). *See Table 1.* **1.NO.2a6 Count two sets to find sums up to 20.** **1.NO.2a8 Decompose a set of up to 20 objects into a group; count the quantity in each group.** **1.NO.2a9 Use manipulatives or representations to write simple addition or subtraction equations within 20 based upon a word problem.** **1.NO.2a10 Use data presented in graphs (i.e., pictorial, object) to solve one step "how many more" or "how many less" word problems.** **1.NO.2a11 Solve word problems within 20.** **1.PRF.1b3 Using objects or pictures respond appropriately to "add __" and "take away ___".** **1.PRF.1c2 Solve one step addition and subtraction word problems where the change or result is unknown (4+_=7) or (4 + 3 = __), within 20 using objects, drawings, pictures.**
	1.OA.A.2	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 (e.g., by using objects, drawings, and/or equations with a symbol for the unknown number to represent the problem). *See Table 1.* **1.PRF.2b2 Create a growing pattern using numbers or objects.**
1.OA.B **Understand and apply properties of operations and the relationship between addition and subtraction.**	**1.OA.B.3**	Apply properties of operations (commutative and associative properties of addition) as strategies to add and subtract within 20. (Students need not use formal terms for these properties.) **1.NO.1i2 Recognize zero as an additive identity.**
	1.OA.B.4	Understand subtraction as an unknown-addend problem within 20 (e.g., subtract 10 - 8 by finding the number that makes 10 when added to 8). **No CCCs developed for this standard.**
1.OA.C **Add and subtract within 10.**	**1.OA.C.5**	Relate counting to addition and subtraction (e.g., by using counting on 2 to add 2). **No CCCs developed for this standard.**
	1.OA.C.6	Fluently add and subtract within 10. **No CCCs developed for this standard.**
1.OA.D **Work with addition and subtraction equations.**	**1.OA.D.7**	Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false (e.g., Which of the following equations are true and which are false? 6 + 1 = 6 - 1, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2). **1.NO.2c1 Identify and apply addition and equal signs.**

	1.OA.D.8	Determine the unknown whole number in an addition or subtraction equation relating three whole numbers (e.g., determine the unknown number that makes the equation true in each of the equations 8 + ☐ = 11, 5 = ☐ - 3, 6 + 6 = ☐). **2.SE.1d2 Represent a "taking away" situation with the – symbol.**
Number and Operations in Base Ten (NBT)		
1.NBT.A **Extend the counting sequence.**	**1.NBT.A.1**	Count to 120 by 1's, 2's, and 10's starting at any number less than 100. In this range, read and write numerals and represent a number of objects with a written numeral. **1.NO.1a6 Rote count up to 100.**
1.NBT.B **Understand place value.**	**1.NBT.B.2**	Understand that the two digits of a two-digit number represent groups of tens and ones. Understand the following as special cases: a. 10 can be thought of as a group of ten ones — called a "ten". b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). **1.NO.1h1 Build representations of numbers up to 19 by creating a group of 10 and some 1s (e.g., 13 = one 10 and three 1s).** **1.NO.1h2 Identify the value of the numbers in the tens and ones place within a given number up to 31.** **2.NO.1h4 Build representations of 3 digit numbers using tens and ones.**
	1.NBT.B.3	Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. **1.NO.1h3 Compare two digit numbers up to 31 using representations and numbers (e.g., identify more tens, less tens, more ones, less ones, larger number, smaller number).**
1.NBT.C **Use place value understanding and properties of operations to add and subtract.**	**1.NBT.C.4**	Demonstrate understanding of addition within 100, connecting objects or drawings to strategies based on place value (including multiples of 10), properties of operations, and/or the relationship between addition and subtraction. Relate the strategy to a written form. *See Table 1.* **No CCCs developed for this standard.**
	1.NBT.C.5	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count. **No CCCs developed for this standard.**
	1.NBT.C.6	Subtract multiples of 10 in the range of 10 to 90 (positive or zero differences), using objects or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy to a written form. **2.NO.2c4 Decompose tens into ones and/or hundreds into tens in subtraction situations.**

Measurement and Data (MD)		
1.MD.A **Measure lengths indirectly and by iterating length units.**	**1.MD.A.1**	Order three objects by length. Compare the lengths of two objects indirectly by using a third object. **1.ME.1b3_ Order up to 3 objects based on a measurable attribute (height, weight, length).** **1.ME.1b4 Order three objects by length; compare the lengths of two objects indirectly by using a third object.**
	1.MD.A.2	Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. (Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.) **1.ME.2a1 Measure using copies of 1 object to measure another.** **1.ME.2b1 Express length of an object as a whole number of lengths unit by laying multiple copies of a shorter object end to end.** **1.ME.1c1 Compare 2 units of measurement and identify which unit would require more or less when measuring a selected object. (I can measure with paper clips or markers, which unit will requ more to measure the table?).**
1.MD.B **Work with time and money.**	**1.MD.B.3a**	Tell and write time in hours and half-hours using analog and digital clocks. **1.ME.2a2 Use time to sequence up to 3 events using a digital or analog clock.** **1.ME.1a5 Tell time to the nearest ½ hour using digital clocks.**
	1.MD.B.3b	Identify coins by name and value (pennies, nickels, dimes and quarters). **No CCCs developed for this standard.**
1.MD.C **Represent and interpret data.**	**1.MD.C.4**	Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. **1.DPS.1a2 Select questions that ask about "How many" and represent up to three categories that can be concretely represented.** **1.DPS.1a3 Identify 2 categories resulting from a selected question.** **1.DPS.1a4 Analyze data by sorting into 2 categories; answer questions about the total number of data points and how many in each category.** **1.DPS.1c1 Using a picture graph, represent each object/person counted on the graph (1:1 correspondence) for 2 or more categories.** **1.DPS.1d1 Interpret a picture graph to answer questions about how many in each category.** **1.DPS.1e1 Compare the values of the 2 categories of data in terms of more or less.**

Geometry (G)		
1.G.A	**1.G.A.1**	Distinguish between defining attributes (triangles are closed and 3 sided) versus non-defining attributes (color, orientation, overall size) for two-dimensional shapes; build and draw shapes that possess defining attributes.

Reason with shapes and their attributes.		**1.GM.1b2 Distinguish two-dimensional shapes based upon their defining attributes (i.e., size, corners, and points).**
	1.G.A.2	Compose two-dimensional shapes or three-dimensional shapes to create a composite shape. **2.GM.1d1 Compose three- dimensional shapes.** **1.GM.1c2 Compose two- and three-- dimensional shapes.**
	1.G.A.3	Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters. Describe the whole as two of, or four of the shares. Understand that decomposing into more equal shares creates smaller shares. **1.GM.1f1 Partition circles and rectangles into 2 and 4 equal parts.**

Standards for Mathematical Practice	
1.MP.1	**Make sense of problems and persevere in solving them.** Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, “Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.
1.MP.2	**Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.
1.MP.3	**Construct viable arguments and critique the reasoning of others.** Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming or questioning the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes

	sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.
1.MP.4	**Model with mathematics.** Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

1.MP.5	**Use appropriate tools strategically.** Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.
1.MP.6	**Attend to precision.** Mathematically proficient students clearly communicate to others using appropriate mathematical terminology, and craft explanations that convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.
1.MP.7	**Look for and make use of structure.** Mathematically proficient students use structure and patterns to assist in making connections among mathematical ideas or concepts when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.
1.MP.8	**Look for and express regularity in repeated reasoning.** Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.

Table 1. Common Addition and Subtraction Problem Types/Situations.[1]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together / Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean ***makes*** or ***results*** in but always does mean ***is the same quantity as***.

[3]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

Core Content Connectors

Second Grade

ARIZONA DEPARTMENT OF EDUCATION
HIGH ACADEMIC STANDARDS FOR STUDENTS
December, 2016

Second Grade Overview

Operations and Algebraic Thinking (OA)

- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.
- Work with equal groups of objects to gain foundations for multiplication.

Number and Operations in Base Ten (NBT)

- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data (MD)

- Measure and estimate lengths in standard units.
- Relate addition and subtraction to length.
- Work with time and money.
- Represent and interpret data.

Geometry (G)

- Reason with shapes and their attributes.

Standards for Mathematical Practices (MP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Second Grade: Critical Areas

In second grade, instructional time should focus on three critical areas:

1. Extend place value understanding of whole number relationships and place value, including grouping in hundreds, tens and ones.
2. Build fluency of addition, subtraction, and strategies for addition and subtraction.
3. Develop understanding of standard units of measure.

More learning time in 2nd grade should be devoted to working with whole numbers and place value than any other topic.

(1) Students extend their understanding of place value using the base-ten system. This includes ideas of counting by ones, fives, tens, and hundreds as well as understanding number relationships involving these units, including comparing. Students understand multi-digit numbers through 1000 written in base-ten notation, recognizing that the digits in each place represent amounts of hundreds, tens, or ones.

(2) Students use their understanding of addition to develop fluency with addition and subtraction within 20. They demonstrate understanding of addition and subtraction within 1000 with the use of models. They develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole number using base-ten notation, understanding of place value, and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences.

(3) Students develop understanding for standard units of measure (centimeter and inch), and they use rulers and other measurement tools with the understanding that linear measurement involves an iteration (repetition) of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

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Operations and Algebraic Thinking (OA)		
2.OA.A **Represent and solve problems involving addition and subtraction.**	**2.OA.A.1**	Use addition and subtraction within 100 to solve one- and two-step word problems. Represent a word problem as an equation with a symbol for the unknown. *See Table 1.* **2.SE.1d1 Represent addition of 2 sets when shown the + symbol** **2.NO.2a16 Solve word problems within 20.** **2.NO.2a17 Solve word problems within 100.** **2.PRF.1c3 Solve one or two step addition and subtraction problems, and add and subtract within 100, using objects, drawings, pictures.** **2.PRF.1c4 Use pictures, drawings or objects represent the steps of a problem.** **2.PRF.1c5 Write or select an equation representing the problem and its solution.**
2.OA.B **Add and subtract within 20.**	**2.OA.B.2**	Fluently add and subtract within 20. By the end of Grade 2, know from memory all sums of two one-digit numbers. **2.PRF.2c2 Identify the rule of arithmetic patterns that are increasing.**
2.OA.C **Work with equal groups of objects to gain foundations for multiplication.**	**2.OA.C.3**	Determine whether a group of objects (up to 20) has an odd or even number of members (e.g., by pairing objects or counting them by 2's). **2.NO.1e7 Identify numbers as odd or even.**
	2.OA.C.4	Use addition to find the total number of objects arranged in rectangular arrays (with up to 5 rows and 5 columns). Write an equation to express the total as a sum of equal addends. **3.NO.2d1 Find the total number of objects when given the number of identical groups and the number of objects in each group neither number larger than 5.** **3.NO.2d2 Find total number inside an array with neither number in the columns or rows larger than 5.**

Number and Operations in Base Ten (NBT)		
2.NBT.A **Understand place value.**	**2.NBT.A.1**	Understand that the three digits of a three-digit number represent groups of hundreds, tens, and ones (e.g., 706 equals 7 hundreds, 0 tens, and 6 ones and also equals 70 tens and 6 ones). Understand the following as special cases: a. 100 can be thought of as a group of ten tens—called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). **2.NO.1h4 Build representations of 3 digit numbers using tens and ones.** **2.NO.1h5 Build representations of 3 digit numbers using hundreds, tens, and ones.**
	2.NBT.A.2	Count within 1000; skip count by 5's, 10's and 100's. **2.NO.1e4 Skip count by 5s** **2.NO.1e5 Skip count by 10s.**

		2.NO.1e6 Skip count by 100s. **2.NO.3c1 Solve real world problems by using mental math (such as skip counting by 2s within 20, 5s within 50, and 10s within 100).**
	2.NBT.A.3	Read and write numbers up to 1000 using base-ten numerals, number names, and expanded form. **2.NO.1d5 Identify numerals 0-100.** **2.NO.1d6 Identify the numeral between 0 and 100 when presented the name.** **2.NO.1e3 Write or select the numerals 0-100.** **2.NO.1h8 Write or select expanded form for any 2 digit number.** **2.NO.1h9 Write or select expanded form for any 3 digit number.** **2.NO.1i3 Explain what the zero represents in place value (hundreds, tens, ones) in a number.** **3.NO.1j2 Write or select the expanded form for up to 3 digit number.**
	2.NBT.A.4	Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons. **2.NO.1f6 Compare (greater than, less than, equal to) 2 numbers up to 100.** **2.NO.1h6 Compare 2 digit numbers using representations and numbers (e.g., identify more tens, less tens, more ones, less ones, larger number, smaller number).** **2.NO.1h7 Compare 3 digit numbers using representations and numbers (e.g., identify more hundreds, less hundreds, more tens, less tens, more ones, less ones, larger number, smaller number).** **3.NO.1h1 Compare 3 digit numbers using representations and numbers (e.g., identify more hundreds, less hundreds, more tens, less tens, more ones, less ones, larger number, smaller number).**
2.NBT.B **Use place value understanding and properties of operations to add and subtract.**	**2.NBT.B.5**	Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. **2.NO.2a12 Model addition and subtraction with base 10 blocks within 20.** **2.NO.2a13 Model addition and subtraction with base 10 blocks within 50.** **2.NO.2a14 Model addition and subtraction with base 10 blocks within 100.**
	2.NBT.B.6	Add up to three two-digit numbers using strategies based on place value and properties of operations. **2.NO.2a19 Combine up to 3 sets of 20 or less.**
	2.NBT.B.7	Demonstrate understanding of addition and subtraction within 1000, connecting objects or drawings to strategies based on place value (including multiples of 10), properties of operations, and/or the relationship between addition and subtraction. Relate the strategy to a written form. *See Table 1.* **2.NO.2c3 Decompose tens into ones and/or hundreds into tens in subtraction situations.** **2.NO.2c4 Decompose tens into ones and/or hundreds into tens in subtraction situations.** **2.NO.2a18 Use diagrams and number lines to solve addition or subtraction problems.**
	2.NBT.B.8	Mentally add 10 or 100 to a given number in the range of 100 and 900, and mentally subtract 10 or 100 from a given number in the range of 100 and 900.

		2.NO.1e8 Mentally add or subtract 10 from a given set from the 10s family (e.g., what is 10 more than 50? What is 10 less than 70?). **2.NO.1e9 Mentally add or subtract 100 from a given set from the 100s family (e.g., what is 100 more than 500? What is 100 less than 700?).** **3.NO.1e2 Mentally add or subtract 100 from a given set from the 100s family (e.g., what is 100 more than 500? What is 100 less than 700?).**
	2.NBT.B.9	Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.) **No CCCs developed for this standard.**
Measurement and Data (MD)		
2.MD.A **Measure and estimate lengths in standard units.**	**2.MD.A.1**	Measure the length of an object by selecting and using appropriate tools (e.g., ruler, meter stick, yardstick, measuring tape). ***2.ME.1a3*** **Select appropriate tool and unit of measurement to measure an object(ruler or yard stick; inches or feet).** ***2.ME.2b2 Select appropriate tools and demonstrate or identify appropriate measuring techniques.***
	2.MD.A.2	Measure the length of an object twice, using different standard length units for the two measurements; describe how the two measurements relate to the size of the unit chosen. Understand that depending on the size of the unit, the number of units for the same length varies. **2.ME.1c2 Measure the attributes (length, width, height) of an object using 2 different size units.**
	2.MD.A.3	Estimate lengths using units of inches, feet, centimeters, and meters. **2.ME.1c3 Recognize that standard measurement units can be decomposed into smaller units.** **2.ME.2a3 Estimate the length of an object using units of feet and inches**
	2.MD.A.4	Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. **2.ME.1b5 Solve word problems involving the difference in standard length units.** **2.ME.2a4 Solve one step subtraction problems involving the difference of the lengths of 2 objects in standard length units.**
2.MD.B **Relate addition and subtraction to length.**	**2.MD.B.5**	Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same unit. *See Table 1.* **No CCCs developed for this standard.**
	2.MD.B.6	Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences within 100 on a number line diagram. **No CCCs developed for this standard.**

2.MD.C **Work with time and money.**	**2.MD.C.7**	Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. **3.ME.1a1 Tell time to the nearest 5 minutes using a digital clock.**
	2.MD.C.8	Solve word problems involving collections of money, including dollar bills, quarters, dimes, nickels, and pennies. Record the total using $ and ¢ appropriately. *See Table 1.* **2.ME.1a4 Solve word problems using dollar bills, quarters, dimes, nickels or pennies.**
2.MD.D **Represent and interpret data.**	**2.MD.D.9**	Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. **2.DPS.1c3 Organize data by representing continuous data on a line plot.**
	2.MD.D.10	Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in the graph. *See Table 1.* **2.DPS.1d2 Identify the value of each category represented on picture graph and bar graph or each point on a line plot.** **2.DPS.1a7 Analyze data by sorting into categories established by each question** **2.DPS.1c2 Organize data by representing categorical data on a pictorial graph or bar graph.** **2.DPS.1e2 Compare the information shown in a bar graph or picture graph with up to 4 categories. Solve simple comparisons of how many more or how many less.**
Geometry (G)		
2.G.A **Reason with shapes and their attributes.**	**2.G.A.1**	Identify and describe specified attributes of two-dimensional and three-dimensional shapes, according to the number and shape of faces, number of angles, and the number of sides and/or vertices. Draw two-dimensional shapes based on the specified attributes (e.g., triangles, quadrilaterals, pentagons, and hexagons). **2.GM.1a4 Identify two- dimensional shapes such as rhombus, pentagons, hexagons, octagon, ovals, equilateral, isosceles, and scalene triangles.** **2.GM.1b3 Distinguish two- or three- dimensional shapes based upon their attributes (i.e., #of sides, equal or different lengths of sides, # of faces, # of corners).** **2.GM.1e1 Draw two- dimensional shapes with specific attributes.**
	2.G.A.2	Partition a rectangle into rows and columns of same-size rectangles and count to find the total number of rectangles. **No CCCs developed for this standard.**
	2.G.A.3	Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, fourths, half of, third of, fourth of, and describe the whole as two halves, three thirds, or four fourths. Recognize that equal shares of identical wholes need not have the same shape. **2.GM.1f2 Partition circles and rectangles into 2 and 4 equal parts.**

		2.GM.1f3 Label a partitioned shape (e.g., one whole rectangle was separated into 2 halves, one whole circle was separated into three thirds).

Standards for Mathematical Practice	
2.MP.1	**Make sense of problems and persevere in solving them.** Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, "Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.
2.MP.2	**Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.
2.MP.3	**Construct viable arguments and critique the reasoning of others.** Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming or questioning the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.
2.MP.4	**Model with mathematics.** Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

2.MP.5	**Use appropriate tools strategically.** Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.
2.MP.6	**Attend to precision.** Mathematically proficient students clearly communicate to others using appropriate mathematical terminology, and craft explanations that convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.
2.MP.7	**Look for and make use of structure.** Mathematically proficient students use structure and patterns to assist in making connections among mathematical ideas or concepts when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.
2.MP.8	**Look for and express regularity in repeated reasoning.** Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.

Table 1. Common Addition and Subtraction Problem Types/Situations.[1]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together / Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean ***makes*** or ***results*** in but always does mean ***is the same quantity as***.

[3]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

Core Content Connectors

Third Grade

ARIZONA DEPARTMENT OF EDUCATION
HIGH ACADEMIC STANDARDS FOR STUDENTS
December, 2016

Third Grade Overview

Operations and Algebraic Thinking (OA)
Note: Grade 3 expectations in this domain are limited to multiplication through 10 x 10 and division with both quotients and divisors less than or equal to 10.

- Represent and solve problems involving whole number multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Number and Operations in Base Ten (NBT)
Note: A range of algorithms may be used.

- Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions (NF)
Note: Grade 3 expectations are limited to fractions with denominators: 2, 3, 4, 6, 8

- Understand fractions as numbers.

Measurement and Data (MD)

- Solve problems involving measurement.
- Represent and interpret data.
- Geometric measurement: Understand concepts of area and perimeter.

Geometry (G)

- Reason with shapes and their attributes.

Standards for Mathematical Practices (MP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Third Grade: Critical Areas

In third grade, instructional time should focus on three critical areas:

1. Develop competency in multiplication and division and strategies for multiplication and division within 100.
2. Develop understanding of fractions as numbers, especially unit fractions.
3. Develop understanding of the structure of rectangular arrays and of area.

(1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models as described in *Table 2.* Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division. By the end of 3^{rd} grade, students are fluent in multiplication and division within 100.

(2) Students develop an understanding of fractions as numbers, beginning with unit fractions. Students understand that the size of a fractional part is relative to the size of the whole. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on recognizing equal numerators or denominators.

(3) Students develop an understanding of area as an attribute of two-dimensional regions by making connections among standards in the domains OA, NBT and G. They measure the area of a shape by finding the total number of same-size units required to cover the shape without gaps or overlaps. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By working with arrays, students connect area to multiplication and justify using multiplication to determine the area.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Operations and Algebraic Thinking (OA) *Note: Grade 3 expectations in this domain are limited to whole number multiplication through 10 x 10 and whole number division with both quotients and divisors less than or equal to 10.*		
3.OA.A Represent and solve problems involving whole number multiplication and division.	**3.OA.A.1**	Interpret products of whole numbers as the total number of objects in equal groups (e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each). **3.NO.2d1 Find the total number of objects when given the number of identical groups and the number of objects in each group neither number larger than 5.** **3.NO.2d2 Find total number inside an array with neither number in the columns or rows larger than 5.** **3. NO.2d3 Solve multiplication problems with neither number greater than 5.** **3.PRF.1d1 Use objects to model multiplication and division situations involving up to 5 groups with up to 5 objects in each group and interpret the results.** **4.NO.2d6 Find total number inside an array with neither number in the columns or rows larger than 10.** **4.NO.2d8 Match an accurate addition and multiplication equation to a representation.** **4. PRF.1d2 Use objects to model multiplication and division situations involving up to 10 groups with up to 5 objects in each group and interpret the results.**
	3.OA.A.2	Interpret whole number quotients of whole numbers (e.g., interpret 56 ÷ 8 as the number of objects in each group when 56 objects are partitioned equally into 8 groups, or as a number of groups when 56 objects are partitioned into equal groups of 8 objects each). *See Table 2.* **3.NO.2d4 Determine how many objects go into each group when given the total number of objects and the number of groups where the number in each group or number of groups is not greater than 5.** **3.NO.2d5 Determine the number of groups given the number of total number of objects and the number of objects in each group where the number in each group and the number of groups is not greater than 5.** **3.PRF.1d1 Use objects to model multiplication and division situations involving up to 5 groups with up to 5 objects in each group and interpret the results.**
	3.OA.A.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities. *See Table 2*. **No CCCs developed for this standard**.
	3.OA.A.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers *For example, determine the unknown number that makes the equation true in each of the equations 8 x ☐ = 48, 5 = ☐ ÷ 3, 6 x 6 = ☐. See Table 2*.

		No CCCs developed for this standard.
3.OA.B **Understand properties of multiplication and the relationship between multiplication and division.**	**3.OA.B.5**	Apply properties of operations as strategies to multiply and divide. Properties include commutative and associative properties of multiplication and the distributive property. (Students do not need to use the formal terms for these properties.) **3.PRF.2d2 Apply properties of operations as strategies to multiply and divide.**
	3.OA.B.6	Understand division as an unknown-factor problem (e.g., find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8). **No CCCs developed for this standard.**
3.OA.C **Multiply and divide within 100.**	**3.OA.C.7**	Fluently multiply and divide within 100. By the end of Grade 3, know from memory all multiplication products through 10 x 10 and division quotients when both the quotient and divisor are less than or equal to 10. **No CCCs developed for this standard.**
3.OA.D **Solve problems involving the four operations, and identify and explain patterns in arithmetic.**	**3.OA.D.8**	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Utilize understanding of the Order of Operations when there are no parentheses. **3.NO.2e1 solve or solve and check one or two step word problems requiring addition, subtraction or multiplication with answers up to 100.**
	3.OA.D.9	Identify patterns in the addition table and the multiplication table and explain them using properties of operations (e.g. observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends). **3.PRF.1e1 Describe the rule for a numerical pattern (e.g., increase by 2, 5, or 10).** **3.PRF.1e2 Select or name the three next terms in a numerical pattern where numbers increase by 2, 5 or 10.** **3. PRF.2d1 Identify multiplication patterns in a real word setting.**
	3.OA.D.10	When solving problems, assess the reasonableness of answers using mental computation and estimation strategies including rounding. **No CCCs developed for this standard.**

Number and Operations in Base Ten (NBT) *Note: A range of algorithms may be used.*		
3.NBT.A **Use place value understanding and properties of operations to perform multi-digit arithmetic.**	**3.NBT.A.1**	Use place value understanding to round whole numbers to the nearest 10 or 100. **3.NO.1j3 Use place value to round to the nearest 10 or 100.** **3.NO.1j4 Use rounding to solve word problems.**
	3.NBT.A.2	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. **3.NO.2c1 Solve multi-step addition and subtraction problems up to 100.** **3. NO.2b1 Use the relationships between addition and subtraction to solve problems.**
	3.NBT.A.3	Multiply one-digit whole numbers by multiples of 10 in the range 10 to 90 using strategies based on place value and the properties of operations (e.g., 9 x 80, 5 x 60). **No CCCs developed for this standard.**
Number and Operations – Fractions (NF) *Note: Grade 3 expectations are limited to fractions with denominators: 2,3,4,6,8.*		
3.NF.A **Understand fractions as numbers.**	**3.NF.A.1**	Understand a fraction (1/*b*) as the quantity formed by one part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by *a* parts of size 1/*b*. **3. NO.1l1 Identify the number of highlighted parts (numerator) of a given representation (rectangles and circles).** **3. NO.1l2 Identify the total number of parts (denominator) of a given representation (rectangles and circles).** **3. NO.1l3 Identify the fraction that matches the representation (rectangles and circles; halves, fourths, and thirds, eighths).**
	3.NF.A.2	Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Understand that each part has size 1/*b* and that the end point of the ased at 0 locates the number 1/*b* on the number line. b. Represent a fraction *a/b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Understand that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line including values greater than 1. **3. NO.1l4 Identify that a part of a rectangle can be represented as a fraction that has a value between 0 and 1.** **3.NO.1l5 Locate given common unit fractions (i.e., ½, ¼, 1/8,) on a number line or ruler.** **4. NO.1l6 Locate fractions on a number line.** **4. NO.1l7 Order fractions on a number line.**

		c. Understand a fraction 1/b as a special type of fraction that can be referred to as a unit fraction (e.g. 1/2, 1/4). **3. NO.1l1 Identify the number of highlighted parts (numerator) of a given representation (rectangles and circles).** **3. NO.1l2 Identify the total number of parts (denominator) of a given representation (rectangles and circles).** **3. NO.1l3 Identify the fraction that matches the representation (rectangles and circles; halves, fourths, and thirds, eighths).**
	3.NF.A.3	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent if they have the same relative size compared to 1 whole. b. Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Understand that comparisons are valid only when the two fractions refer to the same whole. Record results of comparisons with the symbols >, =, or <, and justify conclusions. ***3. SE.1g1 Use =, <, or > to compare 2 fractions with the same numerator or denominator.*** ***4.SE.1h1 Express whole numbers as fractions.*** **4. NO.1m1, part c Determine equivalent fractions.** ***4.NO.2h3 Solve word problems involving addition and subtraction of fractions with like denominators (2, 3, 4, or 8).***

Measurement and Data (MD)		
3.MD.A **Solve problems involving measurement.**	**3.MD.A.1a**	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes (e.g., representing the problem on a number line diagram). **3.ME.1a2 Solve word problems involving the addition and subtraction of time intervals of whole hours or within an hour (whole hours: 5:00 to 8:00, within hours: 7:15 to 7:45).** **3.PRF.1f1 Determine the equivalence between number of minutes and the fraction of the hour (e.g., 30 minutes = hour).** **3.PRF.1f 2 Determine the equivalence between the number of minutes and the number of hours (e.g., 60 minutes hour).**
	3.MD.A.1b	Solve word problems involving money through $20.00, using symbols $, ".", ₵. **No CCCs developed for this standard.**
	3.MD.A.2	Measure and estimate liquid volumes and masses of objects using metric units. (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units. Excludes multiplicative comparison problems (problems involving notions of "times as much"). *See Table 2.* **3.ME.1f1 Select appropriate units for measurement (liquid volume, area, time, money).** **3.ME.1f2 Add to solve one step word problems.** **3.ME.2e1 Select appropriate tool for measurement: liquid volume, area, time, money.** **3.ME.2i1 Estimate liquid volume.**
3.MD.B **Represent and interpret data.**	**3.MD.B.3**	Create a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *See Table 1.* ***3.DPS.1g1*** **Collect data, organize into picture or bar graph.** ***3.DPS.1i1 Select the appropriate statement that describes the data representations based on a given graph (picture, bar, line plots).***
	3.MD.B.4	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch to the nearest quarter-inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters. **3.ME.2e2 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch.** **3.ME.2e3 Measure to solve problems using number lines and ruler to 1 inch, ½ inch, or ¼ of an inch.** **3.DPS.1g2 Organize measurement data into a line plot.**
3.MD.C	**3.MD.C.5**	Understand area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

Geometric measurement: Understand concepts of area and perimeter.		b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. **No CCCs developed for this standard.**
	3.MD.C.6	Measure areas by counting unit squares (e.g., square cm, square m, square in, square ft, and improvised units). **3. ME.1d2 Measure area of rectangles by counting squares.**
	3.MD.C.7	Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. d. Understand that rectilinear figures can be decomposed into non-overlapping rectangles and that the sum of the areas of these rectangles is identical to the area of the original rectilinear figure. Apply this technique to solve problems in real-world contexts. **3.ME.1d1 Use tiling and addition to determine area.** **4.ME.1d3 Use tiling and multiplication to determine area.** **4.ME.2h1 Apply the formulas for area and perimeter to solve real world problems.** **4.PRF.1f3 Apply the distributive property to solve problems with models.**

3.MD.C (cont.)	**3.MD.C.8**	Solve real-world and mathematical problems involving perimeters of plane figures and areas of rectangles, including finding the perimeter given the side lengths, finding an unknown side length. Represent rectangles with the same perimeter and different areas or with the same area and different perimeters. **3.ME.1g1 Identify a figure as getting larger or smaller when the dimensions of the figure changes** **3.ME.2h1 Use addition to find the perimeter of a rectangle** **4.ME.2h1 Apply the formulas for area and perimeter to solve real world problems.**
Geometry (G)		
3.G.A **Reason with shapes and their attributes.**	**3.G.A.1**	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others)may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples quadrilaterals that do not belong to any of these subcategories. **3.GM.1h1 Identify shared attributes of shapes.**
	3.G.A.2	Partition shapes into b parts with equal areas. Express the area of each part as a unit fraction $1/b$ of the whole. (Grade 3 expectations are limited to fractions with denominators $b = 2,3,4,6,8$.) **3.GM.1i1 Partition rectangles into equal parts with equal area.**

Standards for Mathematical Practice	
3.MP.1	**Make sense of problems and persevere in solving them.** Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, “Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.
3.MP.2	**Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.
3.MP.3	**Construct viable arguments and critique the reasoning of others.** Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming or questioning the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.
3.MP.4	**Model with mathematics.** Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

3.MP.5	**Use appropriate tools strategically.** Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.
3.MP.6	**Attend to precision.** Mathematically proficient students clearly communicate to others using appropriate mathematical terminology, and craft explanations that convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.
3.MP.7	**Look for and make use of structure.** Mathematically proficient students use structure and patterns to assist in making connections among mathematical ideas or concepts when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.
3.MP.8	**Look for and express regularity in repeated reasoning.** Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.

Table 1. Common Addition and Subtraction Problem Types/Situations.[1]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together / Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean ***makes*** or ***results*** in but always does mean ***is the same quantity as***.

[3]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

Arizona Mathematics Standards 3rd Grade

Table 2. Common Multiplication and Division Problem Types/Situations.[1]

	Unknown Product	**Group Size Unknown** (“How many in each group?” Division)	**Number of Groups Unknown** (“How many groups?” Division)
	3 x 6 = ?	**3 x ? = 18 and 18 ÷ 3 = ?**	**? x 6 = 18 and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? ***Measurement example*.** You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? ***Measurement example*.** You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? ***Measurement example*.** You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[2] Area[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? ***Area example*.** What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? ***Area example*.** A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? ***Area example*.** A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A straw hat costs $6. A baseball hat costs 3 times as much as the straw hat. How much does the baseball hat cost? ***Measurement example*.** A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A baseball hat costs $18 and that is 3 times as much as a straw hat costs. How much does a blue straw cost? ***Measurement example*.** A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A baseball hat costs $18 and a straw hat costs $6. How many times as much does the baseball hat cost as the straw hat? ***Measurement example*.** A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[1]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

[2]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Core Content Connectors

Fourth Grade

ARIZONA DEPARTMENT OF EDUCATION
HIGH ACADEMIC STANDARDS FOR STUDENTS
December, 2016

Fourth Grade Overview

Operations and Algebraic Thinking (OA)

- Use the four operations with whole numbers to solve problems.
- Gain familiarity with factors and multiples.
- Generate and analyze patterns.

Number and Operations in Base Ten (NBT)

Note: Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

- Generalize place value understanding for multi-digit whole numbers.
- Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions (NF)

Note: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

- Extend understanding of fraction equivalence and ordering.
- Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers.
- Understand decimal notation for fractions, and compare decimal fractions.

Measurement and Data (MD)

- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
- Represent and interpret data.
- Geometric measurement: Understand concepts of angle and measure angles.

Geometry (G)

- Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Standards for Mathematical Practices (MP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Fourth Grade: Critical Areas

In fourth grade, instructional time should focus on three critical areas:

1. Extend understanding of place value to multi-digit numbers and fluently add and subtract multi-digit numbers.
2. Develop competency with multi-digit multiplication, and develop understanding of dividing to find quotients involving multi- digit dividends.
3. Develop understanding of fraction equivalence, addition, and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.

(1) Students generalize their understanding of place value through 1,000,000, and the relative size of numbers in each place. They use their understanding of properties of operations to perform multi-digit arithmetic with multi-digit whole number less than or equal to 1,000,000. They round multi-digit numbers and fluently add and subtract multi-digit whole numbers within 1,000,000.

(2) Students apply their understanding of models for multiplication, place value, and properties of operations, in particular the distributive property, to compute products of multi-digit whole numbers. They develop fluency with efficient strategies for multiplying multi- digit whole numbers through 1,000,000; understand and explain why the strategies work; and use them to solve problems (*Table 2*). Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication to find quotients involving multi-digit dividends.

(3) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., $^{15}/_{9} = ^{5}/_{3}$), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Operations and Algebraic Thinking (OA)		
4.OA.A **Use the four operations with whole numbers to solve problems.**	**4.OA.A.1**	Represent verbal statements of multiplicative comparisons as multiplication equations. Interpret a multiplication equation as a comparison (e.g., 35 is the number of objects in 5 groups, each containing 7 objects, and is also the number of objects in 7 groups, each containing 5 objects). **No CCCs developed for this standard.**
	4.OA.A.2	Multiply or divide within 1000 to solve word problems involving multiplicative comparison (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison). *See Table 2.* **4. NO.2d7 Determine how many objects go into each group when given the total number of objects and the number of groups where the number in each group or number of groups is not greater than 10.** **4.PRF.1e3 Solve multiplicative comparisons with an unknown using up to 2-digit numbers with information presented in a graph or word problem (e.g., an orange hat cost $3. A purple hat cost 2 times as much. How much does the purple hat cost? [3 x 2 = p]).**
	4.OA.A.3	Solve multistep word problems using the four operations, including problems in which remainders must be interpreted. Understand how the remainder is a fraction of the divisor. Represent these problems using equations with a letter standing for the unknown quantity. **4.NO.2e2 Solve or solve and check one or two step word problems requiring addition, subtraction, or multiplication with answers up to 100.**
4.OA.B **Gain familiarity with factors and multiples.**	**4.OA.B.4**	Find all factor pairs for a whole number in the range 1 to 100 and understand that a whole number is a multiple of each of its factors. **4. NO.2f1 Identify multiples for a whole number (e.g., 2= 2, 4, 6, 8, 10).**
4.OA.C **Generate and analyze patterns.**	**4.OA.C.5**	Generate a number pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself and explain the pattern informally (e.g., given the rule “add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers). **4.PRF.2d3 Generate a pattern when given a rule and word problem (I run 3 miles every day, how many miles have I run in 3 days).** **4.PRF.2e1 Extend a numerical pattern when the rule is provided.** **5.PRF.2a1 Generate a pattern that follows the provided rule.**
	4.OA.C.6	When solving problems, assess the reasonableness of answers using mental computation and estimation strategies including rounding. **No CCCs developed for this standard.**
Number and Operations in Base Ten (NBT)		

Note: Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.		
4.NBT.A **Generalize place value understanding for multi-digit whole numbers.**	**4.NBT.A.1**	Apply concepts of place value, multiplication, and division to understand that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. **4.NO.1k1 Compare the value of a number when it is represented in different place values of two 3 digit numbers.**
	4.NBT.A.2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. ***4.NO.1j6 Compare multi-digit numbers using representations and numbers.*** **4.NO.1j7 Write or select the expanded form for a multi-digit number.**
	4.NBT.A.3	Use place value understanding to round multi-digit whole numbers to any place. **4.NO.1j5 Use place value to round to any place (i.e., ones, tens, hundreds, thousands).**

4.NBT.B **Use place value understanding and properties of operations to perform multi-digit arithmetic.**	**4.NBT.B.4**	Fluently add and subtract multi-digit whole numbers using a standard algorithm. **4.NO.2f2 Solve multiplication problems up to two digits by one digit.**
	4.NBT.B.5	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **4.PRF.1f4 Solve a 2-digit by 1-digit multiplication problem using two different strategies.**
	4.NBT.B.6	Demonstrate understanding of division by finding whole-number quotients and remainders with up to four-digit dividends and one-digit divisors. **5.NO.2a2 Separate a group of objects into equal sets when given the number of sets to find the total in each set with the total number less than 50.**

Number and Operations – Fractions (NF)

Note: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

4.NF.A **Extend understanding of fraction equivalence and ordering.**	**4.NF.A.1**	Explain why a fraction *a/b* is equivalent to a fraction (*n x a*)/(*n x b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to understand and generate equivalent fractions. **4.NO.1m1 Determine equivalent fractions.**
	4.NF.A.2	Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators and by comparing to a benchmark fraction). a. Understand that comparisons are valid only when the two fractions refer to the same size whole. b. Record the results of comparisons with symbols >, =, or <, and justify the conclusions. **4. SE.1g2 Use =, <, or > to compare 2 fractions (fractions with a denominator or 10 or less).** **4. NO.1n2 Compare up to 2 given fractions that have different denominators.**
4.NF.B **Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers.**	**4.NF.B.3**	Understand a fraction *a/b* with *a* > 1 as a sum of unit fractions (1/*b*). a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way (e.g., 3/8 = 1/8 + 1/8+1/8; 3/8 = 2/8 + 1/8; 2 1/8 = 1 + 1 + 1/8 + or 2 1/8 = 8/8 + 8/8 + 1/8). c. Add and subtract mixed numbers with like denominators (e.g., by using properties of operations and the relationship between addition and subtraction and/or by replacing each mixed number with an equivalent fraction). d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators. **4. NO.2g1 Using a representation, decompose a fraction into multiple copies of a unit fraction (e.g., ¾ = ¼ + ¼ + ¼).**

		4. NO.2h1 Add and subtract fractions with like denominators of (2,3,4, or 8). ***4. NO.2h2 Add and subtract fractions with like denominators (2,3,4, or 8) using representations.*** ***4.NO.2h3 Solve word problems involving addition and subtraction of fractions with like denominators (2, 3, 4, or 8).***
	4.NF.B.4	Build fractions from unit fractions. a. Understand a fraction $\frac{a}{b}$ as a multiple of a unit fraction $\frac{1}{b}$. In general, $\frac{a}{b}$ = a x $\frac{1}{b}$. b. Understand a multiple of $\frac{a}{b}$ as a multiple of a unit fraction $\frac{1}{b}$, and use this understanding to multiply a whole number by a fraction. In general, n x $\frac{a}{b}$ = $\frac{n\ x\ a}{b}$. c. Solve word problems involving multiplication of a whole number by a fraction. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* **No CCCs developed for these standards.**
4.NF.C **Understand decimal notation for fractions, and compare decimal fractions.**	**4.NF.C.5**	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 (tenths) and 100 (hundredths). *For example, express 3/10 as 30/100, and 3/10 + 4/100 = 34/100.* (Note: Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators, in general, is not a requirement at this grade.) **4.NO.1o2 Find the equivalent decimal for a given fraction.**
	4.NF.C.6	Use decimal notation for fractions with denominators 10 (tenths) or 100 (hundredths), and locate these decimals on a number line. **4.SE.1h2 Identify the equivalent decimal for a fraction** **4.NO.1o1 Match a fraction with a denominator of 10 or 100 as a decimal (5/10 = .5).** **4.NO.1p1 Read, write or select decimals to the tenths place.** **4.NO.1p2 Read, write or select decimals to the hundredths place.** **5. NO.1c1 Rewrite a fraction as a decimal.** **5. NO.1c2 Rewrite a decimal as a fraction.**
	4.NF.C.7	Compare two decimals to hundredths by reasoning about their size. Understand that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <. **4.SE.1g3 Use =, <, or > to compare 2 decimals (decimals in multiples of .10).** **4.NO.1q1 Compare two decimals to the tenths place with a value of less than 1.** **4.NO.1q2 Compare two decimals to the hundredths place with a value of less than 1.**
Measurement and Data (MD)		

4.MD.A **Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**	**4.MD.A.1**	Know relative sizes of measurement units within one system of units which could include km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit and in a smaller unit in terms of a larger unit. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1,12), 2,24), (3,36).* **4.ME.1a1 Identify the appropriate units of measurement for different purposes in a real life context (e.g., measure a wall using feet, not inches).** **4.ME.2f1 Complete a conversion table for length and mass within a single system.** **4.ME.1f3 Select appropriate units for measurement: mass, length, angles.**
	4.MD.A.2	Use the four operations to solve word problems and problems in real-world context involving distances, intervals of time (hr, min, sec), liquid volumes, masses of objects, and money, including decimals and problems involving fractions with like denominators, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using a variety of representations, including number lines that feature a measurement scale. **4. ME.1g2 Solve word problems using perimeter and area where changes occur to the dimensions of a figure.**
	4.MD.A.3	Apply the area and perimeter formulas for rectangles in mathematical problems and problems in real-world contexts including problems with unknown side lengths. Se*e Table 2.* **4.ME.1g2 Solve word problems using perimeter and area where changes occur to the dimensions of a figure.** **4.ME.2h1 Apply the formulas for area and perimeter to solve real world problems.**
4.MD.B **Represent and interpret data.**	**4.MD.B.4**	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. **No CCCs developed for this standard.**
4.MD.C **Geometric measurement: Understand concepts of angle and measure angles.**	**4.MD.C.5**	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. **No CCCs developed for this standard.**
4.MD.C (cont.)	**4.MD.C.6**	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. **4.ME.2e4 Select appropriate tool for measurement: mass, length, angles.** **4.ME.2e5 Construct a given angle.**

		4.ME.2e6 Measure right angles using a tool (e.g., angle ruler, protractor).
	4.MD.C.7	Understand angle measures as additive. (When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.) Solve addition and subtraction problems to find unknown angles on a diagram within mathematical problems as well as problems in real-world contexts. **No CCCs developed for this standard.**
Geometry (G)		
4.G.A **Draw and identify lines and angles, and classify shapes by properties of their lines and angles.**	**4.G.A.1**	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. **4. GM.1j1 Recognize a point, line and line segment, rays in two-dimensional figures.** **4.GM.1j2 Recognize perpendicular and parallel lines in two-dimensional figure.** **4.GM.1j3 Recognize an angle in two-dimensional figures.** **5.GM.1j1 Recognize parallel and perpendicular lines within the context of two-dimensional figures.**
	4.G.A.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size (e.g., understand right triangles as a category, and identify right triangles). **4. GM.1h2 Classify two-dimensional shapes based on attributes (number of angles).** **4. GM.1j4 Categorize angles as right, acute, or obtuse.**
	4.G.A.3	Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry. **4. GM.1k1 Recognize a line of symmetry in a figure.**

Standards for Mathematical Practice	
4.MP.1	**Make sense of problems and persevere in solving them.** Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, "Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.
4.MP.2	**Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.
4.MP.3	**Construct viable arguments and critique the reasoning of others.** Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming or questioning the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.
4.MP.4	**Model with mathematics.** Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

4.MP.5	**Use appropriate tools strategically.** Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.
4.MP.6	**Attend to precision.** Mathematically proficient students clearly communicate to others using appropriate mathematical terminology, and craft explanations that convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.
4.MP.7	**Look for and make use of structure.** Mathematically proficient students use structure and patterns to assist in making connections among mathematical ideas or concepts when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.
4.MP.8	**Look for and express regularity in repeated reasoning.** Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.

Table 2. Common Multiplication and Division Problem Types/Situations.[1]

	Unknown Product 3 x 6 = ?	Group Size Unknown ("How many in each group?" Division) 3 x ? = 18 and 18 ÷ 3 = ?	Number of Groups Unknown ("How many groups?" Division) ? x 6 = 18 and 18 ÷ 6 = ?
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? ***Measurement example*.** You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? ***Measurement example*.** You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? ***Measurement example*.** You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[2] Area[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? ***Area example*.** What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? ***Area example*.** A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? ***Area example*.** A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A straw hat costs \$6. A baseball hat costs 3 times as much as the straw hat. How much does the baseball hat cost? ***Measurement example*.** A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A baseball hat costs \$18 and that is 3 times as much as a straw hat costs. How much does a blue straw cost? ***Measurement example*.** A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A baseball hat costs \$18 and a straw hat costs \$6. How many times as much does the baseball hat cost as the straw hat? ***Measurement example*.** A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[1]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.
[2]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[3]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Core Content Connectors

Fifth Grade

ARIZONA DEPARTMENT OF EDUCATION
HIGH ACADEMIC STANDARDS FOR STUDENTS
December, 2016

Fifth Grade Overview

Operations and Algebraic Thinking (OA)

- Write and interpret numerical expressions.
- Analyze patterns and relationships.

Number and Operations in Base Ten (NBT)

- Understand the place value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations—Fractions (NF)

- Use equivalent fractions to add and subtract fractions.
- Use previous understandings of multiplication and division to multiply and divide fractions.

Measurement and Data (MD)

- Convert like measurement units within a given measurement system.
- Represent and interpret data.
- Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.

Geometry (G)

- Graph points on the coordinate plane to solve mathematical problems as well as problems in real-world context.
- Classify two-dimensional figures into categories based on their properties.

Standards for Mathematical Practices (MP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Fifth Grade: Critical Areas

In fifth grade, instructional time should focus on three critical areas:

1. **Develop competency in dividing and fluency in multiplying whole numbers through the application of understanding of place value and multiplication and division.**
2. **Develop understanding in performing operations with decimals to hundredths and estimating by rounding.**
3. **Develop understanding of multiplication of fractions and division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions).**

(1) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They are fluent with multi-digit multiplication of whole numbers. Students are able to explain patterns associated with multiplication through application of their knowledge of place value such as explaining the pattern in the number of zeros in a product. Students apply their understanding of division to begin working with decimals. They understand and can explain the placement of the decimal point when multiplying or dividing. Students apply their understanding of addition and multiplication of whole numbers (NBT) to foundational understanding of volume (MD).

(2) Students apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations and make reasonable estimates (through rounding) of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (e.g., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths.

(3) Students apply their understanding of fractions and fraction models to efficiently and accurately add and subtract fractions with unlike denominators. Students use their understanding of fractions; make connections to their understanding of multiplication and division, to explain the “why” of multiplying and dividing fractions. (Note: Division of fractions is limited to dividing unit fractions by whole numbers and whole numbers by unit fractions.)

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Operations and Algebraic Thinking (OA)		
5.OA.A **Write and interpret numerical expressions.**	**5.OA.A.1**	Use parentheses and brackets in numerical expressions, and evaluate expressions with these symbols (Order of Operations). **5.SE.1a1 Given a real world problem, write an equation using 1 set of parentheses.**
	5.OA.A.2	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them (e.g., express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product). **No CCCs developed for this standard.**
5.OA.B **Analyze patterns and relationships.**	**5.OA.B.3**	Generate two numerical patterns using two given rules (e.g., generate terms in the resulting sequences). Identify and explain the apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane (e.g., given the rule "add 3" and the starting number 0, and given the rule "add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence). **5.PRF.1b1 Given 2 patterns involving the same context (e.g., collecting marbles) determine the first 5 terms and compare the values.** **5.PRF.1b2 When given a line graph representing two arithmetic patterns, identify the relationship between the two** **5.PRF.2b1 Generate or select a comparison between two graphs from a similar situation.** **5.NO.3b1 Use up to two rules to verify provided responses or select correct answers (e.g., rules: +3, +2 and table lists pairs, 4:5, 7:7, and 10:9).**
	5.OA.B.4	Understand primes have only two factors and decompose numbers into prime factors. **No CCCs developed for this standard.**
Number and Operations in Base Ten (NBT)		
5.NBT.A **Understand the place value system.**	**5.NBT.A.1**	Apply concepts of place value, multiplication, and division to understand that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. **No CCCs developed for this standard.**
	5.NBT.A.2	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. **6.NO.1i1 Identify what an exponent represents (e.g., 8^3= 8 x 8 x 8).**
	5.NBT.A.3	Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form.

		b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **5. NO.1b1 Read, write, or select a decimal to the hundredths place.** **5.NO.1b2 Read, write or select a decimal to the thousandths place.** **5.NO.1b3 Compare two decimals to the thousandths place with a value of less than 1.**
	5.NBT.A.4	Use place value understanding to round decimals to any place. **5.NO.1b4 Round decimals to the next whole number.** **5.NO.1b5 Round decimals to the tenths place.** **5.NO.1b6 Round decimals to the hundredths place.**
5.NBT.B **Perform operations with multi-digit whole numbers and with decimals to hundredths.**	**5.NBT.B.5**	Fluently multiply multi-digit whole numbers using a standard algorithm. **No CCCs developed for this standard.**
	5.NBT.B.6	Apply and extend understanding of division to find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors. **5.NO.2a3 Find whole number quotients up to two dividends and two divisors.** **5.NO.2a4 Find whole number quotients up to four dividends and two divisors.** **5. NO.2a5 Solve word problems that require multiplication or division.**
	5.NBT.B.7	Add, subtract, multiply, and divide decimals to hundredths, connecting objects or drawings to strategies based on place value, properties of operations, and/or the relationship between operations. Relate the strategy to a written form. **5.NO.2c1 Solve 1 step problems using decimals.**

Number and Operations – Fractions (NF)		
5.NF.A **Use equivalent fractions to add and subtract fractions.**	**5.NF.A.1**	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators (e.g., 2/3 + 5/4 = 8/12 + 15/12 = 23/12). **5.NO.2b1 Add and subtract fractions with unlike denominators by replacing fractions with equivalent fractions (identical denominators).** **5.NO.2b2 Add or subtract fractions with unlike denominators.**
	5.NF.A.2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by using a variety of representations, equations, and visual models to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers (e.g. recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2). **5. NO.2c2 Solve word problems involving the addition, subtraction, multiplication or division of fractions.**
5.NF.B **Use previous understandings of multiplication and division to multiply and divide fractions.**	**5.NF.B.3**	Interpret a fraction as the number that results from dividing the whole number numerator by the whole number denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people, each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* **No CCCs developed for this standard.**
	5.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number and a fraction by a fraction. a. Interpret the product (*a/b*) x *q* as *a* parts of a partition of *q* into *b* equal parts. *For example, use a visual fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation.* b. Interpret the product of a fraction multiplied by a fraction (*a/b*) x (*c/d*). Use a visual fraction model and create a story context for this equation. *For example, use a visual fraction model to show (2/3) x (4/5) = 8/15, and create a story context for this equation.* In general, (*a/b*) x (*c/d*) = *ac/bd*. c. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. **5.NO.2b3 Multiply or divide fractions.**
	5.NF.B.5	Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

		b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number; explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $\frac{a}{b} = \frac{n\,x\,a}{n\,x\,b}$ to the effect of multiplying $\frac{a}{b}$ by 1. **5.PRF.1a1 Determine whether the product will increase or decrease based on the multiplier.** **5.PRF.1a2 Determine whether or not the quotient will increase or decrease based on the divisor.**
5.NF.B (cont.)	**5.NF.B.6**	Solve problems in real-world contexts involving multiplication of fractions, including mixed numbers, by using a variety of representations including equations and models. **No CCCs developed for this standard.**
	5.NF.B.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. Use the relationship between multiplication and division to justify conclusions. b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient.* Use the relationship between multiplication and division to justify conclusions (e.g., 4 ÷ (1/5) = 20 because 20 x (1/5) = 4). c. Solve problems in real-world context involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, using a variety of representations. **6.NO.2c4 Solve word problems involving the addition, subtraction, multiplication or division of fractions.**
Measurement and Data (MD)		
5.MD.A **Convert like measurement units within a given measurement system.**	**5.MD.A.1**	Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step, real-world problems. **5.ME.1b1 Convert measurements of time.** **5. ME.1b2 Convert standard measurements of length.** **5.ME.1b3 Convert standard measurements of mass.** **5. ME.2a1 Solve problems involving conversions of standard measurement units when finding area, volume, time lapse, or mass.**
5.MD.B **Represent and interpret data.**	**5.MD.B.2**	Make a line plot to display a data set of measurements in fractions of a unit (1/8, 1/2, 3/4). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.* **5.DPS.1c1 Collect and graph data: bar graph, line plots, picture graph (e.g., average height among 3 classrooms, # of boys and girls).**

5.MD.C **Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.**	**5.MD.C.3**	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. **No CCCs developed for this standard.**
	5.MD.C.4	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. **5.ME.2b1 Use filling and multiplication to determine volume.**

5.MD.C (cont.)	**5.MD.C.5**	Relate volume to the operations of multiplication and addition and solve mathematical problems and problems in real-world contexts involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes (e.g., to represent the associative property of multiplication). **No CCCs developed for this standard.** b. Understand and use the formulas $V = l \times w \times h$ and $V = B \times h$, where in this case B is the area of the base ($B = l \times w$), for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths to solve mathematical problems and problems in real-world contexts. **No CCCs developed for this standard.** c. Understand volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms, applying this technique to solve mathematical problems and problems in real-world contexts. **5.ME.2b2 Apply formula to solve one step problems involving volume.**
Geometry (G)		
5.G.A **Graph points on the coordinate plane to solve mathematical problems as well as problems in real-world context.**	**5.G.A.1**	Understand and describe a coordinate system as perpendicular number lines, called axes, that intersect at the origin (0 , 0). Identify a given point in the first quadrant of the coordinate plane using an ordered pair of numbers, called coordinates. Understand that the first number (*x*) indicates the distance traveled on the horizontal axis, and the second number (*y*) indicates the distance traveled on the vertical axis. **5.GM.1c1 Locate the x and y axis on a graph.** **5.GM.1c2 Locate points on a graph.** **5.GM.1c3 Use order pairs to graph given points.**
	5.G.A.2	Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. **6.GM.1c6 Find coordinate values of points in the context of a situation.**
5.G.B **Classify two-dimensional figures into categories based on their properties.**	**5.G.B.3**	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. **5.GM.1a1 Recognize properties of simple plane figures.**
	5.G.B.4	Classify two-dimensional figures in a hierarchy based on properties. **5.GM.1b1 Distinguish plane figures by their properties.**

Standards for Mathematical Practice	
5.MP.1	**Make sense of problems and persevere in solving them.** Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, "Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.
5.MP.2	**Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.
5.MP.3	**Construct viable arguments and critique the reasoning of others.** Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming or questioning the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.
5.MP.4	**Model with mathematics.** Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5.MP.5	**Use appropriate tools strategically.** Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.
5.MP.6	**Attend to precision.** Mathematically proficient students clearly communicate to others using appropriate mathematical terminology, and craft explanations that convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.
5.MP.7	**Look for and make use of structure.** Mathematically proficient students use structure and patterns to assist in making connections among mathematical ideas or concepts when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.
5.MP.8	**Look for and express regularity in repeated reasoning.** Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.